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Stability and superstability of generalized quadratic ternary derivations on non-Archimedean ternary Banach algebras: a fixed point approach

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Abstract

Using fixed point method, we prove the Hyers-Ulam stability and the superstability of generalized quadratic ternary derivations on non-Archimedean ternary Banach algebras. Indeed, we investigate the Hyers-Ulam stability and the superstability of the system of functional equations

$$\begin{cases} f([abc]) = \left[f(a)b^2c^2\right] + \left[a^2f(b)c^2\right] + \left[a^2b^2f(c)\right]; \\ g([abc]) = \left[g(a)b^2c^2\right] + [a^2f(b)c^2] + [a^2b^2f(c)]; \\ g(ux + vy) + g(ux - vy) = 2u^2g(x) + 2v^2g(y); \\ f(ux + vy) + f(ux - vy) = 2u^2f(x) + 2v^2f(y) \end{cases}$$

in non-Archimedean ternary Banach algebras.

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1. Introduction and preliminaries

The stability problem of functional equations had been first raised by Ulam [1]. This problem solved by Hyers [2] in the framework of Banach spaces. In 1978, Th.M. Rassias [3] provided a generalization of the Hyers' theorem by proving the existence of unique linear mappings near approximate additive mappings. Găvruta [4] obtained generalized result of the Th.M. Rassias' theorem which allows the Cauchy difference to be controlled by a general unbounded function.

Bourgin [5] proved the stability of ring homomorphisms between two unital Banach algebras and Badora [6] gave a generalization of the Bourgin result. The stability result concerning derivations between operator algebras was first obtained by Šemrl [7]. In [8], Badora proved the stability of functional equation

$$f(xy) = xf(y) + f(x)y,$$

where f is a mapping on normed algebra A with the unit. Park et al. proved the stability of homomorphisms and derivations in Banach algebras, Banach ternary algebras, C^*-algebras, Lie C^*-algebras and C^*-ternary algebras (see [9-14]).

Let $\mathcal{A}$ be a ternary algebra. A mapping $f : \mathcal{A} \to \mathcal{A}$ is called a *quadratic ternary derivation* if f is a quadratic mapping satisfies

$$f\left(\left[a, b, c\right]\right) = \left[f(a), b^2, c^2\right] + \left[a^2, f(b), c^2\right] + \left[a^2, b^2, f(c)\right]$$

for all $a, b, c \in \mathcal{A}$.

A mapping $g : \mathcal{A} \to \mathcal{A}$ is called a *generalized quadratic ternary derivation* if there exists a quadratic ternary derivation $f : \mathcal{A} \to \mathcal{A}$ such that

$$g\left(\left[a, b, c\right]\right) = \left[g(a), b^2, c^2\right] + \left[a^2, f(b), c^2\right] + \left[a^2, b^2, f(c)\right]$$

for all $a, b, c \in \mathcal{A}$.

Let $\mathbb{K}$ denote a field and function (valuation absolute) $|\cdot|$ from $\mathbb{K}$ into $[0, \infty)$. A *non-Archimedean valuation* is a function $|\cdot|$ that satisfies the strong triangle inequality, namely,

$$|x + y| \leq \max\left\{|x|, |y|\right\} \leq |x| + |y|$$

for all $x, y \in \mathbb{K}$. The associated field $\mathbb{K}$ is referred to as a *non-Archimedean field*. Clearly, $|1| = |-1| = 1$ and $|n| \leq 1$ for all $n \geq 1$. A trivial example of a non-Archimedean valuation is the function $|\cdot|$ taking everything except 0 into 1 and $|0| = 0$. We always assume in addition that $|\cdot|$ is non trivial, i.e., there exists $z \in \mathbb{K}$ such that $|z| \neq 0, 1$.

Let X be a linear space over a field $\mathbb{K}$ with a non-Archimedean nontrivial valuation $|\cdot|$. A function $\|\cdot\| : X \to [0, \infty)$ is said to be a *non-Archimedean norm* if it is a norm over $\mathbb{K}$ with the strong triangle inequality (ultrametric), namely,

$$\|x + y\| \leq \max\left\{\|x\|, \|y\|\right\}$$

for all $x, y \in X$. Then $(X, \|\cdot\|)$ is called a *non-Archimedean space*. In any such a space a sequence $\{x_n\}_{n\in\mathbb{N}}$ is a Cauchy sequence if and only if $\{x_{n+1}-x_n\}_{n\in\mathbb{N}}$ converges to zero. By a *complete non-Archimedean space* we mean one in which every Cauchy sequence is convergent.

A *non-Archimedean ternary Banach algebra* is a complete non-Archimedean space $\mathcal{A}$ equipped with a ternary product $(x,y,z) \to [x,y,z]$ of $\mathcal{A}^3$ into $\mathcal{A}$ which is $\mathcal{K}$-linear in each variables and associative in the sense that

$$[x, y, [z, w, v]] = [x, [w, z, y], v] = [[x, y, z], w, v]$$

and satisfies the following:

$$\|[x, y, z]\| \leq \|x\| \cdot \|y\| \cdot \|z\|$$

(see [15-19]).

Arriola and Beyer [20] initiated the stability of functional equations in non-Archimedean spaces. In fact, they established stability of Cauchy functional equations over p-adic fields. After their results some papers (see, for instance, [21-27]) on the stability of other equations in such spaces have been published. Although different methods are known for establishing the stability of functional equations, almost all proofs depend on Hyers' method in [2]. In 2003, Radu [28] employed the alternative fixed point theorem, due to Diaz and Margolis [29], to prove the stability of Cauchy additive functional equation. Subsequently, this method was applied to investigate the Hyers-Ulam stability for Jensen functional equation [30], as well as for the Cauchy functional equation [31], by considering a general control function $\phi(x, y)$, with suitable properties. Using such an elegant idea, several authors applied the method to investigate the stability of some functional equations (see [12,32-34]).

Recently, Eshaghi Gordji and Khodaei [35] proved the Hyers-Ulam stability of the following quadratic functional equation

$$f(ax+by)+f(ax-by)=2a^2f(x)+2b^2f(y) \tag{1.1}$$

for nonzero fixed integers a, b. Recently, Eshaghi Gordji and Alizadeh [36,37] proved the Hyers-Ulam stability of homomorphisms and derivations on non-Archimedean Banach algebras.

In this paper, by using fixed point method, we establish the stability of generalized quadratic ternary derivations related to the quadratic functional equation (1.1) over non-Archimedean ternary Banach algebras.

In 1897, Hensel [38] discovered the p-adic numbers as a number theoretical analogue of power series in complex analysis. During the last three decades p-adic numbers have gained the interest of physicists for their research, in particular, in the problems coming from quantum physics, p-adic strings and superstrings [39,40]. A key property of p-adic numbers is that they do not satisfy the Archimedean axiom: For any $x, y > 0$, there exists $n \in \mathbb{N}$ such that $x < ny$ (see [41,42]).

2. Main results

Using the strong triangle inequality in the proof of the main result of [29], we get the following result:

Theorem 2.1. (Non-Archimedean Alternative Contraction Principle) *Let* (Ω,d) *be a non-Archimedean generalized complete metric space and* $T:\Omega \to \Omega$ *a strictly contractive mapping* (*that is,* $d(T(x),T(y)) \le Ld(x, y)$ *for all* $x, y \in T$ *and a Lipschitz constant* $L < 1$). *Let* $x \in \Omega$. *If either*

(a) $d(T^n(x),T^{n+1}(x)) = \infty$ *for all* $n \ge 0$, *or*
(b) *there exists some* $n_0 \ge 0$ *such that* $d(T^n(x),T^{n+1}(x)) < \infty$ *for all* $n \ge n_0$, *then the sequence* $\{T^n(x)\}$ *is convergent to a unique fixed point* x^* *of* T *in the set*

$$\Lambda = \left\{y \in \Omega : d\left(T^{n_0}(x), y\right) < \infty\right\}$$

and $d(y, x^*) \le d(y,T(y))$ *for all* y *in this set.*

From now on, we assume that $\left(\mathcal{A}, [\cdot,\cdot,\cdot]\right)$ is a non-Archimedean ternary Banach algebra and $\ell \in \{-1,1\}$ is fixed. Also, we suppose that $|4| < 1$ and that $4 \neq 0$ in $\mathbb{K}$ (i.e., the characteristic of $\mathbb{K}$ is not 4). We denote $[a, b, c]$ by $[abc]$ in ternary Banach algebra $\mathcal{A}$.

Theorem 2.2. *Let* $g, f : \mathcal{A} \to \mathcal{A}$ *be two mappings with* $g(0) = f(0) = 0$ *for which there exists a function* $\varphi : \mathcal{A}^8 \to [0,\infty)$ *such that*

$$\begin{aligned}
&\left\|h(ax+by)+h(ax-by)-2a^2h(x)-2b^2h(y)\right\| \\
&+\left\|f([uvw])-[f(u)v^2w^2]-[u^2f(v)w^2]-[u^2v^2f(w)]\right\| \\
&+\left\|g([rst])-\left[g(r)s^2t^2\right]-\left[r^2f(x)t^2\right]-\left[r^2s^2f(t)\right]\right\| \\
&\leq \varphi(x,y,u,v,w,r,s,t)
\end{aligned} \tag{2.1}$$

for all $h \in \{f,g\}$, $x,y,u,v,w,r,s,t \in \mathcal{A}$ *and nonzero fixed integers a, b. Suppose that there exists* $L < 1$ *such that*

$$\varphi(x,y,u,v,w,r,s,t) \leq |4|^{\ell(\ell+2)} L\varphi\left(\frac{x}{2^\ell},\frac{y}{2^\ell},\frac{u}{2^\ell},\frac{v}{2^\ell},\frac{w}{2^\ell},\frac{r}{2^\ell},\frac{s}{2^\ell},\frac{t}{2^\ell}\right) \tag{2.2}$$

for all $x,y,u,v,w,r,s,t \in \mathcal{A}$. *Then there exist a unique quadratic ternary derivation* $d:\mathcal{A}\to\mathcal{A}$ *and a unique generalized quadratic ternary derivation* $D:\mathcal{A}\to\mathcal{A}$ *(respected to d) such that*

$$\max\left\{\left\|g(x)-D(x)\right\|,\left\|f(x)-d(x)\right\|\right\} \leq \frac{L^{\frac{1-\ell}{2}}}{|4|}\psi(x) \tag{2.3}$$

for all $x \in \mathcal{A}$, *where*

$$\begin{aligned}
\psi(x):\max\Big\{&\varphi\left(\frac{x}{a},\frac{x}{b},0,0,0,0,0,0\right),\varphi\left(\frac{x}{a},0,0,0,0,0,0,0\right),\frac{1}{|2b^2|}\varphi(x,x,0,0,0,0,0,0),\\
&\frac{1}{|2b^2|}\varphi(x,-x,0,0,0,0,0,0),\varphi\left(0,\frac{x}{b},0,0,0,0,0,0\right)\Big\}
\end{aligned}$$

for all $x \in \mathcal{A}$.

Proof. By (2.2), one can show that

$$\lim_{n\to\infty}\frac{1}{|4|^{\ell(\ell+2)n}}\varphi\left(2^{\ell n}x,2^{\ell n}y,2^{\ell n}u,2^{\ell n}v,2^{\ell n}w,2^{\ell n}r,2^{\ell n}s,2^{\ell n}t\right)=0 \tag{2.4}$$

for all $x,y,u,v,w,r,s,t \in \mathcal{A}$. Putting $h = g$ in (2.1) and letting $u = v = w = r = s = t = 0$ in (2.1), we get

$$\left\|g(ax+by)+g(ax-by)-2a^2g(x)-2b^2g(y)\right\| \leq \varphi(x,y,0,0,0,0,0,0) \tag{2.5}$$

for all $x,y \in \mathcal{A}$. Putting $y = 0$ in (2.5), we get

$$\left\|2g(ax)-2a^2g(x)\right\| \leq \varphi(x,0,0,0,0,0,0,0) \tag{2.6}$$

for all $x \in \mathcal{A}$. Setting $y = -y$ in (2.5), we get

$$\left\|g(ax-by)+g(ax+by)-2a^2g(x)-2b^2g(-y)\right\| \leq \varphi(x,-y,0,0,0,0,0,0) \tag{2.7}$$

for all $x,y \in \mathcal{A}$. It follows from (2.5) and (2.7) that

$$\left\|2b^2g(y)-2b^2h(-y)\right\| \leq \max\left\{\varphi(x,y,0,0,0,0,0,0),\varphi(x,-y,0,0,0,0,0,0)\right\} \tag{2.8}$$

for all $x,y \in \mathcal{A}$. Putting $y = by$ in (2.8), we get

$$\left\|g(by)-g(-by)\right\| \leq \max\left\{\frac{1}{|2b^2|}\varphi(x,by,0,0,0,0,0,0),\frac{1}{|2b^2|}\varphi(x,-by,0,0,0,0,0,0)\right\} \tag{2.9}$$

for all $x, y \in \mathcal{A}$. Setting $x = 0$ in (2.5), we get

$$\left\| g(by) + g(-by) - 2b^2 g(y) \right\| \leq \varphi\left(0, y, 0, 0, 0, 0, 0, 0\right) \tag{2.10}$$

for all $y \in \mathcal{A}$. It follows from (2.9) and (2.10) that

$$\begin{aligned} &\left\| 2g(by) - 2b^2 g(y) \right\| \\ &\leq \max\left\{ \frac{1}{|2b^2|}\varphi(x, by, 0, 0), \frac{1}{|2b^2|}\varphi\left(x, -by, 0, 0, 0, 0, 0, 0\right), \varphi\left(0, y, 0, 0, 0, 0, 0, 0\right) \right\} \end{aligned} \tag{2.11}$$

for all $x, y \in \mathcal{A}$. Replacing x and y by $\frac{x}{a}$ and $\frac{x}{b}$ in (2.5), respectively, we get

$$\left\| g(2x) - 2a^2 g\left(\frac{x}{a}\right) - 2b^2 g\left(\frac{x}{b}\right) \right\| \leq \varphi\left(\frac{x}{a}, \frac{x}{b}, 0, 0, 0, 0, 0, 0\right) \tag{2.12}$$

for all $x \in \mathcal{A}$. Setting $x = \frac{x}{a}$ in (2.6), we get

$$\left\| 2a^2 g\left(\frac{x}{a}\right) - 2g(x) \right\| \leq \varphi\left(\frac{x}{a}, 0, 0, 0, 0, 0, 0, 0\right) \tag{2.13}$$

for all $x \in \mathcal{A}$. Putting $y = \frac{x}{b}$ in (2.11), we get

$$\begin{aligned} &\left\| 2b^2 g\left(\frac{x}{b}\right) - 2g(x) \right\| \\ &\leq \max\left\{ \frac{1}{|2b^2|}\varphi\left(x, x, 0, 0, 0, 0, 0, 0\right), \frac{1}{|2b^2|}\varphi\left(x, -x, 0, 0, 0, 0, 0, 0\right), \varphi\left(0, \frac{x}{b}, 0, 0, 0, 0, 0, 0\right) \right\} \end{aligned} \tag{2.14}$$

for all $x \in \mathcal{A}$. It follows from (2.12), (2.13) and (2.14) that

$$\left\| g(2x) - 4g(x) \right\| \leq \psi(x)$$

for all $x \in \mathcal{A}$. Let $\Omega := \left\{ g' | g' : \mathcal{A} \to \mathcal{A}, g'(0) = 0 \right\}$. For every g', $h' \in \Omega$, define

$$\rho\left(g', h'\right) := \inf\left\{ C \in (0, \infty) : \left\| g'(x) - h'(x) \right\| \leq C\psi(x), \forall x \in \mathcal{A} \right\}.$$

It is easy to show that ρ is a complete generalized non-Archimedean metric on Ω (see [30,31,34]). We define $J : \Omega \to \Omega$ by $J(g')(x) = \frac{1}{4^\ell} g'\left(2^\ell x\right)$ for all $x \in \mathcal{A}$ and all $g' \in \Omega$. One can show that

$$\rho\left(Jg', Jh'\right) \leq L\rho\left(g', h'\right).$$

Hence J is a strictly contractive mapping on Ω with Lipschitz constant L. It follows from Theorem 2.1 that J has a unique fixed point $D : \mathcal{A} \to \mathcal{A}$ in the set $\Lambda = \{g' \in \Omega : \rho(g, g') < \infty\}$, where d is defined by

$$D(x) = \lim_{n \to \infty} J^n g(x) = \lim_{n \to \infty} \frac{1}{4^{\ell n}} g\left(2^{\ell n} x\right) \tag{2.15}$$

for all $x \in \mathcal{A}$. It follows from (2.4) and (2.15) that

$$\begin{aligned} &\left\| D\left(ax + by\right) + D\left(ax - by\right) - 2a^2 D(x) - 2b^2 D(y) \right\| \\ &= \lim_{n \to \infty} \frac{1}{|4|^{\ell n}} \left\| g\left(2^{\ell n} ax + 2^{\ell n} by\right) + g\left(2^{\ell n} ax - 2^{\ell n} by\right) - 2a^2 g(2^{\ell n} x) - 2b^2 g\left(2^{\ell n} y\right) \right\| \\ &\leq \lim_{n \to \infty} \frac{1}{|4|^{\ell n}} \varphi\left(2^{\ell n} x, 2^{\ell n} y, 0, 0, 0, 0, 0, 0\right) \\ &\leq \lim_{n \to \infty} \frac{1}{|4|^{\ell n(\ell + 2)}} \varphi\left(2^{\ell n} x, 2^{\ell n} y, 0, 0, 0, 0, 0, 0\right) = 0 \end{aligned}$$

for all $x, y \in \mathcal{A}$. This shows that D is quadratic.

If $D' : \mathcal{A} \to \mathcal{A}$ is another quadratic mapping which satisfies (2.3), then D' is a fixed point of J in Λ. The uniqueness of the fixed point of J in Λ implies that $D = D'$. Putting $h = f$, $u = v = w = r = s = t = 0$ in (2.4), we get

$$\left\| f(ax + by) + f(ax - by) - 2a^2 f(x) - 2b^2 f(y) \right\| \le \varphi\left(x, y, 0, 0, 0, 0, 0, 0\right)$$

for all $x, y \in \mathcal{A}$. By the same reasoning as above, we can show that the limit

$$d(x) =: \lim_{n\to\infty} \frac{1}{4^{\ell n}} f(2^{\ell n} x)$$

exists for all $x \in \mathcal{A}$. Moreover, we can show that d is a unique quadratic mapping on $\mathcal{A}$ satisfying (2.3).

On the other hand, we have

$$\begin{aligned}
&\left\| d([uvw]) - \left[d(u)vw\right] - \left[ud(v)w\right] - \left[uvd(w)\right] \right\| \\
&= \lim_{n\to\infty} \frac{1}{|4|^{2\ell n}} \left\| f\left(4^{\ell n}[uvw]\right) - \left[f\left(2^{\ell n}u\right)vw\right] - \left[uf\left(2^{\ell n}v\right)w\right] - \left[uvf\left(2^{\ell n}w\right)\right] \right\| \\
&\le \lim_{n\to\infty} \frac{1}{|4|^{2\ell n}} \varphi\left(0, 0, 2^{\ell n}u, 2^{\ell n}v, 2^{\ell n}w, 0, 0, 0\right) \\
&\le \lim_{n\to\infty} \frac{1}{|4|^{\ell(\ell+2)n}} \varphi\left(0, 0, 2^{\ell n}u, 2^{\ell n}v, 2^{\ell n}w, 0, 0, 0\right) = 0
\end{aligned}$$

for all $u, v, w \in \mathcal{A}$. Therefore, d is a quadratic ternary derivation on $\mathcal{A}$. Also, we have

$$\begin{aligned}
&\left\| D([rst]) - \left[D(r)st\right] - \left[rd(s)t\right] - \left[rsd(t)\right] \right\| \\
&= \lim_{n\to\infty} \frac{1}{|4|^{2\ell n}} \left\| g\left(4^{\ell n}[rst]\right) - \left[g(2^{\ell n}r)st\right] - \left[rf\left(2^{\ell n}s\right)t\right] - \left[rsf\left(2^{\ell n}t\right)\right] \right\| \\
&\le \lim_{n\to\infty} \frac{1}{|4|^{2\ell n}} \varphi\left(0, 0, 0, 0, 0, 2^{\ell n}r, 2^{\ell n}s, 2^{\ell n}t\right) \\
&\le \lim_{n\to\infty} \frac{1}{|4|^{\ell(\ell+2)n}} \varphi\left(0, 0, 0, 0, 0, 2^{\ell n}r, 2^{\ell n}s, 2^{\ell n}t\right) = 0
\end{aligned}$$

for all $r, s, t \in \mathcal{A}$. It follows that D is a generalized quadratic ternary derivation (related to d) on $\mathcal{A}$.This completes the proof.

From now on, we use the following abbreviation for any mappings $g, f : \mathcal{A} \to \mathcal{A}$:

$$\begin{aligned}
\Delta\left(g, f\right)\left(x_1, \cdots, x_8\right) := &\left\| f\left(ax_1 + bx_2\right) + f\left(ax_1 - bx_2\right) - 2a^2 h(x_1) - 2b^2 f(x_2) \right\| \\
&+ \left\| g\left(ax_1 + bx_2\right) + g\left(ax_1 - bx_2\right) - 2a^2 g(x_1) - 2b^2 g(x_2) \right\| \\
&+ \left\| f\left([x_3x_4x_5]\right) - \left[f\left(x_3x_4^2x_5^2\right)\right] - \left[x_3^2 f\left(x_4\right) x_5^2\right] - \left[x_3^2x_4^2 f(x_5)\right] \right\| \\
&+ \left\| g\left([x_6x_7x_8]\right) - \left[g(x_6)x_7^2x_8^2\right] - \left[x_6^2 f(x_7) x_8^2\right] - \left[x_6^2x_7^2 f(x_8)\right] \right\|.
\end{aligned}$$

Remark. Let $\mathbb{K} = \mathbb{Q}_2$ be the 2-adic number field. Let $\mathcal{A}$ be a non-Archimedean Banach algebra on $\mathbb{K}$. Let ε be a nonnegative real number and let s be a real number such that $s > 6$ if $\ell = 1$ and $0 < s < 2$ if $\ell = -1$. Suppose that the mappings $g, f : \mathcal{A} \to \mathcal{A}$ satisfy $g(0) = f(0) = 0$ and

$$\Delta(g, f)(x_1, \cdots, x_8) \le \varepsilon \max\left\{ \|x_i\|^s : 1 \le i \le 8 \right\}$$

for all $x_1, x_2, \cdots, x_8 \in \mathcal{A}$. Then there exist a unique quadratic ternary derivation $d : \mathcal{A} \to \mathcal{A}$ and a unique generalized quadratic ternary derivation $D : \mathcal{A} \to \mathcal{A}$ (respected to d) such that

$$\max\{||g(x) - D(x)||, ||f(x) - d(x)||\}$$

$$\leq |2|^{\frac{\ell(4-s)+s}{2}} \varepsilon \|x\|^s \begin{cases} 2, & \gcd(a,2) = \gcd(b,2) = 1; \\ \max\left\{2^{is}, 2\right\}, & a = k2^i,\ \gcd(b,2) = 1; \\ \max\left\{2^{js}, 2^{2j+1}\right\}, & \gcd(a,2) = 1, b = m2^j \ \vee a = k2^i,\ b = m2^j (j \geq i); \\ \max\left\{2^{is}, 2^{2j+1}\right\}, & a = k2^i,\ \ b = m2^j (i \geq j); \end{cases}$$

for all $x \in \mathcal{A}$, where $i,j,k,m \geq 1$ are integers and $\gcd(k, 2) = \gcd(m, 2) = 1$.

Now, we have the following result on superstability of generalized quadratic ternary derivations on non-Archimedean ternary Banach algebras:

Corollary 2.3. *Let $p > 0$ be a nonnegative real number such that $|2|^{(2\ell+4)p} \geq 1$ and let $j \in \{3, 4, ..., 8\}$ be fixed. Suppose that the mappings $g, f : \mathcal{A} \to \mathcal{A}$ satisfy $g(0) = f(0) = 0$ and*

$$\Delta(g, f)(x_1, ..., x_8) \leq \left(\sum_{i=1}^{8} \|x_i\|^p\right) \|x_j\|^p$$

for all $x_1, \cdots, x_8 \in \mathcal{A}$, where a, b are positive fixed integers. Then f is a quadratic ternary derivation and g is a generalized quadratic ternary derivation related to f.

Proof. It follows from Theorem 2.2 by taking

$$\varphi(x_1, x_2, \cdots, x_8) = \left(\sum_{i=1}^{8} \|x_i\|^p\right) \|x_j\|^p$$

for all $x_1, \cdots, x_8 \in \mathcal{A}$ and putting $L = |2|^{-(2\ell+4)p}$.

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Authors' contributions
All authors conceived of the study, participated in its design and coordination, drafted the manuscript, participated in the sequence alignment, and read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

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