Vibration reduction Design of Permanent Magnet Motor Using Level Set Based Shape Optimization Method

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Abstract

This paper presents a new optimization method to design a rotor structure for a permanent magnet motor that can reduce vibration. The optimization problem is formulated with a multi-objective function to minimize the fluctuation of the radial magnetic force and the torque ripple. To obtain the optimal rotor shape consisting of a permanent magnet and ferromagnetic material, a multiple level set model is employed to express the structural boundaries and magnetic properties of each material. The updating process of the level set function based on the adjoint sensitivity and the time evolutional equation makes it possible to obtain a novel rotor configuration of the motor. To verify the usefulness of the proposed method, a rotor design example of the surface-mounted permanent magnet motor for electric power steering system is performed.

Keywords: Permanent magnet motor, vibration reduction, rotor design, magnet force, torque ripple, level set method

1 Introduction

Since mechanical vibration is harmful to the robustness of an integration system and generates audible noise, which is a critical factor in human sensitivity, vibration reduction has become an important issue in motor design [1]-[3]. Many studies have noted that the main sources of motor vibration are fluctuation of the magnetic force and mechanical deformation of the motor structure [4]-[5]. Hence, several design techniques such as modification of the motor shape [6]-[7] and pole-slot combination [8] have been proposed to minimize them. Particularly in the surface-mounted permanent magnet (SPM) motor where the permanent magnet (PM) dominates the magnetic flux path, the shape change of the rotor is the key to vibration reduction [9]. Unfortunately, since the magnetic forces that cause the motor's vibration are essential to satisfy the output power, it is very difficult to design a low-vibration rotor shape by using previous approaches because they depend on experimental data and engineering intuition.

In this paper, an optimization method that incorporates localized geometrical change [10] is introduced to get an innovative rotor design that guarantees a decrease in the motor vibration. The level set functions are employed to express the clear boundaries of the rotor and to calculate the magnetic properties of the PM and the ferromagnetic material (FM) [11]. The optimization problem is formulated to minimize the fluctuation of the magnetic forces, such as the local force on the stator and the driving torque, which can cause mechanical deformation of the stator. The volume fraction constraints for each level set function are added to the problem formulation for limiting the material usage. The optimization is performed by solving the time evolutional equation that can lead to initial-independent results. The design example of a SPM motor is provided to investigate the effectiveness

of the proposed method and achieve an optimum rotor design that promises reduction of the vibration without deterioration of the output torque.

2 Problem Formulation

2.1 Material Modelling and Analysis

To perform a magnetic analysis, two level set functions (ϕ_1, ϕ_2) are employed as design variables for representing the structural boundaries and material properties of PM, FM and air in the rotor. Each level set function has a sign for distinguishing the different material domains (Ω_i) and the zero points for representing the structural boundaries ($\partial\Omega_i$) as follows:

$$\begin{cases} \phi_i(\mathbf{x}) > 0 & \text{for } x \in \Omega_i^+ \\ \phi_i(\mathbf{x}) = 0 & \text{for } x \in \partial\Omega_i \qquad i = 1, 2 \\ \phi_i(\mathbf{x}) < 0 & \text{for } x \in \Omega_i^- \end{cases} \quad (1)$$

Fig. 1 shows how to separate each material domain and express its boundaries. The characteristic function of χ_i has a value of 1 or 0 according to the sign of each level set function as follows:

$$\chi_i = \begin{cases} 0 & \text{for } \phi_i < 0 \\ 1 & \text{for } \phi_i \geq 0 \end{cases} \qquad i = 1, 2 \quad (2)$$

The magnetic properties (p) such as the relative magnetic reluctivity (ν_r) and the remanent magnetic flux of PM ($\mathbf{B}_r$) are defined by the characteristic function as follows:

$$p(\chi_1, \chi_2) = \chi_1 \left[p_{PM} \chi_2 + p_{FM} (1 - \chi_2) \right] + p_{air} (1 - \chi_1) \quad (3)$$

To calculate the magnetic force causes a deformation of the stator, the nonlinear magnetostatic analysis is performed by solving the following equation:

$$\begin{aligned} -\nu_0 \nu_r (\chi_1, \chi_2) \nabla^2 \mathbf{A} \\ = \mathbf{J} + \nu_0 \nu_r (\chi_1, \chi_2) \left[\nabla \times \mathbf{B}_r (\chi_1, \chi_2) \right] \end{aligned} \quad (4)$$
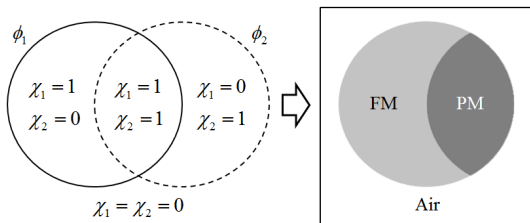

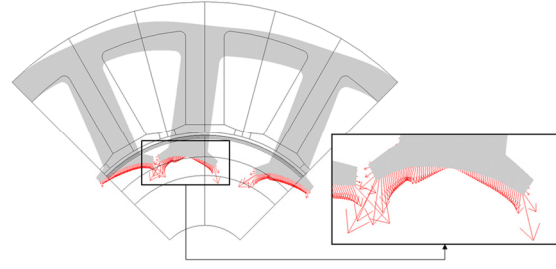Figure 1: Expression of multiple material domains

Figure 2: Deformation of the stator due to the local magnetic force

where $\mathbf{A}$ and $\mathbf{J}$ are the magnetic vector potential and the input current matrix, respectively. By using the discrete values of the characteristic function, each material region can express only a single material. Hence the magnetic analysis can be solved with clear structural boundaries and the magnetic properties of each material. The magnetic local forces calculated from the governing equation bring out the mechanical deformation of the stator, as shown in Fig. 2.

2.2 Optimization Problem Formulation

The objective function (F_j) for vibration reduction is composed of two values: the fluctuation of the magnetic force on the stator (f_j) and the driving torque (T_j) as follows:

$$\begin{aligned} F_j(\phi_1, \phi_2) &= w_f F_{f,j}(\phi_1, \phi_2) + w_T F_{T,j}(\phi_1, \phi_2) \\ &= w_f \left[\frac{f_j(\phi_1, \phi_2) - f_{avg,j}}{f_{avg,j}} \right]^2 + w_T \left[\frac{T_j(\phi_1, \phi_2) - T_{0,j}}{T_{0,j}} \right]^2 \end{aligned} \quad (5)$$

where f_{avg} and T_0 are the average magnetic force and the target average torque, respectively, and j a certain rotor position. It is noted that each objective function is combined with the weighting factors (w).

The optimization problem is formulated to minimize F with the volume constraints of PM (VF_{PM}) and FM (VF_{FM}) as follows:

$$\begin{cases} \underset{\phi_1, \phi_2}{\text{Minimize }} F(\phi_1, \phi_2) = \sum_{j=1}^{n} F_j(\phi_1, \phi_2) \\ \text{subject to } G_1(\phi_1, \phi_2) = \int_\Omega \chi_1 (1 - \chi_2) d\Omega \big/ \int_\Omega d\Omega \leq VF_{FM} \\ \qquad G_2(\phi_2) = \int_\Omega \chi_2 d\Omega \big/ \int_\Omega d\Omega \leq VF_{PM} \end{cases} \quad (6)$$

where n is the total number of rotating position and Ω is the total design domain. By minimizing the objective function, we can reduce the fluctuation of the magnetic force by satisfying the target output torque (T_0). The average magnetic force on the stator (f_{avg}) is updated for minimizing

both the fluctuation of the force and its amplitude in each design step.

2.3 Optimization Process

The design sensitivities of the two objective functions calculated by the adjoint variable method for reducing the computational time are combined as follows:

$$\frac{\delta F_j}{\delta \phi_i} = w_f \frac{\delta F_{f,j}/\delta \phi_i}{\max\left(\delta F_{f,j}/\delta \phi_i\right)} + w_T \frac{\delta F_{T,j}/\delta \phi_i}{\max\left(\delta F_{T,j}/\delta \phi_i\right)} \quad (7)$$

It is noted that the design sensitivity for each objective function is normalized to the same scale by dividing its maximum value. Then the level set functions are updated by solving the time evolutional equation as follows:

$$\frac{\partial \phi_i}{\partial t} = -\left(\sum_{j=1}^{n} \frac{\delta F_j}{\delta \phi_i} - \lambda_i\right) \quad (8)$$

where t is the fictitious time interval for the moving boundaries and λ is the Lagrange multiplier to satisfy the volume constraint. These processes are continued until the convergence condition is satisfied.

3 Design Example

The proposed method is applied to the rotor design of an 8-pole 12-slot SPM motor. The initial design illustrated in Fig. 3 is developed for the EPS system of which the important design target is vibration reduction.

The average torque of the initial design is 3.18 Nm and its torque ripple (T_{ripple}) which is calculated by the following equation is 3.79% at the maximum rated speed of 2500 rpm.

$$T_{\mathrm{ripple}} = \frac{T_{\max} - T_{\min}}{T_{\max} + T_{\min}} \times 100 \ [\%] \quad (9)$$

The design domain is the upper side of the rotor since the lower part is bordered with the rotating shaft as shown in Fig. 3. It shows that the analysis model is set to a 1/4 partial model and has the periodic boundary conditions for the magnetic analysis. The target torque is assigned to 3.2 Nm for maintaining the output power.
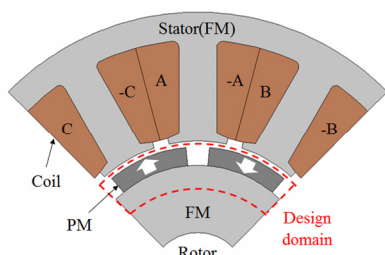

Figure 3: Initial design of the SPM motor

3.1 Optimal Results according to the PM Volume

The optimizations are performed with different volume fractions of PM ($\mathrm{VF_{PM}}$) and the volume constraint of FM ($\mathrm{VF_{FM}}$) is set to the same value (0.45) as the initial design. The weighting factors for each objective function (w_f, w_T) are fixed to 0.5 for applying the effect of each design sensitivity to the optimal shape equally.

Fig. 4 shows the optimal distribution of each material according to the PM volume, with the initial design noticed by the solid lines. It is noted that the outer surface of the PM becomes an uneven shape for adjusting the path of the magnetic flux and the airgap is increased for minimizing the magnetic force on the stator.
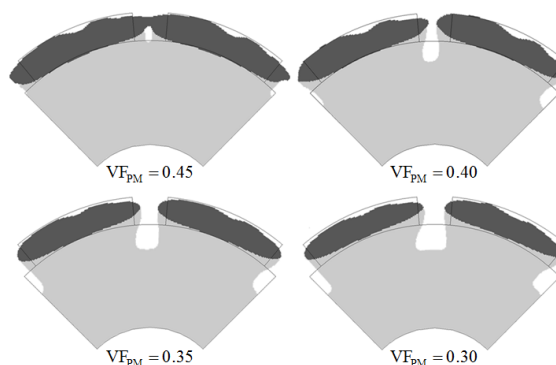

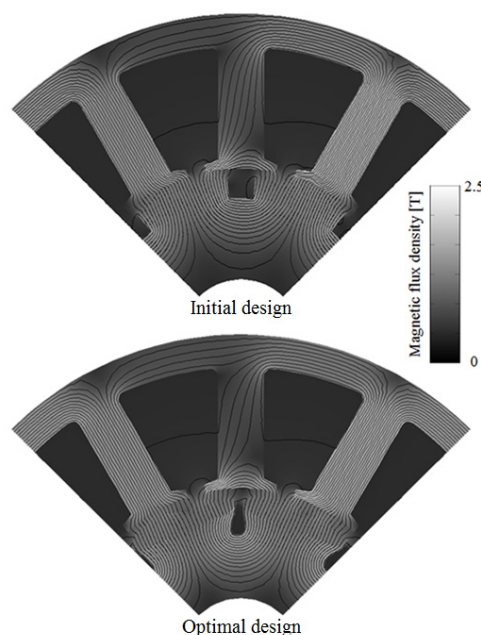Figure 4: Optimal rotor design of the SPM motor according to the PM volume

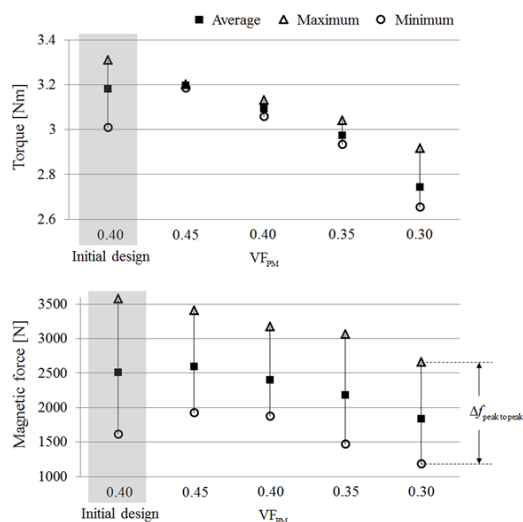Figure 5: Magnetic flux in the SPM motor

Figure 6: Magnetic performance of the SPM motor according to the PM volume

The notches in the middle part of the PM prevent the concentration of the magnetic flux for reducing the fluctuation of the magnetic force. Instead of a clearly separated shape for each pole as the initial design, the optimal PM shapes are the long and thin shape along the tangential direction of the rotor for providing the uniform amount of the magnetic flux regardless of the rotating angle as illustrated in Fig. 5. As the volume fraction has decreased, the PM becomes divided into two parts as illustrated in Fig. 4, and yet they still maintain their long, thin shape.

These optimal shapes are useful to minimize both the fluctuation and the average amount of the magnetic force (f_{avg}), as shown in Fig. 6. The fluctuation of the magnetic force ($\Delta f_{peak\,to\,peak}$) decreases more than 20% in all optimization cases. The torque ripple (T_{ripple}) also decreases greatly except for one case with few usage of PM ($VF_{PM} = 0.30$).

Unfortunately, such optimal shapes of PM also bring a reduction of the output power because of the wide airgap. Even if we increase the volume fraction of the PM compared to the initial design as shown in Fig. 4, the optimal design barely maintain a similar level of the output torque as the initial design.

3.2   Effect of the Weighting Factors

Hence, additional optimizations are performed for satisfying the target output torque. To increase the effect of $F_{T,j}$, the different value of w_T more than 0.5 is applied to the optimization.

VF_{PM} and VF_{FM} are set to 0.40 and 0.45 the same as the initial design.

Fig. 7 shows the optimal rotor shape at different value for w_T. When w_T is 2.0, the boundaries of the PM become a rounded shape and the central thickness increases to provide the magnetic flux to the stator sufficiently, as illustrated in Fig. 7. The FM around the edge of the PM is eliminated to reduce the leakage flux. It is confirmed that these shapes of the PM and FM help to maximize the average torque (T_{avg}) as well as to minimize the torque ripple. As w_T is decreased, the thickness of the PM is reduced and the notches appear to minimize $F_{f,j}$.

Fig. 8 shows the torque profile and fluctuation of the magnetic force on single teeth according to the weight function value. Although both the reduction of the force fluctuation and the output torque occur at the same time of necessity, we can provide several compromised designs. It is confirmed that the optimal rotor designs provide a large decrease in both the torque ripple and fluctuation of the magnetic force with only a slight loss of the output power.
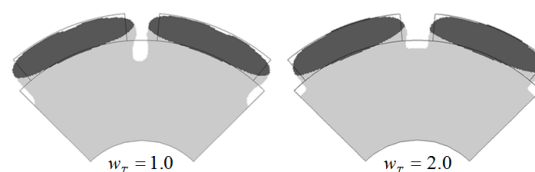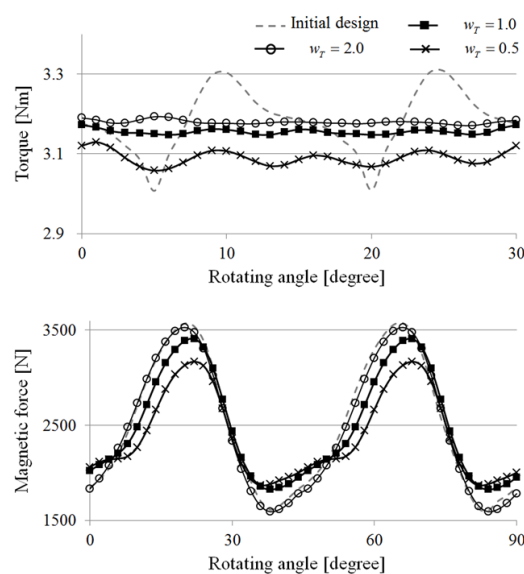

Figure 7: Effect of the weighting factor

Figure 8: Comparison of magnetic performance

4 Conclusion

This paper proposes a new design optimization method to reduce the mechanical vibration of the SPM motor. The design problem is formulated to minimize fluctuation of the magnetic force while maintaining the output torque. The design example shows a way to design a low-vibration rotor shape. It is expected that this proposed method will provide suitable design candidates at the conceptual design stage.

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