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Comment on "Functional inequalities associated with Jordan-von Neumann type additive functional equations"

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Abstract

Park et al. proved the Hyers-Ulam stability of some additive functional inequalities. There is a fatal error in the proof of Theorem 3.1. We revise the statements of the main theorems and prove the revised theorems.
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1 Introduction and preliminaries

Ulam [1] gave a talk before the Mathematics Club of the University of Wisconsin in which he discussed a number of unsolved problems. Among these was the following question concerning the stability of homomorphisms.

We are given a group G and a metric group G' with metric $\rho(\cdot,\cdot)$. *Given* $\epsilon > 0$, *does there exist a* $\delta > 0$ *such that if* $f : G \to G'$ *satisfies* $\rho(f(xy), f(x)f(y)) < \delta$ *for all* $x,y \in G$, *then a homomorphism* $h : G \to G'$ *exists with* $\rho(f(x), h(x)) < \epsilon$ *for all* $x \in G$?

Hyers [2] considered the case of approximately additive mappings $f: E \to E'$, where E and E' are Banach spaces and f satisfies *Hyers' inequality*

$$\|f(x+y) - f(x) - f(y)\| \le \varepsilon$$

for all $x,y \in E$. It was shown that the limit

$$L(x) = \lim_{n\to\infty} \frac{f(2^n x)}{2^n}$$

exists for all $x \in E$ and that $L : E \to E'$ is the unique additive mapping satisfying

$$\|f(x) - L(x)\| \le \varepsilon.$$

Rassias [3] provided a generalization of Hyers' Theorem which allows the *Cauchy difference to be unbounded.*

Theorem 1.1. (Rassias). *Let* $f : E \to E'$ *be a mapping from a normed vector space E into a Banach space E' subject to the inequality*

$$\|f(x+y) - f(x) - f(y)\| \le \varepsilon(\|x\|^p + \|y\|^p) \tag{1.1}$$

for all $x,y \in E$, where ϵ and p are constants with $\epsilon > 0$ and $p < 1$. Then the limit

$$L(x) = \lim_{n\to\infty} \frac{f(2^n x)}{2^n}$$

exists for all $x \in E$ and $L : E \to E'$ is the unique additive mapping which satisfies

$$\left\| f(x) - L(x) \right\| \le \frac{2\varepsilon}{2-2^p} \|x\|^p \tag{1.2}$$

for all $x \in E$. If $p < 0$ then inequality (1.1) *holds for $x,y \neq 0$ and* (1.2) *for $x \neq 0$.*

Rassias [4] during the 27th International Symposium on Functional Equations asked the question whether such a theorem can also be proved for $p \ge 1$. Gajda [5] following the same approach as in Rassias [3] gave an affirmative solution to this question for $p > 1$. It was shown by Gajda [5], as well as by Rassias and Šemrl [6] that one cannot prove a Rassias' type theorem when $p = 1$ (cf. the books of Czerwik [7] and Hyers et al. [8]).

Rassias [9] followed the innovative approach of Rassias' theorem [3] in which he replaced the factor $\|x\|^p + \|y\|^p$ by $\|x\|^p \cdot \|y\|^q$ for $p,q \in \mathbb{R}$ with $p + q \neq 1$. Găvruta [10] provided a further generalization of Rassias' theorem. During the last two decades a number of papers and research monographs have been published on various generalizations and applications of the Hyers-Ulam stability to a number of functional equations and mappings (see [11-13]).

Throughout this article, let G be a 2-divisible abelian group. Assume that X is a normed space with norm $|| \cdot ||_X$ and that Y is a Banach space with norm $|| \cdot ||_Y$.

Gilányi [14] showed that if f satisfies the functional inequality

$$\left\| 2f(x) + 2f(y) - f(xy^{-1}) \right\| \le \left\| f(xy) \right\| \tag{1.3}$$

then f satisfies the Jordan-von Neumann functional equation

$$2f(x) + 2f(y) = f(xy) + f(xy^{-1}).$$

See also [15]. Gilányi [16] and Fechner [17] proved the Hyers-Ulam stability of the functional inequality (1.3).

Park et al. [18] proved the Hyers-Ulam stability of the following functional inequalities

$$\left\| f(x) + f(y) + f(z) \right\| \le \left\| 2f\left(\frac{x+y+z}{2}\right) \right\|, \tag{1.4}$$

$$\left\| f(x) + f(y) + f(z) \right\| \le \left\| f(x+y+z) \right\|, \tag{1.5}$$

$$\left\| f(x) + f(y) + 2f(z) \right\| \le \left\| 2f\left(\frac{x+y}{2} + z\right) \right\|. \tag{1.6}$$

But there is an error in the 8th line on the 6th page in the proof of [18, Theorem 3.1]. We revise the statements of the main theorems and prove the revised theorems.

In Section 2, we prove the Hyers-Ulam stability of the functional inequality (1.4).

In Section 3, we prove the Hyers-Ulam stability of the functional inequality (1.5).

In Section 4, we prove the Hyers-Ulam stability of the functional inequality (1.6).

2 Stability of a functional inequality associated with a 3-variable Jensen additive functional equation

Proposition 2.1. [18, Proposition 2.1] *Let $f : G \to Y$ be a mapping such that*

$$\|f(x) + f(y) + f(z)\|_Y \le \left\|2f\left(\frac{x+y+z}{2}\right)\right\|_Y$$

for all $x, y, z \in G$. Then f is Cauchy additive.

We prove the Hyers-Ulam stability of a functional inequality associated with a Jordan-von Neumann type 3-variable Jensen additive functional equation.

Theorem 2.2. *Let $r > 1$ and θ be nonnegative real numbers, and let $f : X \to Y$ be an odd mapping such that*

$$\|f(x) + f(y) + f(z)\|_Y \le \left\|2f\left(\frac{x+y+z}{2}\right)\right\|_Y + \theta(\|x\|_X^r + \|y\|_X^r + \|z\|_X^r) \tag{2.1}$$

for all $x, y, z \in X$. Then there exists a unique Cauchy additive mapping $h : X \to Y$ such that

$$\|f(x) - h(x)\|_Y \le \frac{2^r + 2}{2^r - 2}\theta \|x\|_X^r \tag{2.2}$$

for all $x \in X$.

Proof. Letting $y = x$ and $z = -2x$ in (2.1), we get

$$\|2f(x) - f(2x)\|_Y = \|2f(x) + f(-2x)\|_Y \le (2 + 2^r)\theta \|x\|_X^r \tag{2.3}$$

for all $x \in X$. So

$$\left\|f(x) - 2f\left(\frac{x}{2}\right)\right\|_Y \le \frac{2 + 2^r}{2^r}\theta \|x\|_X^r$$

for *all* $x \in X$. Hence

$$\begin{aligned}\left\|2^l f\left(\frac{x}{2^l}\right) - 2^m f\left(\frac{x}{2^m}\right)\right\|_Y &\le \sum_{j=l}^{m-1} \left\|2^j f\left(\frac{x}{2^j}\right) - 2^{j+1} f\left(\frac{x}{2^{j+1}}\right)\right\|_Y \\ &\le \frac{2 + 2^r}{2^r} \sum_{j=l}^{m-1} \frac{2^j}{2^{rj}}\theta \|x\|_X^r\end{aligned} \tag{2.4}$$

for all nonnegative integers m and l with $m > l$ and all $x \in X$.

It follows from (2.4) that the sequence $\{2^n f(\frac{x}{2^n})\}$ is a Cauchy sequence for all $x \in X$. Since Y is complete, the sequence $\{2^n f(\frac{x}{2^n})\}$ converges. So one can define the mapping $h : X \to Y$ by

$$h(x) := \lim_{n\to\infty} 2^n f\left(\frac{x}{2^n}\right)$$

for all $x \in X$. Moreover, letting $l = 0$ and passing the limit $m \to \infty$ in (2.4), we get (2.2).

It follows from (2.1) that

$$\begin{aligned}\left\|h(x)+h(y)+h(z)\right\|_Y &= \lim_{n\to\infty} 2^n \left\|f\left(\frac{x}{2^n}\right)+f\left(\frac{y}{2^n}\right)+f\left(\frac{z}{2^n}\right)\right\|_Y \\ &\le \lim_{n\to\infty} 2^n \left\|2f\left(\frac{x+y+z}{2^{n+1}}\right)\right\|_Y + \lim_{n\to\infty}\frac{2^n\theta}{2^{nr}}(\|x\|_X^r+\|y\|_X^r+\|z\|_X^r) \\ &= \left\|2h\left(\frac{x+y+z}{2}\right)\right\|_Y\end{aligned}$$

for all $x, y, z \in X$. So

$$\left\|h(x)+h(y)+h(z)\right\|_Y \le \left\|2h\left(\frac{x+y+z}{2}\right)\right\|_Y$$

for all $x, y, z \in X$. By Proposition 2.1, the mapping $h : X \to Y$ is Cauchy additive.

Now, let $T : X \to Y$ be another Cauchy additive mapping satisfying (2.2). Then we have

$$\begin{aligned}\left\|h(x)-T(x)\right\|_Y &= 2^n\left\|h\left(\frac{x}{2^n}\right)-T\left(\frac{x}{2^n}\right)\right\|_Y \\ &\le 2^n\left(\left\|h\left(\frac{x}{2^n}\right)-f\left(\frac{x}{2^n}\right)\right\|_Y+\left\|T\left(\frac{x}{2^n}\right)-f\left(\frac{x}{2^n}\right)\right\|_Y\right) \\ &\le \frac{2(2^r+2)2^n}{(2^r-2)2^{nr}}\theta\,\|x\|_X^r,\end{aligned}$$

which tends to zero as $n \to \infty$ for all $x \in X$. So we can conclude that $h(x) = T(x)$ for all $x \in X$. This proves the uniqueness of h. Thus the mapping $h : X \to Y$ is a unique Cauchy additive mapping satisfying (2.2).

Theorem 2.3. *Let $r < 1$ and θ be positive real numbers, and let $f : X \to Y$ be an odd mapping satisfying* (2.1). *Then there exists a unique Cauchy additive mapping $h : X \to Y$ such that*

$$\left\|f(x)-h(x)\right\|_Y \le \frac{2+2^r}{2-2^r}\theta\,\|x\|_X^r \tag{2.5}$$

for all $x \in X$.

Proof. It follows from (2.3) that

$$\left\|f(x)-\frac{1}{2}f(2x)\right\|_Y \le \frac{2+2^r}{2}\theta\,\|x\|_X^r$$

for all $x \in X$. Hence

$$\begin{aligned}\left\|\frac{1}{2^l}f(2^lx)-\frac{1}{2^m}f(2^mx)\right\|_Y &\le \sum_{j=l}^{m-1}\left\|\frac{1}{2^j}f(2^jx)-\frac{1}{2^{j+1}}f(2^{j+1}x)\right\|_Y \\ &\le \frac{2+2^r}{2}\sum_{j=l}^{m-1}\frac{2^{rj}}{2^j}\theta\,\|x\|_X^r\end{aligned} \tag{2.6}$$

for all nonnegative integers m and l with $m > l$ and all $x \in X$.

It follows from (2.6) that the sequence $\{\frac{1}{2^n}f(2^nx)\}$ is a Cauchy sequence for all $x \in X$. Since Y is complete, the sequence $\{\frac{1}{2^n}f(2^nx)\}$ converges. So one can define the mapping $h : X \to Y$ by

$$h(x) := \lim_{n\to\infty} \frac{1}{2^n} f(2^n x)$$

for all $x \in X$. Moreover, letting $l = 0$ and passing the limit $m \to \infty$ in (2.6), we get (2.5).

The rest of the proof is similar to the proof of Theorem 2.2.

Theorem 2.4. *Let $r > \frac{1}{3}$and θ be nonnegative real numbers, and let $f : X \to Y$ be an odd mapping such that*

$$\left\|f(x) + f(y) + f(z)\right\|_Y \le \left\|2f\left(\frac{x+y+z}{2}\right)\right\|_Y + \theta \cdot \|x\|_X^r \cdot \|y\|_X^r \cdot \|z\|_X^r \tag{2.7}$$

for all x, y, $z \in X$. Then there exists a unique Cauchy additive mapping $h : X \to Y$ such that

$$\left\|f(x) - h(x)\right\|_Y \le \frac{2^r\theta}{8^r - 2} \|x\|_X^{3r} \tag{2.8}$$

for all $x \in X$.

Proof. Letting $y = x$ and $z = -2x$ in (2.7), we get

$$\left\|2f(x) - f(2x)\right\|_Y = \left\|2f(x) + f(-2x)\right\|_Y \le 2^r\theta \|x\|_X^{3r} \tag{2.9}$$

for all $x \in X$. So

$$\left\|f(x) - 2f\left(\frac{x}{2}\right)\right\|_Y \le \frac{2^r}{8^r}\theta \|x\|_X^{3r}$$

for all $x \in X$. Hence

$$\begin{aligned} \left\|2^l f\left(\frac{x}{2^l}\right) - 2^m f\left(\frac{x}{2^m}\right)\right\|_Y &\le \sum_{j=l}^{m-1} \left\|2^j f\left(\frac{x}{2^j}\right) - 2^{j+1} f\left(\frac{x}{2^{j+1}}\right)\right\|_Y \\ &\le \frac{2^r}{8^r} \sum_{j=l}^{m-1} \frac{2^j}{8^{rj}}\theta \|x\|_X^{3r} \end{aligned} \tag{2.10}$$

for all nonnegative integers m and l with $m > l$ and all $x \in X$.

It follows from (2.10) that the sequence $\{2^n f(\frac{x}{2^n})\}$ is a Cauchy sequence for all $x \in X$. Since Y is complete, the sequence $\{2^n f(\frac{x}{2^n})\}$ converges. So one can define the mapping $h : X \to Y$ by

$$h(x) := \lim_{n\to\infty} 2^n f(\frac{x}{2^n})$$

for all $x \in X$. Moreover, letting $l = 0$ and passing the limit $m \to \infty$ in (2.10), we get (2.8).

The rest of the proof is similar to the proof of Theorem 2.2.

Theorem 2.5. *Let $r < \frac{1}{3}$and θ be positive real numbers, and let $f : X \to Y$ be an odd mapping satisfying* (2.7). *Then there exists a unique Cauchy additive mapping $h:X \to Y$ such that*

$$\left\|f(x) - h(x)\right\|_Y \le \frac{2^r\theta}{2-8^r}\, \|x\|_X^{3r} \tag{2.11}$$

for all $x \in X$.

Proof. It follows from (2.9) that

$$\left\|f(x) - \frac{1}{2}f(2x)\right\|_Y \le \frac{2^r}{2}\theta\, \|x\|_X^{3r}$$

for all $x \in X$. Hence

$$\begin{aligned}\left\|\frac{1}{2^l}f(2^l x) - \frac{1}{2^m}f(2^m x)\right\|_Y &\le \sum_{j=l}^{m-1}\left\|\frac{1}{2^j}f(2^j x) - \frac{1}{2^{j+1}}f(2^{j+1}x)\right\|_Y \\ &\le \frac{2^r}{2}\sum_{j=l}^{m-1}\frac{8^{rj}}{2^j}\theta\, \|x\|_X^r\end{aligned} \tag{2.12}$$

for all nonnegative integers m and l with $m > l$ and all $x \in X$.

It follows from (2.12) that the sequence $\{\frac{1}{2^n}f(2^n x)\}$ is a Cauchy sequence for all $x \in X$. Since Y is complete, the sequence $\{\frac{1}{2^n}f(2^n x)\}$ converges. So one can define the mapping $h : X \to Y$ by

$$h(x) := \lim_{n\to\infty}\frac{1}{2^n}f(2^n x)$$

for all $x \in X$. Moreover, letting $l = 0$ and passing the limit $m \to \infty$ in (2.12), we get (2.11).

The rest of the proof is similar to the proof of Theorem 2.2.

3 Stability of a functional inequality associated with a 3-variable Cauchy additive functional equation

Proposition 3.1. [18, Proposition 2.2] *Let $f : G \to Y$ be a mapping such that*

$$\left\|f(x) + f(y) + f(z)\right\|_Y \le \left\|f(x+y+z)\right\|_Y$$

for all $x, y, z \in G$. Then f is Cauchy additive.

We prove the Hyers-Ulam stability of a functional inequality associated with a Jordan-von Neumann type 3-variable Cauchy additive functional equation.

Theorem 3.2. *Let $r > 1$ and θ be nonnegative real numbers, and let $f : X \to Y$ be an odd mapping such that*

$$\left\|f(x) + f(y) + f(z)\right\|_Y \le \left\|f(x+y+z)\right\|_Y + \theta(\|x\|_X^r + \|y\|_X^r + \|z\|_X^r) \tag{3.1}$$

for all $x, y, z \in X$. Then there exists a unique Cauchy additive mapping $h : X \to Y$ such that

$$\left\|f(x) - h(x)\right\|_Y \le \frac{2^r+2}{2^r-2}\theta\, \|x\|_X^r$$

for all $x \in X$.

Proof. Letting $y = x$ and $z = -2x$ in (3.1), we get

$$\|2f(x) - f(2x)\|_Y = \|2f(x) + f(-2x)\|_Y \le (2 + 2^r)\theta \, \|x\|_X^r \tag{3.2}$$

for all $x \in X$.

The rest of the proof is the same as in the proof of Theorem 2.2.

Theorem 3.3. *Let $r < 1$ and θ be positive real numbers, and let $f : X \to Y$ be an odd mapping satisfying* (3.1). *Then there exists a unique Cauchy additive mapping $h:X \to Y$ such that*

$$\|f(x) - h(x)\|_Y \le \frac{2 + 2^r}{2 - 2^r}\theta \, \|x\|_X^r$$

for all $x \in X$.

Proof. It follows from (3.2) that

$$\left\|f(x) - \frac{1}{2}f(2x)\right\|_Y \le \frac{2 + 2^r}{2}\theta \, \|x\|_X^r$$

for all $x \in X$.

The rest of the proof is the same as in the proofs of Theorems 2.2 and 2.3.

Theorem 3.4. *Let $r > \frac{1}{3}$ and θ be nonnegative real numbers, and let $f : X \to Y$ be an odd mapping such that*

$$\|f(x) + f(y) + f(z)\|_Y \le \|f(x + y + z)\|_Y + \theta \cdot \|x\|_X^r \cdot \|y\|_X^r \cdot \|z\|_X^r \tag{3.3}$$

for all $x, y, z \in X$. Then there exists a unique Cauchy additive mapping $h : X \to Y$ such that

$$\|f(x) - h(x)\|_Y \le \frac{2^r\theta}{8^r - 2} \, \|x\|_X^{3r}$$

for all $x \in X$.

Proof Letting $y = x$ and $z = -2x$ in (3.3), we get

$$\|2f(x) - f(2x)\|_Y = \|2f(x) + f(-2x)\|_Y \le 2^r\theta \, \|x\|_X^{3r} \tag{3.4}$$

for all $x \in X$.

The rest of the proof is the same as in the proofs of Theorems 2.2 and 2.4.

Theorem 3.5. *Let $r < \frac{1}{3}$ and θ be positive real numbers, and let $f : X \to Y$ be an odd mapping satisfying* (3.3). *Then there exists a unique Cauchy additive mapping $h : X \to Y$ such that*

$$\|f(x) - h(x)\|_Y \le \frac{2^r\theta}{2 - 8^r} \, \|x\|_X^{3r}$$

for all $x \in X$.

Proof. It follows from (3.4) that

$$\left\|f(x) - \frac{1}{2}f(2x)\right\|_Y \le \frac{2^r}{2}\theta \, \|x\|_X^{3r}$$

for all $x \in X$.

The rest of the proof is the same as in the proofs of Theorems 2.2 and 2.5.

4 Stability of a functional inequality associated with the Cauchy-Jensen functional equation

Proposition 4.1. [18, Proposition 2.3] *Let* $f: G \to Y$ *be a mapping such that*

$$\|f(x) + f(y) + 2f(z)\|_Y \le \left\|2f\left(\frac{x+y}{2} + z\right)\right\|_Y$$

for all $x, y, z \in G$. *Then* f *is Cauchy additive.*

We prove the Hyers-Ulam stability of a functional inequality associated with a Jordan-von Neumann type Cauchy-Jensen functional equation.

Theorem 4.2. *Let* $r > 1$ *and* θ *be nonnegative real numbers, and let* $f: X \to Y$ *be an odd mapping such that*

$$\|f(x) + f(y) + 2f(z)\|_Y \le \left\|2f\left(\frac{x+y}{2} + z\right)\right\|_Y + \theta(\|x\|_X^r + \|y\|_X^r + \|z\|_X^r) \tag{4.1}$$

for all $x, y, z \in X$. *Then there exists a unique Cauchy additive mapping* $h: X \to Y$ *such that*

$$\|f(x) - h(x)\|_Y \le \frac{2^r + 1}{2^r - 2}\theta \|x\|_X^r$$

for all $x \in X$.

Proof. Replacing x by $2x$ and letting $y = 0$ and $z = -x$ in (4.1), we get

$$\|f(2x) - 2f(x)\|_Y = \|f(2x) + 2f(-x)\|_Y \le (1 + 2^r)\theta \|x\|_X^r \tag{4.2}$$

for all $x \in X$. So

$$\left\|f(x) - 2f\left(\frac{x}{2}\right)\right\|_Y \le \frac{1 + 2^r}{2^r}\theta \|x\|_X^r$$

for all $x \in X$.

The rest of the proof is similar to the proof of Theorem 2.2.

Theorem 4.3. *Let* $r < 1$ *and* θ *be positive real numbers, and let* $f: X \to Y$ *be an odd mapping satisfying* (4.1). *Then there exists a unique Cauchy additive mapping* $h: X \to Y$ *such that*

$$\|f(x) - h(x)\|_Y \le \frac{1 + 2^r}{2 - 2^r}\theta \|x\|_X^r$$

for all $x \in X$.

Proof. It follows from (4.2) that

$$\left\|f(x) - \frac{1}{2}f(2x)\right\|_Y \le \frac{1 + 2^r}{2}\theta \|x\|_X^r$$

for all $x \in X$.

The rest of the proof is similar to the proofs of Theorems 2.2 and 2.3.

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Authors' contributions
All authors conceived of the study, participated in its design and coordination, drafted the manuscript, participated in the sequence alignment, and read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

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