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Generalized Ulam–Hyers stability of random homomorphisms in random normed algebras associated with the Cauchy functional equation

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ABSTRACT

Using the fixed point method, we prove the generalized Ulam–Hyers stability of random homomorphisms in random normed algebras associated with the Cauchy functional equation.

1. Introduction

Fuzzy set theory is a powerful tool set for modeling uncertainty and vagueness in various problems arising in the field of science and engineering. It has also very useful applications in various fields, e.g., population dynamics [1], chaos control [2], computer programming [3], etc. Recently, the fuzzy topology has proved to be a very useful tool for dealing with situations where the use of classical theories breaks down.

In the sequel, we adopt the usual terminology, notation and conventions of the theory of random normed spaces, as in [4–8]. Throughout this work, Δ^+ is the space of distribution functions, that is, the space of all mappings $F : \mathbb{R}\cup\{-\infty, \infty\} \to [0, 1]$ such that F is left-continuous and non-decreasing on $\mathbb{R}$, $F(0) = 0$ and $F(+\infty) = 1$. D^+ is a subset of Δ^+ consisting of all functions $F \in \Delta^+$ for which $l^-F(+\infty) = 1$, where $l^-f(x)$ denotes the left limit of the function f at the point x, that is, $l^-f(x) = \lim_{t\to x^-} f(t)$. The space Δ^+ is partially ordered by the usual pointwise ordering of functions, i.e., $F \le G$ if and only if $F(t) \le G(t)$ for all t in $\mathbb{R}$. The maximal element for Δ^+ in this order is the distribution function ε_0 given by

$$\varepsilon_0(t) = \begin{cases} 0, & \text{if } t \le 0, \\ 1, & \text{if } t > 0. \end{cases}$$

Definition 1.1 (*[7]*). A mapping $T : [0, 1] \times [0, 1] \to [0, 1]$ is a continuous triangular norm (for short, a continuous t-norm) if T satisfies the following conditions:

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(a) T is commutative and associative;
(b) T is continuous;
(c) $T(a, 1) = a$ for all $a \in [0, 1]$;
(d) $T(a, b) \le T(c, d)$ whenever $a \le c$ and $b \le d$ for all $a, b, c, d \in [0, 1]$.

Typical examples of continuous t-norms are $T_P(a, b) = ab$, $T_M(a, b) = \min(a, b)$ and $T_L(a, b) = \max(a + b - 1, 0)$ (the Lukasiewicz t-norm). Recall (see [9,10]) that if T is a t-norm and $\{x_n\}$ is a given sequence of numbers in [0, 1], then $T_{i=1}^{n} x_i$ is defined recurrently by $T_{i=1}^{1} x_i = x_1$ and $T_{i=1}^{n} x_i = T(T_{i=1}^{n-1} x_i, x_n)$ for $n \ge 2$. $T_{i=n}^{\infty} x_i$ is defined as $T_{i=1}^{\infty} x_{n+i-1}$. It is known [10] that for the Lukasiewicz t-norm the following implication holds:

$$\lim_{n\to\infty} (T_L)_{i=1}^{\infty} x_{n+i-1} = 1 \Longleftrightarrow \sum_{n=1}^{\infty} (1 - x_n) < \infty.$$

Definition 1.2 ([8]). A *random normed space* (for short, RN-space) is a triple (X, μ, T_M), where X is a vector space and μ is a mapping from X into D^+ such that the following conditions hold:

(RN_1) $\mu_x(t) = \varepsilon_0(t)$ for all $t > 0$ if and only if $x = 0$;
(RN_2) $\mu_{\alpha x}(t) = \mu_x\left(\frac{t}{|\alpha|}\right)$ for all $x \in X$, $\alpha \neq 0$;
(RN_3) $\mu_{x+y}(t + s) \ge T_M(\mu_x(t), \mu_y(s))$ for all $x, y \in X$ and all $t, s > 0$.

Every normed space $(X, \|\cdot\|)$ defines a random normed space (X, μ, T_M), where

$$\mu_x(t) = \frac{t}{t + \|x\|}$$

for all $t > 0$. This space is called the induced random normed space.

Definition 1.3. Let (X, μ, T) be an RN-space.

(1) A sequence $\{x_n\}$ in X is said to be *convergent* to x in X if, for every $\epsilon > 0$ and $\lambda > 0$, there exists a positive integer N such that $\mu_{x_n - x}(\epsilon) > 1 - \lambda$ whenever $n \ge N$.
(2) A sequence $\{x_n\}$ in X is called a *Cauchy sequence* if, for every $\epsilon > 0$ and $\lambda > 0$, there exists a positive integer N such that $\mu_{x_n - x_m}(\epsilon) > 1 - \lambda$ whenever $n \ge m \ge N$.
(3) An RN-space (X, μ, T) is said to be *complete* if and only if every Cauchy sequence in X is convergent to a point in X.

Theorem 1.4 ([7]). *If (X, μ, T) is an RN-space and $\{x_n\}$ is a sequence such that $x_n \to x$, then $\lim_{n\to\infty} \mu_{x_n}(t) = \mu_x(t)$ almost everywhere.*

Definition 1.5. A *random normed algebra* is a random normed space with algebraic structure such that (RN_4)$\mu_{xy}(ts) \ge \mu_x(t)\mu_y(s)$ for all $x, y \in X$ and all $t, s > 0$.

Example 1.6. Every normed algebra $(X, \|.\|)$ defines a random normed algebra (X, μ, T_M), where

$$\mu_x(t) = \frac{t}{t + \|x\|}$$

for all $t > 0$. This space is called the induced random normed algebra.

Definition 1.7. Let (X, μ, T_M) and (Y, μ, T_M) be random normed algebras. An $\mathbb{R}$-linear mapping $f : X \to Y$ is called a *random homomorphism* if $f(xy) = f(x)f(y)$ for all $x, y \in X$.

Let X be a set. A function $d : X \times X \to [0, \infty]$ is called a *generalized metric* on X if d satisfies

(1) $d(x, y) = 0$ if and only if $x = y$;
(2) $d(x, y) = d(y, x)$ for all $x, y \in X$;
(3) $d(x, z) \le d(x, y) + d(y, z)$ for all $x, y, z \in X$.

We recall a fundamental result in fixed point theory.

Theorem 1.8 ([11,12]). *Let (X, d) be a complete generalized metric space and let $J : X \to X$ be a strictly contractive mapping with Lipschitz constant $L < 1$. Then for each given element $x \in X$, either*

$$d(J^n x, J^{n+1} x) = \infty$$

for all nonnegative integers n or there exists a positive integer n_0 such that

(1) $d(J^n x, J^{n+1} x) < \infty, \ \forall n \ge n_0$;

(2) *the sequence* $\{J^n x\}$ *converges to a fixed point* y^* *of* J;
(3) y^* *is the unique fixed point of* J *in the set* $Y = \{y \in X \mid d(J^{n_0}x, y) < \infty\}$;
(4) $d(y, y^*) \le \frac{1}{1-L} d(y, Jy)$ *for all* $y \in Y$.

The stability problem of functional equations originated from a question of Ulam [13] concerning the stability of group homomorphisms. Hyers [14] gave a first affirmative partial answer to the question of Ulam for Banach spaces. Hyers' theorem was generalized by Aoki [15] for additive mappings and by Rassias [16] for linear mappings by considering an unbounded Cauchy difference. The paper of Rassias [16] has had a lot of influence in the development of what we call *generalized Ulam–Hyers stability* of functional equations. A generalization of the Rassias theorem was obtained by Găvruta [17] by replacing the unbounded Cauchy difference by a general control function in the spirit of the approach of Rassias.

On the other hand, in 1982–1998, Rassias generalized the Ulam–Hyers stability result by presenting a weaker condition controlled by a product of different powers of norms.

Theorem 1.9 (*[18–24]*)**.** *Assume that there exist constants* $\Theta \ge 0$ *and* $p_1, p_2 \in \mathbb{R}$ *such that* $p = p_1 + p_2 \ne 1$, *and* $f : E \to E'$ *is a mapping from a normed space* E *into a Banach space* E' *such that the inequality*

$$\|f(x+y) - f(x) - f(y)\| \le \epsilon \|x\|^{p_1} \|y\|^{p_2}$$

for all $x, y \in E$. *Then there exists a unique additive mapping* $T : E \to E'$ *such that*

$$\|f(x) - L(x)\| \le \frac{\Theta}{2 - 2^p} \|x\|^p$$

for all $x \in E$.

The control function $\|x\|^p \cdot \|y\|^q + \|x\|^{p+q} + \|y\|^{p+q}$ was introduced by Rassias [25] and was used in several papers (see [26–31]). The stability problems of several functional equations have been extensively investigated by a number of authors and there are many interesting results concerning this problem (see [32–41]).

In 1996, Isac and Rassias [42] were the first to provide applications of stability theory of functional equations for the proof of new fixed point theorems with applications. By using fixed point methods, the stability problems of several functional equations have been extensively investigated by a number of authors (see [5,43,44]).

The generalized Ulam–Hyers stability of different functional equations in random normed and fuzzy normed spaces has been studied recently in [6,45].

Using the fixed point method, we prove the generalized Ulam–Hyers stability of random homomorphisms in random normed algebras, associated with the Cauchy functional equation $f(x+y) = f(x) + f(y)$.

Throughout this work, assume that (X, μ, T_M) is a random normed algebra and that (Y, μ, T_M) is a complete random normed algebra.

2. Generalized Ulam–Hyers stability of random homomorphisms in random normed algebras

Using the fixed point method, we prove the generalized Ulam–Hyers stability of random homomorphisms associated with the Cauchy functional equation.

Theorem 2.1. *Let* $\varphi : X^2 \to [0, \infty)$ *be a function such that there exists a constant* $0 < L < \frac{1}{2}$ *with*

$$\varphi(x, y) \le \frac{L}{2} \varphi(2x, 2y)$$

for all $x, y \in X$. *Let* $f : X \to Y$ *be a mapping satisfying*

$$\mu_{f(rx+ry) - rf(x) - rf(y)}(t) \ge \frac{t}{t + \varphi(x, y)}, \tag{2.1}$$

$$\mu_{f(xy) - f(x)f(y)}(t) \ge \frac{t}{t + \varphi(x, y)} \tag{2.2}$$

for all $r \in \mathbb{R}$, *all* $x, y \in X$ *and all* $t > 0$. *Then*

$$H(x) := \lim_{n \to \infty} 2^n f\left(\frac{x}{2^n}\right)$$

exists for each $x \in X$ *and defines a random homomorphism* $H : X \to Y$ *such that*

$$\mu_{f(x) - H(x)}(t) \ge \frac{(2 - 2L)t}{(2 - 2L)t + L\varphi(x, x)} \tag{2.3}$$

for all $x \in X$ *and all* $t > 0$.

Proof. Letting $y = x$ and $r = 1$ in (2.1), we get

$$\mu_{f(2x)-2f(x)}(t) \geq \frac{t}{t+\varphi(x,x)}$$

for all $x \in X$ and all $t > 0$. So

$$\mu_{f(x)-2f\left(\frac{x}{2}\right)}(t) \geq \frac{t}{t+\varphi\left(\frac{x}{2},\frac{x}{2}\right)} \geq \frac{2t}{2t+L\varphi(x,x)} \tag{2.4}$$

for all $x \in X$ and all $t > 0$.

Consider the set

$$S := \{g : X \to Y\}$$

and introduce a generalized metric on S:

$$d(g,h) = \inf\left\{\nu \in \mathbb{R}_+ : \mu_{g(x)-h(x)}(\nu t) \geq \frac{t}{t+\varphi(x,x)},\ \forall x \in X, \forall t > 0\right\},$$

where, as usual, $\inf \phi = +\infty$. It is easy to show that (S, d) is complete (see the proof of [6, Lemma 2.1]).

Now we consider the linear mapping $J : S \to S$ such that

$$Jg(x) := 2g\left(\frac{x}{2}\right)$$

for all $x \in X$.

Let $g, h \in S$ be given such that $d(g,h) = \varepsilon$. Then

$$\mu_{g(x)-h(x)}(\varepsilon t) \geq \frac{t}{t+\varphi(x,x)}$$

for all $x \in X$ and all $t > 0$. Hence

$$\begin{aligned}\mu_{Jg(x)-Jh(x)}(L\varepsilon t) &= \mu_{2g\left(\frac{x}{2}\right)-2h\left(\frac{x}{2}\right)}(L\varepsilon t) = \mu_{g\left(\frac{x}{2}\right)-h\left(\frac{x}{2}\right)}\left(\frac{L}{2}\varepsilon t\right)\\ &\geq \frac{\frac{Lt}{2}}{\frac{Lt}{2}+\varphi\left(\frac{x}{2},\frac{x}{2}\right)} \geq \frac{\frac{Lt}{2}}{\frac{Lt}{2}+\frac{L}{2}\varphi(x,x)} = \frac{t}{t+\varphi(x,x)}\end{aligned}$$

for all $x \in X$ and all $t > 0$. So $d(g,h) = \varepsilon$ implies that $d(Jg, Jh) \leq L\varepsilon$. This means that

$$d(Jg, Jh) \leq Ld(g,h)$$

for all $g, h \in S$.

It follows from (2.4) that

$$\mu_{f(x)-2f\left(\frac{x}{2}\right)}\left(\frac{L}{2}t\right) \geq \frac{t}{t+\varphi(x,x)}$$

for all $x \in X$ and all $t > 0$. So $d(f, Jf) \leq \frac{L}{2}$.

By Theorem 1.8, there exists a mapping $H : X \to Y$ satisfying the following:

(1) H is a fixed point of J, i.e.,

$$H\left(\frac{x}{2}\right) = \frac{1}{2}H(x) \tag{2.5}$$

for all $x \in X$. The mapping H is a unique fixed point of J in the set

$$M = \{g \in S : d(f,g) < \infty\}.$$

This implies that H is a unique mapping satisfying (2.5) such that there exists a $\nu \in (0,\infty)$ satisfying

$$\mu_{f(x)-H(x)}(\nu t) \geq \frac{t}{t+\varphi(x,x)}$$

for all $x \in X$ and all $t > 0$.

(2) $d(J^n f, H) \to 0$ as $n \to \infty$. This implies the equality

$$\lim_{n\to\infty} 2^n f\left(\frac{x}{2^n}\right) = H(x)$$

for all $x \in X$.

(3) $d(f, H) \le \frac{1}{1-L} d(f, Jf)$, which implies the inequality

$$d(f, H) \le \frac{L}{2-2L}.$$

This implies that the inequality (2.3) holds.

Let $r = 1$ in (2.1). By (2.1),

$$\mu_{2^n f\left(\frac{x}{2^n}+\frac{y}{2^n}\right)-2^n f\left(\frac{x}{2^n}\right)-2^n f\left(\frac{y}{2^n}\right)}\left(2^n t\right) \ge \frac{t}{t+\varphi\left(\frac{x}{2^n}, \frac{y}{2^n}\right)}$$

for all $x, y \in X$, all $t > 0$ and all $n \in \mathbb{N}$. So

$$\mu_{2^n f\left(\frac{x}{2^n}+\frac{y}{2^n}\right)-2^n f\left(\frac{x}{2^n}\right)-2^n f\left(\frac{y}{2^n}\right)}(t) \ge \frac{\frac{t}{2^n}}{\frac{t}{2^n}+\frac{L^n}{2^n}\varphi(x, y)}$$

for all $x, y \in X$, all $t > 0$ and all $n \in \mathbb{N}$. Since $\lim_{n\to\infty} \frac{\frac{t}{2^n}}{\frac{t}{2^n}+\frac{L^n}{2^n}\varphi(x,y)} = 1$ for all $x, y \in X$ and all $t > 0$,

$$\mu_{H(x+y)-H(x)-H(y)}(t) = 1$$

for all $x, y \in X$ and all $t > 0$. Thus the mapping $H : X \to Y$ is Cauchy additive.

Let $y = 0$ in (2.1). By (2.1),

$$\mu_{2^n f\left(\frac{rx}{2^n}\right)-2^n r f\left(\frac{x}{2^n}\right)}\left(2^n t\right) \ge \frac{t}{t+\varphi\left(\frac{x}{2^n}, 0\right)}$$

for all $r \in \mathbb{R}$, all $x \in X$, all $t > 0$ and all $n \in \mathbb{N}$. So

$$\mu_{2^n f\left(\frac{rx}{2^n}\right)-2^n r f\left(\frac{x}{2^n}\right)}(t) \ge \frac{\frac{t}{2^n}}{\frac{t}{2^n}+\frac{L^n}{2^n}\varphi(x, 0)}$$

for all $r \in \mathbb{R}$, all $x \in X$, all $t > 0$ and all $n \in \mathbb{N}$. Since $\lim_{n\to\infty} \frac{\frac{t}{2^n}}{\frac{t}{2^n}+\frac{L^n}{2^n}\varphi(x,0)} = 1$ for all $x \in X$ and all $t > 0$,

$$\mu_{H(rx)-rH(x)}(t) = 1$$

for all $r \in \mathbb{R}$, all $x \in X$ and all $t > 0$. Thus the additive mapping $H : X \to Y$ is $\mathbb{R}$-linear.

By (2.2),

$$\mu_{4^n f\left(\frac{x}{2^n}\cdot\frac{y}{2^n}\right)-2^n f\left(\frac{x}{2^n}\right)\cdot 2^n f\left(\frac{y}{2^n}\right)}\left(4^n t\right) \ge \frac{t}{t+\varphi\left(\frac{x}{2^n}, \frac{y}{2^n}\right)}$$

for all $x, y \in X$, all $t > 0$ and all $n \in \mathbb{N}$. So

$$\mu_{4^n f\left(\frac{x}{2^n}\cdot\frac{y}{2^n}\right)-2^n f\left(\frac{x}{2^n}\right)\cdot 2^n f\left(\frac{y}{2^n}\right)}(t) \ge \frac{\frac{t}{4^n}}{\frac{t}{4^n}+\frac{L^n}{2^n}\varphi(x, y)}$$

for all $x, y \in X$, all $t > 0$ and all $n \in \mathbb{N}$. Since $\lim_{n\to\infty} \frac{\frac{t}{4^n}}{\frac{t}{4^n}+\frac{L^n}{2^n}\varphi(x,y)} = 1$ for all $x, y \in X$ and all $t > 0$,

$$\mu_{H(xy)-H(x)H(y)}(t) = 1$$

for all $x, y \in X$ and all $t > 0$. Thus the mapping $H : X \to Y$ is multiplicative.

Therefore, there exists a unique random homomorphism $H : X \to Y$ satisfying (2.3). □

Similarly, we can obtain the following. We will omit the proof.

Theorem 2.2. *Let $\varphi : X^2 \to [0, \infty)$ be a function such that there exists a constant $0 < L < 1$ with*

$$\varphi(x, y) \le 2L\varphi\left(\frac{x}{2}, \frac{y}{2}\right)$$

for all $x, y \in X$. Let $f : X \to Y$ be a mapping satisfying (2.1) and (2.2). Then

$$H(x) := \lim_{n\to\infty} \frac{1}{2^n} f\left(2^n x\right)$$

exists for each $x \in X$ and defines a random homomorphism $H : X \to Y$ such that

$$\mu_{f(x)-H(x)}(t) \ge \frac{(2-2L)t}{(2-2L)t+\varphi(x, x)}$$

for all $x \in X$ and all $t > 0$.

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