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Approximate n-Jordan $*$-derivations on C^*-algebras and JC^*-algebras

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Abstract
In this paper, we investigate the superstability and the Hyers-Ulam stability of n-Jordan $*$-derivations on C^*-algebras and JC^*-algebras.
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Keywords: n-Jordan $*$-derivation; C^*-algebra; JC^*-algebra; Hyers-Ulam stability

1 Introduction and preliminaries

The stability of functional equations was first introduced by Ulam [1] in 1940. More precisely, he proposed the following problem. Given a group H_1, a metric group (H_2, d) and $\epsilon > 0$, does there exist a $\delta > 0$ such that if a mapping $f : H_1 \to H_2$ satisfies the inequality $d(f(xy), f(x)f(y)) < \delta$ for all $x, y \in H_1$, then there exists a homomorphism $T : H_1 \to H_2$ such that $d(f(x), T(x)) < \epsilon$ for all $x \in H_1$? As mentioned above, when this problem has a solution, we say that the homomorphisms from H_1 to H_2 are stable. In 1941, Hyers [2] gave a partial solution of the Ulam problem for the case of approximate additive mappings under the assumption that H_1 and H_2 are Banach spaces. In 1950, Aoki [3] generalized the Hyers theorem for approximately additive mappings. In 1978, Rassias [4] generalized the theorem of Hyers by considering the stability problem with unbounded Cauchy differences. During the last decades, several stability problems of functional equations have been investigated by many mathematicians (see [5–20]).

Let n be an integer greater than one and A be a ring, and let B be an A-module. An additive mapping $D : A \to B$ is called an n-Jordan derivation (resp. n-ring derivation) if

$$D(a^n) = D(a)a^{n-1} + aD(a)a^{n-2} + \cdots + a^{n-2}D(a)a + a^{n-1}D(a)$$

for all $a \in A$

$$\text{(resp.}\quad D\left(\prod_{i=1}^{n} a_i\right) = D(a_1)a_2 \cdots a_n + a_1 D(a_2)a_3 \cdots a_n + \cdots + a_1 a_2 \cdots a_{n-1} D(a_n)$$

for all $a_1, a_2, \ldots, a_n \in A$). The concept of n-Jordan derivations was studied by Eshaghi Gordji [21] (see also [22–25]).

Definition 1.1 Let A be a C^*-algebra. A $\mathbb{C}$-linear mapping $D : A \to A$ is called an n-Jordan $*$-derivation if

$$D(a^n) = D(a)a^{n-1} + aD(a)a^{n-2} + \cdots + a^{n-2}D(a)a + a^{n-1}D(a),$$
$$D(a^*) = D(a)^*$$

for all $a \in A$.

A C^*-algebra A, endowed with the Jordan product $a \circ b = \frac{ab+ba}{2}$ on A, is called a JC^*-algebra (see [26]).

Definition 1.2 Let A be a JC^*-algebra. A $\mathbb{C}$-linear mapping $D : A \to A$ is called an n-Jordan $*$-derivation if

$$D(a^n) = D(a) \circ a^{n-1} + a \circ D(a) \circ a^{n-2} + \cdots + a^{n-2} \circ D(a) \circ a + a^{n-1} \circ D(a),$$
$$D(a^*) = D(a)^*$$

for all $a \in A$.

This paper is organized as follows. In Section 2, we investigate the superstability of n-Jordan $*$-derivations on C^*-algebras associated with the following functional inequality:

$$\|f(x) + f(y) + f(z)\| \le \left\| 2f\left(\frac{x+y+z}{2}\right)\right\|. \tag{1.1}$$

We, moreover, prove the Hyers-Ulam stability of n-Jordan $*$-derivations on C^*-algebras associated with the following functional equation:

$$2f\left(\frac{x+y}{2}\right) = f(x) + f(y). \tag{1.2}$$

In Section 3, we investigate the superstability of n-Jordan $*$-derivations on JC^*-algebras associated with the functional inequality (1.1) and prove the Hyers-Ulam stability of n-Jordan $*$-derivations on JC^*-algebras associated with the functional equation (1.2).

In this paper, assume that n is an integer greater than one.

2 n-Jordan $*$-derivations on C^*-algebras

Throughout this section, assume that A is a C^*-algebra, and that n_0 is a fixed positive integer.

Lemma 2.1 *Let $f : A \to A$ be a mapping such that*

$$\|\mu f(x) + \mu f(y) + f(\mu z)\| \le \left\| 2f\left(\mu\frac{x+y+z}{2}\right)\right\| \tag{2.1}$$

for all $\mu \in \mathbb{T}^1_{n_0} := \{e^{i\theta} \in \mathbb{C} : 0 \le \theta \le \frac{2\pi}{n_0}\}$ and all $x, y, z \in A$. Then the mapping $f : A \to A$ is $\mathbb{C}$-linear.

Proof Letting $\mu = 1$ and $x = y = z = 0$ in (2.1), we get

$$\|3f(0)\| \le 2\|f(0)\|.$$

So, $f(0) = 0$.

Letting $\mu = 1$, $y = -x$ and $z = 0$ in (2.1), we get

$$\|f(x) + f(-x)\| \le \|2f(0)\| = 0$$

and so $f(-x) = -f(x)$ for all $x \in A$.

Letting $\mu = 1$ and $z = -x - y$ in (2.1), we get

$$\|f(x) + f(y) + f(-x - y)\| \le \|2f(0)\| = 0$$

and so

$$f(x + y) = -f(-x - y) = f(x) + f(y)$$

for all $x, y \in A$. Hence, the mapping $f : A \to A$ is additive.

Letting $y = 0$ and $z = -x$ in (2.1), we get

$$\|\mu f(x) + f(-\mu x)\| \le \|2f(0)\| = 0$$

for all $x \in A$ and all $\mu \in \mathbb{T}^1_{n_0}$. So, $f(\mu x) = \mu f(x)$ for all $x \in A$ and all $\mu \in \mathbb{T}^1_{n_0}$. By [27, Lemma 2.1], the mapping $f : A \to A$ is $\mathbb{C}$-linear. □

We prove the superstability of n-Jordan $*$-derivations on C^*-algebras.

Theorem 2.2 *Let $f : A \to A$ be a mapping satisfying* (2.1) *and*

$$\|f(z^n) - f(z)z^{n-1} - zf(z)z^{n-2} - \cdots - z^{n-2}f(z)z - z^{n-1}f(z) + f(w^*) - f(w)^*\| \le \varphi(z, w) \tag{2.2}$$

for all $z, w \in A$, where $\varphi : A \times A \to [0, \infty)$ satisfies

$$\sum_{i=0}^{\infty} \frac{1}{2^i}\varphi(2^i z, 2^i w) < \infty \quad or \quad \sum_{i=0}^{\infty} 2^i \varphi\left(\frac{z}{2^i}, \frac{w}{2^i}\right) < \infty \tag{2.3}$$

for all $z, w \in A$. Then the mapping $f : A \to A$ is an n-Jordan $*$-derivation.

Proof By Lemma 2.1, the mapping $f : A \to A$ is $\mathbb{C}$-linear.

Assume that $\varphi : A \times A \to [0, \infty)$ satisfies $\sum_{i=0}^{\infty} \frac{1}{2^i}\varphi(2^i z, 2^i w) < \infty$ for all $z, w \in A$.

It follows from (2.2) that

$$\begin{aligned}&\left\|f(z^n)-f(z)z^{n-1}-zf(z)z^{n-2}-\cdots-z^{n-2}f(z)z-z^{n-1}f(z)\right\|\\&\quad\leq\frac{1}{2^i}\left\|f(2^iz^n)-f(2^iz)z^{n-1}-zf(2^iz)z^{n-2}-\cdots-z^{n-2}f(2^iz)z-z^{n-1}f(2^iz)\right\|\\&\quad\leq\frac{1}{2^i}\varphi(2^iz,0),\end{aligned}$$

which tends to zero as $i\to\infty$ for all $z\in A$. So,

$$f(z^n)=f(z)z^{n-1}+zf(z)z^{n-2}+\cdots+z^{n-2}f(z)z+z^{n-1}f(z)$$

for all $z\in A$.

Similarly, one can show that

$$f(w^*)=f(w)^*$$

for all $w\in A$.

Therefore, the mapping $f:A\to A$ is an n-Jordan $*$-derivation.

Assume that $\varphi:A\times A\to[0,\infty)$ satisfies $\sum_{i=0}^{\infty}2^i\varphi(\frac{z}{2^i},\frac{w}{2^i})<\infty$ for all $z,w\in A$. By the same reasoning as in the previous case, one can prove that the mapping $f:A\to A$ is an n-Jordan $*$-derivation. □

Corollary 2.3 *Let $p\neq 1$ and θ be nonnegative real numbers. Let $f:A\to A$ be a mapping satisfying* (2.1) *and*

$$\begin{aligned}&\left\|f(z^n)-f(z)z^{n-1}-zf(z)z^{n-2}-\cdots-z^{n-2}f(z)z-z^{n-1}f(z)+f(w^*)-f(w)^*\right\|\\&\quad\leq\theta\left(\|z\|^p+\|w\|^p\right)\end{aligned}$$

for all $z,w\in A$. Then the mapping $f:A\to A$ is an n-Jordan $*$-derivation.

Now we prove the Hyers-Ulam stability of n-Jordan derivations on C^*-algebras.

Theorem 2.4 *Let $f:A\to A$ be a mapping with $f(0)=0$ for which there exists a function $\varphi:A^4\to[0,\infty)$ such that*

$$\Phi(x,y,z,w):=\sum_{i=1}^{\infty}\frac{1}{2^i}\varphi(2^ix,2^iy,2^iz,2^iw)<\infty,\tag{2.4}$$

$$\begin{aligned}&\left\|2f\left(\frac{\mu x+\mu y}{2}\right)-\mu f(x)-\mu f(y)+f(z^n)-f(z)z^{n-1}-zf(z)z^{n-2}-\cdots\right.\\&\quad\left.-z^{n-2}f(z)z-z^{n-1}f(z)+f(w^*)-f(w)^*\right\|\leq\varphi(x,y,z,w)\end{aligned}\tag{2.5}$$

for all $x,y,z,w\in A$ and all $\mu\in\mathbb{T}^1_{n_0}$. Then there exists a unique n-Jordan $*$-derivation $D:A\to A$ such that

$$\left\|f(x)-D(x)\right\|\leq\Phi(x,0,0,0)\tag{2.6}$$

for all $x\in A$.

Proof Letting $\mu = 1$, $y = z = w = 0$ and replacing x by $2x$ in (2.5), we get

$$\left\| f(x) - \frac{1}{2} f(2x) \right\| \le \frac{1}{2}\varphi(2x,0,0,0) \tag{2.7}$$

for all $x \in A$.

Using the induction method, we have

$$\left\| f(x) - \frac{1}{2^l} f\left(2^l x\right) \right\| \le \sum_{i=1}^{l} \frac{1}{2^i}\varphi\left(2^i x,0,0,0\right) \tag{2.8}$$

for all $x \in A$. Replacing x by $2^m x$ in (2.8) and multiplying by $\frac{1}{2^m}$, we have

$$\left\| \frac{1}{2^m} f\left(2^m x\right) - \frac{1}{2^{l+m}} f\left(2^{l+m} x\right) \right\| \le \sum_{i=m+1}^{l+m} \frac{1}{2^i}\varphi\left(2^i x,0,0,0\right)$$

for all $x \in A$. Hence, $\{\frac{1}{2^l} f(2^l x)\}$ is a Cauchy sequence. Since A is complete,

$$D(x) = \lim_{l\to\infty} \frac{1}{2^l} f\left(2^l x\right)$$

exists for all $x \in A$.

Taking the limit as $l \to \infty$ in (2.8), we obtain the inequality (2.6).

It follows from (2.4) and (2.5) that

$$\begin{aligned}\left\| 2D\left(\frac{x+y}{2}\right) - D(x) - D(y) \right\| &= \lim_{l\to\infty} \frac{1}{2^l} \left\| 2f\left(2^{l-1}(x+y)\right) - f\left(2^l x\right) - f\left(2^l y\right) \right\| \\ &\le \lim_{l\to\infty} \frac{1}{2^l}\varphi\left(2^l x, 2^l y, 0, 0\right)\end{aligned}$$

for all $x, y \in A$. So,

$$2D\left(\frac{x+y}{2}\right) = D(x) + D(y)$$

for all $x, y \in A$. Since $f(0) = 0$, $D(0) = 0$. Thus, D is additive.

To prove the uniqueness, let L be another additive mapping satisfying (2.6). Then we have, for any positive integer k,

$$\begin{aligned}\left\| D(x) - L(x) \right\| &\le \frac{1}{2^k}\left(\left\| D\left(2^k x\right) - f\left(2^k x\right) \right\| + \left\| f\left(2^k x\right) - L\left(2^k x\right) \right\| \right) \\ &\le \frac{2}{2^k}\Phi\left(2^k x,0,0,0\right),\end{aligned}$$

which tends to zero as $k \to \infty$. So, we conclude that $D(x) = L(x)$ for all $x \in A$.

On the other hand, we have

$$D(\mu x) - \mu D(x) = \lim_{l\to\infty} \frac{1}{2^l} \left\| f\left(2^l \mu x\right) - \mu f\left(2^l x\right) \right\| \le \lim_{l\to\infty} \frac{1}{2^{l+1}}\varphi\left(2^l x, 2^l x, 0, 0\right) = 0$$

for all $\mu \in \mathbb{T}^1_{n_0}$ and all $x \in A$. By [27, Lemma 2.1], the mapping $D : A \to A$ is $\mathbb{C}$-linear.

It follows from (2.4) and (2.5) that

$$\begin{aligned}
&\left\|D(z^n) - D(z)z^{n-1} - zD(z)z^{n-2} - \cdots - z^{n-2}D(z)z - z^{n-1}D(z)\right\| \\
&\quad = \lim_{m\to\infty}\left\|\frac{1}{2^{mn}}\Big[f\big((2^m z)^n\big) - f(2^m z)(2^m z)^{n-1} - (2^m z)f(2^m z)(2^m z)^{n-2} - \cdots \\
&\qquad - (2^m z)^{n-2}f(2^m z)(2^m z) - (2^m z)^{n-1}f(2^m z)\Big]\right\| \\
&\quad \le \lim_{m\to\infty}\frac{1}{2^{mn}}\varphi(0,0,2^m z,0) \le \lim_{m\to\infty}\frac{1}{2^m}\varphi(0,0,2^m z,0) = 0
\end{aligned}$$

and

$$\left\|D(w^*) - D(w)^*\right\| = \lim_{m\to\infty}\left\|\frac{1}{2^m}f(2^m w^*) - \frac{1}{2^m}\big(f(2^m w)\big)^*\right\| \le \lim_{m\to\infty}\frac{1}{2^m}\varphi(0,0,0,2^n w) = 0$$

for all $z, w \in A$. Hence, $D : A \to A$ is a unique n-Jordan $*$-derivation. □

Corollary 2.5 *Let $f : A \to A$ be a mapping with $f(0) = 0$ for which there exist positive constants θ and $p < 1$ such that*

$$\begin{aligned}
&\left\|2f\left(\frac{\mu x + \mu y}{2}\right) - \mu f(x) - \mu f(y) + f(z^n) - f(z)z^{n-1} - zf(z)z^{n-2} - \cdots \right. \\
&\qquad \left. - z^{n-2}f(z)z - z^{n-1}f(z) + f(w^*) - f(w)^*\right\| \le \theta\left(\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p\right) \qquad (2.9)
\end{aligned}$$

for all $x, y, z, w \in A$ and all $\mu \in \mathbb{T}^1_{n_0}$. Then there exists a unique n-Jordan $*$-derivation D : $A \to A$ such that

$$\left\|f(x) - D(x)\right\| \le \frac{2^p\theta}{2 - 2^p}\|x\|^p$$

for all $x \in A$.

Proof Letting $\varphi(x, y, z, w) = \theta(\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p)$ in Theorem 2.4, we have

$$\left\|f(x) - D(x)\right\| \le \frac{2^p\theta}{2 - 2^p}\|x\|^p$$

for all $x \in A$, as desired. □

Theorem 2.6 *Let $f : A \to A$ be a mapping for which there exists a function $\varphi : A^4 \to [0,\infty)$ satisfying* (2.5) *and*

$$\sum_{i=0}^{\infty} 2^{ni}\varphi\left(\frac{x}{2^i}, \frac{y}{2^i}, \frac{z}{2^i}, \frac{w}{2^i}\right) < \infty \qquad (2.10)$$

for all $x, y, z, w \in A$. Then there exists a unique n-Jordan $*$-derivation $D : A \to A$ such that

$$\left\|f(x) - D(x)\right\| \le \sum_{i=0}^{\infty} 2^i\varphi\left(\frac{x}{2^i}, 0, 0, 0\right) \qquad (2.11)$$

for all $x \in A$.

Proof It follows from (2.7) that

$$\left\| f(x) - 2f\left(\frac{x}{2}\right) \right\| \le \varphi(x, 0, 0, 0)$$

for all $x \in A$.

The rest of the proof is similar to the proof of Theorem 2.4. □

Corollary 2.7 *Let $f : A \to A$ be a mapping with $f(0) = 0$ for which there exist positive constants θ and $p > n$ satisfying* (2.9). *Then there exists a unique n-Jordan $*$-derivation $D : A \to A$ such that*

$$\| f(x) - D(x) \| \le \frac{2^p \theta}{2^p - 2} \|x\|^p$$

for all $x \in A$.

Proof Letting $\varphi(x, y, z, w) = \theta(\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p)$ in Theorem 2.6, we have

$$\| f(x) - D(x) \| \le \frac{2^p \theta}{2^p - 2} \|x\|^p$$

for all $x \in A$, as desired. □

3 *n*-Jordan $*$-derivations on JC^*-algebras

Throughout this section, assume that A is a JC^*-algebra.

We prove the superstability of n-Jordan $*$-derivations on JC^*-algebras.

Theorem 3.1 *Let $f : A \to A$ be a mapping satisfying* (2.1) *and*

$$\begin{aligned} &\| f(z^n) - f(z) \circ z^{n-1} - z \circ f(z) \circ z^{n-2} - \cdots - z^{n-2} \circ f(z) \circ z - z^{n-1} \circ f(z) \\ &\quad + f(w^*) - f(w)^* \| \le \varphi(z, w) \end{aligned}$$

for all $z, w \in A$, where $\varphi : A \times A \to [0, \infty)$ satisfies (2.3). *Then the mapping $f : A \to A$ is an n-Jordan $*$-derivation.*

Proof The proof is similar to the proof of Theorem 2.2. □

Corollary 3.2 *Let $p \ne 1$ and θ be nonnegative real numbers. Let $f : A \to A$ be a mapping satisfying* (2.1) *and*

$$\begin{aligned} &\| f(z^n) - f(z) \circ z^{n-1} - z \circ f(z) \circ z^{n-2} - \cdots - z^{n-2} \circ f(z) \circ z - z^{n-1} \circ f(z) \\ &\quad + f(w^*) - f(w)^* \| \le \theta(\|z\|^p + \|w\|^p) \end{aligned}$$

for all $z, w \in A$. Then the mapping $f : A \to A$ is an n-Jordan $*$-derivation.

We prove the Hyers-Ulam stability of n-Jordan derivations on JC^*-algebras.

Theorem 3.3 *Let $f : A \to A$ be a mapping with $f(0) = 0$ for which there exists a function $\varphi : A^4 \to [0, \infty)$ satisfying* (2.4) *and*

$$\left\| 2f\left(\frac{\mu x + \mu y}{2}\right) - \mu f(x) - \mu f(y) + f(z^n) - f(z) \circ z^{n-1} - z \circ f(z) \circ z^{n-2} - \cdots - z^{n-2} \circ f(z) \circ z - z^{n-1} \circ f(z) + f(w^*) - f(w)^* \right\| \leq \varphi(x, y, z, w) \tag{3.1}$$

for all $x, y, z, w \in A$ and all $\mu \in \mathbb{T}^1_{n_0}$. Then there exists a unique n-Jordan $*$-derivation $D : A \to A$ satisfying (2.6).

Proof By the same reasoning as in the proof of Theorem 2.4, there exists a unique $\mathbb{C}$-linear $D : A \to A$ such that

$$\|f(x) - D(x)\| \leq \Phi(x, 0, 0, 0)$$

for all $x \in A$. The mapping $D : A \to A$ is given by

$$D(x) = \lim_{l \to \infty} \frac{1}{2^l} f(2^l x)$$

for all $x \in A$.

The rest of the proof is similar to the proof of Theorem 2.4. □

Corollary 3.4 *Let $f : A \to A$ be a mapping with $f(0) = 0$ for which there exist positive constants θ and $p < 1$ such that*

$$\left\| 2f\left(\frac{\mu x + \mu y}{2}\right) - \mu f(x) - \mu f(y) + f(z^n) - f(z) \circ z^{n-1} - z \circ f(z) \circ z^{n-2} - \cdots - z^{n-2} \circ f(z) \circ z - z^{n-1} \circ f(z) + f(w^*) - f(w)^* \right\| \leq \theta\left(\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p\right) \tag{3.2}$$

for all $x, y, z, w \in A$ and all $\mu \in \mathbb{T}^1_{n_0}$. Then there exists a unique n-Jordan $*$-derivation $D : A \to A$ such that

$$\|f(x) - D(x)\| \leq \frac{2^p \theta}{2 - 2^p} \|x\|^p$$

for all $x \in A$.

Proof Letting $\varphi(x, y, z, w) = \theta(\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p)$ in Theorem 3.3, we have

$$\|f(x) - D(x)\| \leq \frac{2^p \theta}{2 - 2^p} \|x\|^p$$

for all $x \in A$, as desired. □

Theorem 3.5 *Let $f : A \to A$ be a mapping for which there exists a function $\varphi : A^4 \to [0, \infty)$ satisfying* (3.1) *and* (2.10). *Then there exists a unique n-Jordan $*$-derivation $D : A \to A$ satisfying* (2.11).

Proof The proof is similar to the proofs of Theorems 2.4 and 3.3. □

Corollary 3.6 *Let $f : A \to A$ be a mapping with $f(0) = 0$ for which there exist positive constants θ and $p > n$ satisfying* (3.2). *Then there exists a unique n-Jordan $*$-derivation $D : A \to A$ such that*

$$\|f(x) - D(x)\| \le \frac{2^p\theta}{2^p - 2}\|x\|^p$$

for all $x \in A$.

Proof Letting $\varphi(x, y, z, w) = \theta(\|x\|^p + \|y\|^p + \|z\|^p + \|w\|^p)$ in Theorem 3.5, we have

$$\|f(x) - D(x)\| \le \frac{2^p\theta}{2^p - 2}\|x\|^p$$

for all $x \in A$, as desired. □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors conceived of the study, participated in its design and coordination, drafted the manuscript, participated in the sequence alignment, and read and approved the final manuscript.

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