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# An Entropic Understanding of Coulomb Force

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**Abstract.** Exploiting Verlinde's proposal on the entropic understanding of Newton's law, we show that Coulomb force could also be understood as an entropically emergent force (rather than as a fundamental force). We apply Kaluza-Klein idea to Verlinde's formalism to obtain Coulomb interaction in the lower dimensions. The kinematics concerning the Kaluza-Klein momenta separates the interaction due to the momentum flow from the gravitational interaction. The momentum-charge conversion relation results in the precise form of Coulomb interaction.

## 1. Introduction

Gravity has been the last fundamental force to be quantized. (See Ref. [1] for a review and references therein.) People have tried to understand all the known four fundamental forces on a single theoretical ground. As a result, we came to unify those fundamental forces in the framework of quantized gauge theory but the gravity remains untouched yet.

On the other hand, AdS/CFT duality [2] reveals a different role of gravity from other fundamental forces. In this duality, gravity just provides a *classical* background with a holographical boundary that is the playground of other *quantized* fundamental forces. This suggests that the gravity might not be quantized from the beginning and discriminates itself from other fundamental forces [3].

Recent idea of E. Verlinde [4] tried to answer the question, "Is the gravity a fundamental force or a concept emergent from other physics?". His answer is that it is a derived concept from thermodynamics, and identified it as an entropic force, a force that tends to increase the entropy. (For earlier works on the geometry emerging from the thermodynamics, see Refs. [5, 6].) In the case where thermodynamics is obscure, for example as in the interaction between two particles, the force is rather described as an adiabatic reaction force [7].

The question raised at this point is whether the entropic force single out the gravity or be applicable to other forces too. There is no strong reason that the thermodynamic tendency to increase the entropy favors the gravity only. More specifically, one can ask the possibility of understanding the Coulomb force as an entropic force too.

There are some earlier works concerning this issue. In Ref. [8], Coulomb force was obtained on a entropic reason based on the deformed equipartition theorem involving charges. On the other hand in Ref. [9], Freund mentioned the possibility of getting entropic Coulomb force by Kaluza-Klein compactification of higher dimensional entropic gravity. This latter paper

also discussed the extension of the same idea to the other fundamental forces through higher dimensional compactification.

In this contribution, we explain how Coulomb force can be understood as an entropic force. We apply the original entropic force idea to a 5-dimensional system to get the entropic gravity between two lineal mass distributions with their own internal momenta. Standard Kaluza-Klein compactification of the 5-dimensional entropic gravity leads to the entropic Coulomb force in 4-dimensions. This contribution is a short version extracted from a more detailed paper [10] that elaborated on Freund's idea [9] about entropic Coulomb force.

We also show that for equal sign of charges, the entropic Coulomb force acts in the opposite direction to that of the entropic gravity. This is obviously seen when we work in the test-particle rest frame where it is convenient to compute the proper length of 'the Rindler acceleration' the test-particle feels.

This paper is organized as follows. In the next section, we give a brief introduction on Verlinde's entropic gravity. We recapitulate some assumptions of the entropic gravity, which are relevant in our argument. In Sec. 3, we define the hyper-cylindrical holographic screen enclosing a lineal mass distribution in 5-dimensions. In Sec. 4, we consider the entropic interaction of two parallel lineal mass distributions with momenta. In Sec. 5, the entropic Coulomb force is derived via Kaluza-Klein reduction of the result of Sec. 4. Sec. 6 concludes the paper with some remarks on future works.

## 2. Verlinde's entropic force

In this section, we summarize Verlinde's recent idea [4] on the entropic force. In the original paper, several assumptions were made being inspired by familiar properties of known solutions of the conventional gravity. Here we rearrange and consider some of them relevant in our forthcoming argument. They are about the holographic thermodynamic screen, the entropic force, and the entropy change involved in thermalization of a particle, which are in order below.

We first assume a holographic thermodynamic screen due to an unknown source behind the screen. It is characterized by its temperature  $T$ . For a particle on the screen, the temperature is realized as the acceleration  $a = 2\pi c k_B T / \hbar$ . (This is reminiscent of the relation between the Rindler acceleration and the Unruh temperature [11].) Especially if the screen is compact, all the information enclosed by the screen is holographically projected onto finite number of degrees of freedom on the screen. The number is determined by the area in the basic unit;  $N = Ac^3/G\hbar$ . The energy behind the screen is equally distributed over this  $N$  degrees of freedom via  $E = Nk_B T/2$ .

We next assume the entropic force that tends to increase the entropy of the system behind the screen. The 'heat' generating the entropy increase is provided by the work done by the entropic force;  $F\Delta x = T\Delta S$ .

The last assumption is that the entropy change  $\Delta S$  is proportional to the distance  $\Delta x$  the screen shifts due to a test particle of mass  $m$  got thermalized into the screen. The distance is measured in the reduced Compton length  $\lambda = \hbar/mc$  associated with the mass  $m$ . Therefore,  $\Delta S = 2\pi k_B \Delta x / \lambda$ .

Applying the assumptions to a particle of mass  $m$  on a holographic thermodynamic screen of temperature  $T$ , one can obtain Newton's law;

$$\begin{aligned} F\Delta x &= T\Delta S = \left( \frac{\hbar a}{2\pi k_B c} \right) \left( \frac{2\pi k_B m c \Delta x}{\hbar} \right) \\ &= m a \Delta x. \end{aligned} \tag{1}$$

Especially for a spherical screen containing an energy  $E = Mc^2$ , one can specify the acceleration

appearing in the above relation as

$$\begin{aligned} Mc^2 &= \frac{N}{2} k_B T \\ &= \frac{1}{2} \left( \frac{Ac^3}{G\hbar} \right) \left( \frac{\hbar a}{2\pi c} \right) = \frac{r^2 c^2}{G} a. \end{aligned} \quad (2)$$

### 3. Holographic Screen for an Array of Moving Particles

We assume an internal compact direction regarding Kaluza-Klein reduction. A massive particle in lower dimensions corresponds to an array of particles along the compact direction. We first consider a static case where mass  $M_0$  is distributed evenly along the internal circle of radius  $R_0$  forming a massive lineal source with the line density  $\mu_0 = M_0/(2\pi R_0)$ .

The holographic screen enclosing the array takes a hyper-cylindrical shape, that is  $S^2 \times S^1$ . The 2-sphere,  $S^2$ , is embedded in the lower dimensions while the circle,  $S^1$ , denotes the internal compact direction.

Let us apply Verlinde's holographic idea on the entropic force to this system of a lineal source. The energy contained in the cylindrical screen will be  $E_0 = M_0 c^2$ . The equipartition ansatz states that

$$M_0 c^2 = \frac{1}{2} N k_B T_0 = \frac{1}{2} \left( \frac{Ac^3}{G^{(5)}\hbar} \right) \left( \frac{\hbar a_0}{2\pi c} \right) = \frac{r^2 c^2}{G^{(4)}} a_0, \quad (3)$$

where the expression for the area  $A = (4\pi r^2)(2\pi R_0)$  of the hyper-cylinder has been used. The 4-dimensional Newton constant  $G^{(4)}$  is related with the 5-dimensional one by the relation  $G^{(5)} = 2\pi R_0 G^{(4)}$ .

The entropic force of the source acting on a lineal test line of mass  $m_0$  posed along the internal circle direction at the holographic screen will be

$$F_0 = \frac{G^{(4)} M_0 m_0}{r^2}. \quad (4)$$

### 4. Two Parallel Lineal Mass Distributions with Internal Momenta

Let us apply the prescription discussed in the previous section to a more generic non-static case. We consider the force, due to a massive charged source-particle, acting on another massive charged test-particle in 4-dimensions.

According to the Kaluza-Klein proposal, those two massive charged particles correspond to lineal mass distributions carrying different momenta along the internal circle of a radius  $R$ . Let the source-distribution of rest mass  $M_0$  carry momentum  $\gamma M_0 v$ . The test-distribution of rest mass  $m_0$  carries momentum  $\gamma_1 m_0 v_1$ . Here,  $\gamma = 1/\sqrt{1 - v^2/c^2}$  and  $\gamma_1 = 1/\sqrt{1 - v_1^2/c^2}$  are the Lorentz factors concerning the rapidities  $\beta \equiv v/c$  and  $\beta_1 \equiv v_1/c$  respectively.

In order to obtain the entropic force between those two lineal moving arrays of mass, we need to work in the rest frame of the test array. It is the proper acceleration that the 'Unruh temperature'  $T$  corresponds to. Therefore, it is more convenient to work in the frame where the test distribution is static.

We 'unboost' the whole system so that the lineal test distribution of mass become static. The size of the internal circle will expand to

$$R_0 \equiv \gamma_1 R. \quad (5)$$

In the frame, the source distribution carries energy and momentum ( $E'/c$ ,  $p'$ ) as

$$\begin{pmatrix} E'/c \\ p' \end{pmatrix} = \begin{pmatrix} M_0 c \gamma_1 \gamma (1 - \beta_1 \beta) \\ M_0 c \gamma_1 \gamma (\beta - \beta_1) \end{pmatrix}. \quad (6)$$

For notational convenience, let us denote the energy as  $E' = \gamma' M_0 c^2$ , where the Lorentz factor can be recast in terms of the internal momenta of the particles;

$$\begin{aligned}\gamma' &= \gamma_1 \gamma - \sqrt{\gamma_1^2 - 1} \sqrt{\gamma^2 - 1} \\ &= \gamma_1 \gamma - \frac{p_1}{m_0 c} \frac{p}{M_0 c}.\end{aligned}\quad (7)$$

We are ready to apply ‘the equipartition ansatz’ to this system. The total energy  $\gamma' M_0 c^2$  contained in a cylindrical screen is encoded onto the surface of the screen as

$$N' = \frac{A' c^3}{G^{(5)} \hbar} = \frac{4\pi r^2 c^3}{G^{(4)} \hbar} \quad (8)$$

bits of degrees of freedom, each of which carrying the energy  $k_B T'/2$ . The temperature  $T'$  is again realized as the proper acceleration felt by an object on the screen surface;

$$T' = \frac{\hbar a'}{2\pi c k_B}. \quad (9)$$

Hence we have the relation

$$\gamma' M_0 c^2 = \frac{r^2 c^2}{G^{(4)}} a'. \quad (10)$$

The entropic force acting on the static test distribution of mass  $m_0$  touching the screen surface is

$$F' = \gamma' \frac{G^{(4)} M_0 m_0}{r^2}. \quad (11)$$

Since the direction of this entropic force is normal to the holographic screen, its magnitude remains the same even when we go to the original frame by reboosting.

## 5. Coulomb’s Law from the Entropic Force

In lower dimensions, the force (11) concerns the interaction between two massive charged particles. With the notion of the relation between Kaluza-Klein momenta and electric charges

$$e_n = \frac{n l_p q_p}{R} = k_n \frac{\hbar}{c} \sqrt{\frac{G^{(4)}}{k}} = \frac{p_n}{c} \sqrt{\frac{G^{(4)}}{k}}, \quad (12)$$

the expression for the entropic force becomes

$$\begin{aligned}F' &= \left( \gamma_1 \gamma - \frac{p_1}{m_0 c} \frac{p}{M_0 c} \right) \frac{G^{(4)} M_0 m_0}{r^2} \\ &= \frac{G^{(4)} M m}{r^2} - \frac{G^{(4)} p_1 p}{c^2 r^2} = \frac{G^{(4)} M m}{r^2} - \frac{k e_1 e}{r^2}.\end{aligned}\quad (13)$$

In Eq. (12),  $l_p = \sqrt{G^{(4)} \hbar / c^3}$  is the Planck length and  $q_p = \sqrt{\hbar c / k}$  is the Planck charge in 4-dimensions, and  $k = 1/4\pi\epsilon_0$  is the Coulomb constant.

## 6. Discussion and Conclusion

We showed that if the gravitational force is not fundamental rather is a concept emergent entropically, the Coulomb force can be also understood in the same way. We conjoined Kaluza-Klein idea and the entropic force formalism to get this result. The upshot is that the nontrivial kinematic operation (Lorentz factor  $\gamma'$  in (11)) necessary to achieve the momentum flows along the Kaluza-Klein direction is composed of two parts; the mass correction Lorentz factor and the momentum interaction part (See Eq. (7)). Kaluza-Klein reduction generates the Coulombic interaction term in the final expression of the entropic force.

For the equal sign of charges, the entropic Coulomb force acts in the opposite direction to that of the gravitational force. This is due to the combination of Lorentz transforms

$$\Lambda(\beta') = \Lambda^{-1}(\beta_1)\Lambda(\beta) \quad (14)$$

that we introduced to get back to the test-particle rest frame, a convenient frame for computing the proper ‘Rindler acceleration’ the test-particle feels. In order to give relative momenta to the source-particle, we first boosted the source-particle with respect to the test-particle using the transform  $\Lambda(\beta')$ . With another transform  $\Lambda(\beta_1)$  we gave both particles additional momentum. Hence  $\Lambda(\beta) = \Lambda(\beta_1)\Lambda(\beta')$  relates the source-particle rest frame with that of the observer. The different signs of the terms in Eq. (7) comes from the inverse factor,  $\Lambda^{-1}(\beta_1)$ , in the combination (14).

Entropic force idea cannot single out the gravitational force from others. The standard Kaluza-Klein compactification of the entropic gravity results in Coulomb force. Considering higher dimensional internal space, one might get ‘the entropic Yang-Mills interaction’ in the same way. This suggests that the entropic force idea might be a new paradigm applying for all forces rather than be specific only to the gravity.

Lastly we give an outlook on future work regarding the issue dealt with here. The entropic force becomes obscure unless the thermodynamics is physically realizable as in the near region of the black hole. Therefore, we need other substitute for the entropic force as we move off the event horizon. Verlinde discussed this substitute in the name of the adiabatic reaction force [7] in the name of Born-Oppenheimer adiabatic reaction force. In the scheme gauge field appears as Berry connection. The adiabatic reaction force corresponding to the entropic Coulomb force discussed in this paper should be clarified.

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