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On the stability of an AQCQ-functional equation in random normed spaces

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Abstract

In this paper, we prove the Hyers-Ulam stability of the following additive-quadratic-cubic-quartic functional equation

$$\begin{aligned} f(x+2y)+f(x-2y) &= 4f(x+y)+4f(x-y)-6f(x) \\ &\quad +f(2y)+f(-2y)-4f(y)-4f(-y) \end{aligned}$$

in random normed spaces.

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1. Introduction

The stability problem of functional equations originated from a question of Ulam [1] in 1940, concerning the stability of group homomorphisms. Let $(G_1, \cdot)$ be a group and let $(G_2, *, d)$ be a metric group with the metric $d(\cdot\,,\cdot)$. Given $\varepsilon > 0$, does there exist a $\delta > 0$ such that if a mapping $h : G_1 \to G_2$ satisfies the inequality $d(h(x \cdot y), h(x) * h(y)) < \delta$ for all $x, y \in G_1$, then there exists a homomorphism $H : G_1 \to G_2$ with $d(h(x), H(x)) < \varepsilon$ for all $x \in G_1$? In the other words, under what condition does there exists a homomorphism near an approximate homomorphism? The concept of stability for functional equation arises when we replace the functional equation by an inequality which acts as a perturbation of the equation. Hyers [2] gave a first affirmative answer to the question of Ulam for Banach spaces. Let $f : E \to E'$ be a mapping between Banach spaces such that

$$\| f(x+y) - f(x) - f(y) \| \le \delta$$

for all $x, y \in E$ and some $\delta > 0$. Then, there exists a unique additive mapping $T : E \to E'$ such that

$$||f(x) - T(x)|| \le \delta$$

for all $x \in E$. Moreover, if $f(tx)$ is continuous in $t \in \mathbb{R}$ for each fixed $x \in E$, then T is $\mathbb{R}$-linear. In 1978, Th.M. Rassias [3] provided a generalization of the Hyers' theorem that allows the Cauchy difference to be unbounded. In 1991, Gajda [4] answered the question for the case $p > 1$, which was raised by Th.M. Rassias (see [5-11]).

On the other hand, in 1982-1998, J.M. Rassias generalized the Hyers' stability result by presenting a weaker condition controlled by a product of different powers of norms.

Theorem 1.1. ([12-18]). *Assume that there exist constants* $\Theta \geq 0$ *and* $p_1, p_2 \in \mathbb{R}$ *such that* $p = p_1 + p_2 \neq 1$, *and* $f : E \rightarrow E'$ *is a mapping from a normed space E into a Banach space E' such that the inequality*

$$||f(x+y) - f(x) - f(y)|| \leq \varepsilon ||x||^{p_1} ||y||^{p_2}$$

for all $x, y \in E$. *Then, there exists a unique additive mapping* $T : E \rightarrow E'$ *such that*

$$||f(x) - L(x)|| \leq \frac{\Theta}{2 - 2^p} ||x||^p$$

for all $\times \in E$.

The control function $||x||^p \cdot ||y||^q + ||x||^{p+q} + ||y||^{p+q}$ was introduced by Rassias [19] and was used in several papers (see [20-25]).

The functional equation

$$f(x+y) + f(x-y) = 2f(x) + 2f(y) \tag{1.1}$$

is related to a symmetric bi-additive mapping. It is natural that this equation is called a quadratic functional equation. In particular, every solution of the quadratic functional equation (1.1) is said to be a quadratic mapping. It is well known that a mapping f between real vector spaces is quadratic if and only if there exists a unique symmetric bi-additive mapping B such that $f(x) = B(x, x)$ for all x (see [5,26]). The bi-additive mapping B is given by

$$B(x, y) = \frac{1}{4}(f(x+y) - f(x-y)).$$

The Hyers-Ulam stability problem for the quadratic functional equation (1.1) was proved by Skof for mappings $f : A \rightarrow B$, where A is a normed space and B is a Banach space (see [27]). Cholewa [28] noticed that the theorem of Skof is still true if relevant domain A is replaced by an abelian group. In [29], Czerwik proved the Hyers-Ulam stability of the functional equation (1.1). Grabiec [30] has generalized these results mentioned above.

In [31], Jun and Kim considered the following cubic functional equation:

$$f(2x+y) + f(2x-y) = 2f(x+y) + 2f(x-y) + 12f(x). \tag{1.2}$$

It is easy to show that the function $f(x) = x^3$ satisfies the functional equation (1.2), which is called a *cubic functional equation* and every solution of the cubic functional equation is said to be a *cubic mapping*.

In [32], Park and Bae considered the following quartic functional equation

$$f(x+2y) + f(x-2y) = 4[f(x+y) + f(x-y) + 6f(y)] - 6f(x). \tag{1.3}$$

In fact, they proved that a mapping f between two real vector spaces X and Y is a solution of (1:3) if and only if there exists a unique symmetric multi-additive mapping $M : X^4 \rightarrow Y$ such that $f(x) = M(x, x, x, x)$ for all x. It is easy to show that the function $f(x) = x^4$ satisfies the functional equation (1.3), which is called a quartic functional equation (see also [33]). In addition, Kim [34] has obtained the Hyers-Ulam stability for a mixed type of quartic and quadratic functional equation.

It should be noticed that in all these papers, the triangle inequality is expressed by using the strongest triangular norm T_M .

The aim of this paper is to investigate the Hyers-Ulam stability of the additive-quadratic-cubic-quartic functional equation

$$\begin{aligned} f(x+2y)+f(x-2y) &= 4f(x+y)+4f(x-y)-6f(x) \\ &\quad +f(2y)+f(-2y)-4f(y)-4f(-y) \end{aligned} \tag{1.4}$$

in random normed spaces in the sense of Sherstnev under arbitrary continuous t-norms.

In the sequel, we adopt the usual terminology, notations and conventions of the theory of random normed spaces, as in [35-37]. Throughout this paper, Δ^+ is the space of distribution functions, that is, the space of all mappings $F : \mathbb{R} \cup \{-\infty, \infty\} \to [0, 1]$ such that F is left-continuous and non-decreasing on $\mathbb{R}$, $F(0) = 0$ and $F(+\infty) = 1$. D^+ is a subset of Δ^+ consisting of all functions $F \in \Delta^+$ for which $l^- F(+\infty) = 1$, where $l^- f(x)$ denotes the left limit of the function f at the point x, that is, $l^- f(x) = \lim_{t \to x^-} f(t)$. The space Δ^+ is partially ordered by the usual point-wise ordering of functions, i.e., $F \le G$ if and only if $F(t) \le G(t)$ for all t in $\mathbb{R}$. The maximal element for Δ^+ in this order is the distribution function ε_0 given by

$$\varepsilon_0(t) = \begin{cases} 0, & \text{if } t \le 0, \\ 1, & \text{if } t > 0. \end{cases}$$

Definition 1.2. [36]*A mapping* $T : [0, 1] \times [0, 1] \to [0, 1]$ *is a continuous triangular norm (briefly, a continuous t-norm) if T satisfies the following conditions:*

(*a*) *T is commutative and associative;*

(*b*) *T is continuous;*

(*c*) $T(a, 1) = a$ *for all* $a \in [0, 1]$;

(*d*) $T(a, b) \le T(c, d)$ *whenever* $a \le c$ *and* $b \le d$ *for all* $a, b, c, d \in [0, 1]$.

Typical examples of continuous t-norms are $T_P(a, b) = ab$, $T_M(a, b) = \min(a, b)$ and $T_L(a, b) = \max(a+b-1, 0)$ (the Lukasiewicz t-norm). Recall (see [38,39]) that if T is a t-norm and $\{x_n\}$ is a given sequence of numbers in [0, 1], then $T_{i=1}^{n} x_i$ is defined recurrently by $T_{i=1}^{1} x_i = x_1$ and $T_{i=1}^{n} x_i = T(T_{i=1}^{n-1} x_i, x_n)$ for $n \ge 2$. $T_{i=n}^{\infty} x_i$ is defined as $T_{i=1}^{\infty} x_{n+i-1}$. It is known [39] that for the Lukasiewicz t-norm, the following implication holds:

$$\lim_{n\to\infty} (T_L)_{i=1}^{\infty} x_{n+i-1} = 1 \Leftrightarrow \sum_{n=1}^{\infty} (1 - x_n) < \infty$$

Definition 1.3. [37]*A* random normed space *(briefly, RN-space) is a triple* (X, μ, T), *where × is a vector space, T is a continuous t-norm, and* μ *is a mapping from × into* D^+ *such that the following conditions hold:*

(RN_1) $\mu_x(t) = \varepsilon_0(t)$ *for all* $t > 0$ *if and only if* $x = 0$;

(RN_2) $\mu_{\alpha x}(t) = \mu_x(\frac{t}{|\alpha|})$ *for all* $x \in X$, $\alpha \neq 0$;

(RN_3) $\mu_{x+y}(t + s) \ge T(\mu_x(t), \mu_y(s))$ *for all* $x, y \in X$ *and all* $t, s \ge 0$.

Every normed space $(X, ||\cdot||)$ defines a random normed space (X, μ, T_M), where

$$\mu_x(t) = \frac{t}{t + ||x||}$$

for all $t > 0$, and T_M is the minimum t-norm. This space is called the induced random normed space.

Definition 1.4. *Let* (X, μ, T) *be an RN-space.*

(1) *A sequence* $\{x_n\}$ *in* × *is said to be* convergent *to* × *in* × *if, for every* $\varepsilon > 0$ *and* $\lambda > 0$, *there exists a positive integer N such that* $\mu_{x_{n-x}}(\varepsilon) > 1 - \lambda$*whenever* $n \geq N$.

(2) *A sequence* $\{x_n\}$ *in* × *is called a* Cauchy sequence *if, for every* $\varepsilon > 0$ *and* $\lambda > 0$, *there exists a positive integer N such that* $\mu_{x_{n-x_m}}(\varepsilon) > 1 - \lambda$*whenever* $n \geq m \geq N$.

(3) *An RN-space* (X, μ, T) *is said to be* complete *if and only if every Cauchy sequence in* × *is convergent to a point in X.*

Theorem 1.5. [36]*If* (X, μ, T) *is an RN-space and* $\{x_n\}$ *is a sequence such that* $x_n \to x$, *then* $\lim_{n\to\infty} \mu_{x_n}(t) = \mu_x(t)$*almost everywhere.*

Recently, Eshaghi Gordji et al. establish the stability of cubic, quadratic and additive-quadratic functional equations in RN-spaces (see [40-42]).

One can easily show that an odd mapping $f : X \to Y$ satisfies (1.4) if and only if the odd mapping $f : X \to Y$ is an additive-cubic mapping, i.e.,

$$f(x + 2y) + f(x - 2y) = 4f(x + y) + 4f(x - y) - 6f(x).$$

It was shown in [[43], Lemma 2.2] that $g(x) := f(2x) - 8f(x)$ and $h(x) := f(2x) - 2f(x)$ are additive and cubic, respectively, and that $f(x) = \frac{1}{6}h(x) - \frac{1}{6}g(x)$.

One can easily show that an even mapping $f : X \to Y$ satisfies (1.4) if and only if the even mapping $f : X \to Y$ is a quadratic-quartic mapping, i.e.,

$$f(x + 2y) + f(x - 2y) = 4f(x + y) + 4f(x - y) - 6f(x) + 2f(2y) - 8f(y).$$

It was shown in [[44], Lemma 2.1] that $g(x) := f(2x) - 16f(x)$ and $h(x) := f(2x) - 4f(x)$ are quadratic and quartic, respectively, and that $f(x) = \frac{1}{12}h(x) - \frac{1}{12}g(x)$

Lemma 1.6. *Each mapping* $f : X \to Y$ *satisfying* (1.4) *can be realized as the sum of an additive mapping, a quadratic mapping, a cubic mapping and a quartic mapping.*

This paper is organized as follows: In Section 2, we prove the Hyers-Ulam stability of the additive-quadratic-cubic-quartic functional equation (1.4) in RN-spaces for an odd case. In Section 3, we prove the Hyers-Ulam stability of the additive-quadratic-cubic-quartic functional equation (1.4) in RN-spaces for an even case.

Throughout this paper, assume that X is a real vector space and that (X, μ, T) is a complete RN-space.

2.Hyers-Ulam stability of the functional equation (1.4): an odd mapping Case

For a given mapping $f : X \to Y$, we define

$$\begin{aligned} Df(x, y) := {} & f(x + 2y) + f(x - 2y) - 4f(x + y) - 4f(x - y) + 6f(x) \\ & - f(2y) - f(-2y) + 4f(y) + 4f(-y) \end{aligned}$$

for all $x, y \in X$.

In this section, we prove the Hyers-Ulam stability of the functional equation $Df(x, y) = 0$ in complete RN-spaces: an odd mapping case.

Theorem 2.1. *Let* $f : X \to Y$ *be an odd mapping for which there is a* $\rho : X^2 \to D^+$ ($\rho(x, y)$ *is denoted by* $\rho_{x,y}$) *such that*

$$\mu_{Df(x,y)}(t) \geq \rho_{x,y}(t) \tag{2.1}$$

for all $x, y \in X$ *and all* $t > 0$. *If*

$$\lim_{n\to\infty} T_{k=1}^{\infty}(T(\rho_{2^{k+n-1}x,2^{k+n-1}x}(2^{n-3}t), \rho_{2^{k+n}x,2^{k+n-1}x}(2^{n-1}t))) = 1 \tag{2.2}$$

and

$$\lim_{n\to\infty} \rho_{2^n x,2^n y}(2^n t) = 1 \tag{2.3}$$

for all $x, y \in X$ *and all* $t > 0$, *then there exist a unique additive mapping* $A : X \to Y$ *and a unique cubic mapping* $C : X \to Y$ *such that*

$$\mu_{f(2x)-8f(x)-A(x)}(t) \geq T_{k=1}^{\infty}\left(T\left(\rho_{2^{k-1}x,2^{k-1}x}\left(\frac{t}{8}\right), \rho_{2^k x,2^{k-1}x}\left(\frac{t}{2}\right)\right)\right), \tag{2.4}$$

$$\mu_{f(2x)-2f(x)-C(x)}(t) \geq T_{k=1}^{\infty}\left(T\left(\rho_{2^{k-1}x,2^{k-1}x}\left(\frac{t}{8}\right), \rho_{2^k x,2^{k-1}x}\left(\frac{t}{2}\right)\right)\right) \tag{2.5}$$

for all $\times \in X$ *and all* $t > 0$.

Proof. Putting $x = y$ in (2.1), we get

$$\mu_{f(3y)-4f(2y)+5f(y)}(t) \geq \rho_{y,y}(t) \tag{2.6}$$

for all $y \in X$ and all $t > 0$. Replacing x by $2y$ in (2.1), we get

$$\mu_{f(4y)-4f(3y)+6f(2y)-4f(y)}(t) \geq \rho_{2y,y}(t) \tag{2.7}$$

for all $y \in X$ and all $t > 0$. It follows from (2.6) and (2.7) that

$$\begin{aligned}\mu_{f(4x)-10f(2x)+16f(x)}(t) &= \mu_{(4f(3x)-16f(2x)+20f(x))+(f(4x)-4f(3x)+6f(2x)-4f(x))}(t)\\ &\geq T\left(\mu_{4f(3x)-16f(2x)+20f(x)}\left(\frac{t}{2}\right), \mu_{f(4x)-4f(3x)+6f(2x)-4f(x)}\left(\frac{t}{2}\right)\right)\\ &\geq T\left(\rho_{x,x}\left(\frac{t}{8}\right), \rho_{2x,x}\left(\frac{t}{2}\right)\right)\end{aligned} \tag{2.8}$$

for all $x \in X$ and all $t > 0$. Let $g : X \to Y$ be a mapping defined by $g(x) := f(2x) - 8f(x)$. Then we conclude that

$$\mu_{g(2x)-2g(x)}(t) \geq T\left(\rho_{x,x}\left(\frac{t}{8}\right), \rho_{2x,x}\left(\frac{t}{2}\right)\right)$$

for all $x \in X$ and all $t > 0$. Thus, we have

$$\mu_{\frac{g(2x)}{2}-g(x)}(t) \geq T\left(\rho_{x,x}\left(\frac{t}{4}\right), \rho_{2x,x}(t)\right)$$

for all $x \in X$ and all $t > 0$. Hence,

$$\mu_{\frac{g(2^{k+1}x)}{2^{k+1}}-\frac{g(2^k x)}{2^k}}(t) \geq T(\rho_{2^k x,2^k x}(2^{k-2}t), \rho_{2^{k+1}x,2^k x}(2^k t))$$

for all $x \in X$, all $t > 0$ and all $k \in \mathbb{N}$: From $1 > \frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^n}$, it follows that

$$\begin{aligned}\mu_{\frac{g(2^n x)}{2^n} - g(x)}(t) &\geq T_{k=1}^{n}\left(\mu_{\frac{g(2^k x)}{2^k} - \frac{g(2^{k-1} x)}{2^{k-1}}}\left(\frac{t}{2^k}\right)\right) \\ &\geq T_{k=1}^{n}\left(T\left(\rho_{2^{k-1}x, 2^{k-1}x}\left(\frac{t}{8}\right), \rho_{2^k x, 2^{k-1}x}\left(\frac{t}{2}\right)\right)\right)\end{aligned} \tag{2.9}$$

for all $x \in X$ and all $t > 0$. In order to prove the convergence of the sequence $\{\frac{g(2^n x)}{2^n}\}$, replacing x with $2^m x$ in (2.9), we obtain that

$$\begin{aligned}&\mu_{\frac{g(2^{n+m} x)}{2^{n+m}} - \frac{g(2^m x)}{2^m}}(t) \\ &\geq T_{k=1}^{n}(T(\rho_{2^{k+m-1}x, 2^{k+m-1}x}(2^{m-3}t), \rho_{2^{k+m}x, 2^{k+m-1}x}(2^{m-1}t))).\end{aligned} \tag{2.10}$$

Since the right-hand side of the inequality (2.10) tends to 1 as m and n tend to infinity, the sequence $\{\frac{g(2^n x)}{2^n}\}$ is a Cauchy sequence. Thus, we may define $A(x) = \lim_{n\to\infty} \frac{g(2^n x)}{2^n}$ for all $x \in X$.

Now, we show that A is an additive mapping. Replacing x and y with $2^n x$ and $2^n y$ in (2.1), respectively, we get

$$\mu_{\frac{Df(2^n x, 2^n y)}{2^n}}(t) \geq \rho_{2^n x, 2^n y}(2^n t).$$

Taking the limit as $n \to \infty$, we find that $A : X \to Y$ satisfies (1.4) for all $x, y \in X$. Since $f : X \to Y$ is odd, $A : X \to Y$ is odd. By [[43], Lemma 2.2], the mapping $A : X \to Y$ is additive. Letting the limit as $n \to \infty$ in (2.9), we get (2.4).

Next, we prove the uniqueness of the additive mapping $A : X \to Y$ subject to (2.4). Let us assume that there exists another additive mapping $L : X \to Y$ which satisfies (2.4). Since $A(2^n x) = 2^n A(x)$, $L(2^n x) = 2^n L(x)$ for all $x \in X$ and all $n \in \mathbb{N}$, from (2.4), it follows that

$$\begin{aligned}&\mu_{A(x)-L(x)}(2t) = \mu_{A(2^n x)-L(2^n x)}(2^{n+1}t) \\ &\geq T(\mu_{A(2^n x)-g(2^n x)}(2^n t), \mu_{g(2^n x)-L(2^n x)}(2^n t)) \\ &\geq T(T_{k=1}^{\infty}(T(\rho_{2^{n+k-1}x, 2^{n+k-1}x}(2^{n-3}t), \rho_{2^{n+k}x, 2^{n+k-1}x}(2^{n-1}t))), \\ &\quad T_{k=1}^{\infty}(T(\rho_{2^{n+k-1}x, 2^{n+k-1}x}(2^{n-3}t), \rho_{2^{n+k}x, 2^{n+k-1}x}(2^{n-1}t)))\end{aligned} \tag{2.11}$$

for all $x \in X$ and all $t > 0$. Letting $n \to \infty$ in (2.11), we conclude that $A = L$.

Let $h : X \to Y$ be a mapping defined by $h(x) := f(2x) - 2f(x)$. Then, we conclude that

$$\mu_{h(2x)-8h(x)}(t) \geq T\left(\rho_{x,x}\left(\frac{t}{8}\right), \rho_{2x,x}\left(\frac{t}{2}\right)\right)$$

for all $x \in X$ and all $t > 0$. Thus, we have

$$\mu_{\frac{h(2x)}{8} - h(x)}(t) \geq T(\rho_{x,x}(t), \rho_{2x,x}(4t))$$

for all $x \in X$ and all $t > 0$. Hence,

$$\mu_{\frac{h(2^{k+1}x)}{8^{k+1}} - \frac{h(2^k x)}{8^k}}(t) \geq T(\rho_{2^k x, 2^k x}(8^k t), \rho_{2^{k+1}x, 2^k x}(4 \cdot 8^k t))$$

for all $x \in X$, all $t > 0$ and all $k \in \mathbb{N}$: From $1 > \frac{1}{8} + \frac{1}{8^2} + \cdots + \frac{1}{8^n}$, it follows that

$$\begin{aligned}\mu_{\frac{h(2^n x)}{8^n} - h(x)}(t) &\geq T_{k=1}^{n}\left(\mu_{\frac{h(2^k x)}{8^k} - \frac{h(2^{k-1} x)}{8^{k-1}}}\left(\frac{t}{8^k}\right)\right) \\ &\geq T_{k=1}^{n}\left(T\left(\rho_{2^{k-1}x, 2^{k-1}x}\left(\frac{t}{8}\right), \rho_{2^k x, 2^{k-1}x}\left(\frac{t}{2}\right)\right)\right)\end{aligned} \tag{2.12}$$

for all $x \in X$ and all $t > 0$. In order to prove the convergence of the sequence $\{\frac{h(2^n x)}{8^n}\}$, replacing x with $2^m x$ in (2.12), we obtain that

$$\begin{aligned}&\mu_{\frac{h(2^{n+m} x)}{8^{n+m}} - \frac{h(2^m x)}{8^m}}(t) \\ &\geq T_{k=1}^{n}(T(\rho_{2^{k+m-1}x, 2^{k+m-1}x}(8^{m-1}t), \rho_{2^{k+m}x, 2^{k+m-1}x}(4 \cdot 8^{m-1}t))).\end{aligned} \tag{2.13}$$

Since the right-hand side of the inequality (2.13) tends to 1 as m and n tend to infinity, the sequence $\{\frac{h(2^n x)}{8^n}\}$ is a Cauchy sequence. Thus, we may define $C(x) = \lim_{n\to\infty} \frac{h(2^n x)}{8^n}$ for all $x \in X$.

Now, we show that C is a cubic mapping. Replacing x and y with $2^n x$ and $2^n y$ in (2.1), respectively, we get

$$\mu_{\frac{Df(2^n x, 2^n y)}{8^n}}(t) \geq \rho_{2^n x, 2^n y}(8^n t) \geq \rho_{2^n x, 2^n y}(2^n t).$$

Taking the limit as $n \to \infty$, we find that $C : X \to Y$ satisfies (1.4) for all $x, y \in X$. Since $f : X \to Y$ is odd, $C : X \to Y$ is odd. By [[43], Lemma 2.2], the mapping $C : X \to Y$ is cubic. Letting the limit as $n \to \infty$ in (2.12), we get (2.5).

Finally, we prove the uniqueness of the cubic mapping $C : X \to Y$ subject to (2.5). Let us assume that there exists another cubic mapping $L : X \to Y$ which satisfies (2.5). Since $C(2^n x) = 8^n C(x)$, $L(2^n x) = 8^n L(x)$ for all $x \in X$ and all $n \in \mathbb{N}$, from (2.5), it follows that

$$\begin{aligned}&\mu_{C(x)-L(x)}(2t) \\ &= \mu_{C(2^n x)-L(2^n x)}(2 \cdot 8^n t) \\ &\geq T(\mu_{C(2^n x)-h(2^n x)}(8^n t), \mu_{h(2^n x)-L(2^n x)}(8^n t)) \\ &\geq T(T_{k=1}^{\infty}(T(\rho_{2^{n+k-1}x, 2^{n+k-1}x}(8^{n-1}t), \rho_{2^{n+k}x, 2^{n+k-1}x}(4 \cdot 8^{n-1}t))), \\ &\quad T_{k=1}^{\infty}(T(\rho_{2^{n+k-1}x, 2^{n+k-1}x}(8^{n-1}t), \rho_{2^{n+k}x, 2^{n+k-1}x}(4 \cdot 8^{n-1}t))) \\ &\geq T(T_{k=1}^{\infty}(T(\rho_{2^{n+k-1}x, 2^{n+k-1}x}(2^{n-3}t), \rho_{2^{n+k}x, 2^{n+k-1}x}))), \\ &\quad T_{k=1}^{\infty}(T(\rho_{2^{n+k-1}x, 2^{n+k-1}x}(2^{n-3}t), \rho_{2^{n+k}x, 2^{n+k-1}x}(2^{n-1}t)))\end{aligned} \tag{2.14}$$

for all $x \in X$ and all $t > 0$. Letting $n \to \infty$ in (2.14), we conclude that $C = L$, as desired. □

Similarly, one can obtain the following result.

Theorem 2.2. *Let $f : X \to Y$ be an odd mapping for which there is a $\rho : X^2 \to D^+$ ($\rho(x, y)$ is denoted by $\rho_{x, y}$) satisfying* (2.1). *If*

$$\lim_{n\to\infty} T_{k=1}^{\infty}\left(T\left(\rho_{\frac{x}{2^{k+n}}, \frac{x}{2^{k+n}}}\left(\frac{t}{8^{n+2k}}\right), \rho_{\frac{x}{2^{k+n-1}}, \frac{x}{2^{k+n}}}\left(\frac{4t}{8^{n+2k}}\right)\right)\right) = 1$$

and

$$\lim_{n\to\infty} \rho_{\frac{x}{2^n},\frac{y}{2^n}}\left(\frac{t}{8^n}\right) = 1$$

for all x, y ∈ X and all t > 0, then there exist a unique additive mapping $A : X \to Y$ and a unique cubic mapping $C : X \to Y$ such that

$$\mu_{f(2x)-8f(x)-A(x)}(t) \geq T_{k=1}^{\infty}\left(T\left(\rho_{\frac{x}{2^k},\frac{x}{2^k}}\left(\frac{t}{2^{2k+1}}\right), \rho_{\frac{x}{2^{k-1}},\frac{x}{2^k}}\left(\frac{t}{2^{2k-1}}\right)\right)\right),$$
$$\mu_{f(2x)-2f(x)-C(x)}(t) \geq T_{k=1}^{\infty}\left(T\left(\rho_{\frac{x}{2^k},\frac{x}{2^k}}\left(\frac{t}{8^{2k}}\right), \rho_{\frac{x}{2^{k-1}},\frac{x}{2^k}}\left(\frac{4t}{8^{2k}}\right)\right)\right)$$

for all × ∈ X and all t > 0.

3. Hyers-ulam stability of the functional equation (1.4): an even mapping case

In this section, we prove the Hyers-Ulam stability of the functional equation $Df(x, y) = 0$ in complete RN-spaces: an even mapping case.

Theorem 3.1. *Let $f : X \to Y$ be an even mapping for which there is a $\rho : X^2 \to D^+$ ($\rho(x, y)$ is denoted by $\rho_{x,y}$) satisfying $f(0) = 0$ and (2.1). If*

$$\lim_{n\to\infty} T_{k=1}^{\infty}(T(\rho_{2^{k+n-1}x,2^{k+n-1}x}(2\cdot 4^{n-2}t), \rho_{2^{k+n}x,2^{k+n-1}x}(2\cdot 4^{n-1}t))) = 1 \tag{3.1}$$

and

$$\lim_{n\to\infty} \rho_{2^n x,2^n y}(4^n t) = 1 \tag{3.2}$$

for all x, y ∈ X and all t > 0, then there exist a unique quadratic mapping $P : X \to Y$ and a unique quartic mapping $Q : X \to Y$ such that

$$\mu_{f(2x)-16f(x)-P(x)}(t) \geq T_{k=1}^{\infty}\left(T\left(\rho_{2^{k-1}x,2^{k-1}x}\left(\frac{t}{8}\right), \rho_{2^k x,2^{k-1}x}\left(\frac{t}{2}\right)\right)\right), \tag{3.3}$$

$$\mu_{f(2x)-4f(x)-Q(x)}(t) \geq T_{k=1}^{\infty}\left(T\left(\rho_{2^{k-1}x,2^{k-1}x}\left(\frac{t}{8}\right), \rho_{2^k x,2^{k-1}x}\left(\frac{t}{2}\right)\right)\right) \tag{3.4}$$

for all × ∈ X and all t > 0.

Proof. Putting $x = y$ in (2.1), we get

$$\mu_{f(3y)-6f(2y)+15f(y)}(t) \geq \rho_{y,y}(t) \tag{3.5}$$

for all $y \in X$ and all $t > 0$. Replacing x by $2y$ in (2.1), we get

$$\mu_{f(4y)-4f(3y)+4f(2y)+4f(y)}(t) \geq \rho_{2y,y}(t) \tag{3.6}$$

for all $y \in X$ and all $t > 0$. It follows from (3.5) and (3.6) that

$$\begin{aligned}
&\mu_{f(4x)-20f(2x)+64f(x)}(t)\\
&\quad= \mu_{(4f(3x)-24f(2x)+60f(x))+(f(4x)-4f(3x)+4f(2x)+4f(x))}(t)\\
&\quad\geq T\left(\mu_{4f(3x)-24f(2x)+60f(x)}\left(\frac{t}{2}\right), \mu_{f(4x)-4f(3x)+4f(2x)+4f(x)}\left(\frac{t}{2}\right)\right)\\
&\quad\geq T\left(\rho_{x,x}\left(\frac{t}{8}\right), \rho_{2x,x}\left(\frac{t}{2}\right)\right)
\end{aligned} \tag{3.7}$$

for all $x \in X$ and all $t > 0$. Let $g : X \to Y$ be a mapping defined by $g(x) := f(2x) - 16f(x)$. Then we conclude that

$$\mu_{g(2x)-4g(x)}(t) \geq T\left(\rho_{x,x}\left(\frac{t}{8}\right), \rho_{2x,x}\left(\frac{t}{2}\right)\right)$$

for all $x \in X$ and all $t > 0$. Thus, we have

$$\mu_{\frac{g(2x)}{4}-g(x)}(t) \geq T\left(\rho_{x,x}\left(\frac{t}{2}\right), \rho_{2x,x}(2t)\right)$$

for all $x \in X$ and all $t > 0$. Hence,

$$\mu_{\frac{g(2^{k+1}x)}{4^{k+1}}-\frac{g(2^kx)}{4^k}}(t) \geq T(\rho_{2^kx,2^kx}(2\cdot 4^{k-1}t), \rho_{2^{k+1}x,2^kx}(2\cdot 4^kt))$$

for all $x \in X$, all $t > 0$ and all $k \in \mathbb{N}$. From $1 > \frac{1}{4} + \frac{1}{4^2} + \cdots + \frac{1}{4^n}$, it follows that

$$\begin{aligned}
\mu_{\frac{g(2^nx)}{4^n}-g(x)}(t) &\geq T_{k=1}^{n}\left(\mu_{\frac{g(2^kx)}{4^k}-\frac{g(2^{k-1}x)}{4^{k-1}}}\left(\frac{t}{4^k}\right)\right)\\
&\geq T_{k=1}^{n}\left(T\left(\rho_{2^{k-1}x,2^{k-1}x}\left(\frac{t}{8}\right), \rho_{2^kx,2^{k-1}x}\left(\frac{t}{2}\right)\right)\right)
\end{aligned} \tag{3.8}$$

for all $x \in X$ and all $t > 0$. In order to prove the convergence of the sequence $\{\frac{g(2^nx)}{4^n}\}$, replacing x with 2^mx in (3.8), we obtain that

$$\begin{aligned}
&\mu_{\frac{g(2^{n+m}x)}{4^{n+m}}-\frac{g(2^mx)}{4^m}}(t)\\
&\quad\geq T_{k=1}^{n}(T(\rho_{2^{k+m-1}x,2^{k+m-1}x}(2\cdot 4^{m-2}t), \rho_{2^{k+m}x,2^{k+m-1}x}(2\cdot 4^{m-1}t))).
\end{aligned} \tag{3.9}$$

Since the right-hand side of the inequality (3.9) tends to 1 as m and n tend to infinity, the sequence $\{\frac{g(2^nx)}{4^n}\}$ is a Cauchy sequence. Thus, we may define $P(x) = \lim_{n\to\infty} \frac{g(2^nx)}{4^n}$ for all $x \in X$.

Now, we show that P is a quadratic mapping. Replacing x and y with 2^nx and 2^ny in (2.1), respectively, we get

$$\mu_{\frac{Df(2^nx,2^ny)}{4^n}}(t) \geq \rho_{2^nx,2^ny}(4^nt).$$

Taking the limit as $n \to \infty$, we find that $P : X \to Y$ satisfies (1.4) for all $x, y \in X$. Since $f : X \to Y$ is even, $P : X \to Y$ is even. By [[44], Lemma 2.1], the mapping $P : X \to Y$ is quadratic. Letting the limit as $n \to \infty$ in (3.8), we get (3.3).

Next, we prove the uniqueness of the quadratic mapping $P : X \to Y$ subject to (3.3). Let us assume that there exists another quadratic mapping $L : X \to Y$, which satisfies

(3.3). Since $P(2^n x) = 4^n P(x)$, $L(2^n x) = 4^n L(x)$ for all $x \in X$ and all $n \in \mathbb{N}$, from (3.3), it follows that

$$\begin{aligned} &\mu_{P(x)-L(x)}(2t) = \mu_{P(2^n x)-L(2^n x)}(2 \cdot 4^n t) \\ &\geq T(\mu_{P(2^n x)-g(2^n x)}(4^n t), \mu_{g(2^n x)-L(2^n x)}(4^n t)) \\ &\geq T(T_{k=1}^{\infty}(T(\rho_{2^{n+k-1}x,2^{n+k-1}x}(2 \cdot 4^{n-2}t), \rho_{2^{n+k}x,2^{n+k-1}x}(2 \cdot 4^{n-1}t))), \\ &\quad T_{k=1}^{\infty}(T(\rho_{2^{n+k-1}x,2^{n+k-1}x}(2 \cdot 4^{n-2}t), \rho_{2^{n+k}x,2^{n+k-1}x}(2 \cdot 4^{n-1}t)))) \end{aligned} \tag{3.10}$$

for all $x \in X$ and all $t > 0$. Letting $n \to \infty$ in (3.10), we conclude that $P = L$.

Let $h : X \to Y$ be a mapping defined by $h(x) := f(2x) - 4f(x)$. Then, we conclude that

$$\mu_{h(2x)-16h(x)}(t) \geq T\left(\rho_{x,x}\left(\frac{t}{8}\right), \rho_{2x,x}\left(\frac{t}{2}\right)\right)$$

for all $x \in X$ and all $t > 0$. Thus, we have

$$\mu_{\frac{h(2x)}{16}-h(x)}(t) \geq T(\rho_{x,x}(2t), \rho_{2x,x}(8t))$$

for all $x \in X$ and all $t > 0$. Hence,

$$\mu_{\frac{h(2^{k+1}x)}{16^{k+1}}-\frac{h(2^k x)}{16^k}}(t) \geq T(\rho_{2^k x,2^k x}(2 \cdot 16^k t), \rho_{2^{k+1}x,2^k x}(8 \cdot 16^k t))$$

for all $x \in X$, all $t > 0$ and all $k \in \mathbb{N}$. From $1 > \frac{1}{16} + \frac{1}{16^2} + \cdots + \frac{1}{16^n}$, it follows that

$$\begin{aligned} \mu_{\frac{h(2^n x)}{16^n}-h(x)}(t) &\geq T_{k=1}^{n}\left(\mu_{\frac{h(2^k x)}{16^k}-\frac{h(2^{k-1}x)}{16^{k-1}}}\left(\frac{t}{16^k}\right)\right) \\ &\geq T_{k=1}^{n}\left(T\left(\rho_{2^{k-1}x,2^{k-1}x}\left(\frac{t}{8}\right), \rho_{2^k x,2^{k-1}x}\left(\frac{t}{2}\right)\right)\right) \end{aligned} \tag{3.11}$$

for all $x \in X$ and all $t > 0$. In order to prove the convergence of the sequence $\{\frac{h(2^n x)}{16^n}\}$, replacing x with $2^m x$ in (3.11), we obtain that

$$\begin{aligned} &\mu_{\frac{h(2^{n+m}x)}{16^{n+m}}-\frac{h(2^m x)}{16^m}}(t) \\ &\geq T_{k=1}^{n}(T(\rho_{2^{k+m-1}x,2^{k+m-1}x}(2 \cdot 16^{m-1}t), \rho_{2^{k+m}x,2^{k+m-1}x}(8 \cdot 16^{m-1}t))). \end{aligned} \tag{3.12}$$

Since the right-hand side of the inequality (3.12) tends to 1 as m and n tend to infinity, the sequence $\{\frac{h(2^n x)}{16^n}\}$ is a Cauchy sequence. Thus, we may define $Q(x) = \lim_{n\to\infty} \frac{h(2^n x)}{16^n} x \in X$.

Now, we show that Q is a quartic mapping. Replacing x and y with $2^n x$ and $2^n y$ in (2.1), respectively, we get

$$\mu_{\frac{Df(2^n x,2^n y)}{16^n}}(t) \geq \rho_{2^n x,2^n y}(16^n t) \geq \rho_{2^n x,2^n y}(4^n t).$$

Taking the limit as $n \to \infty$, we find that $Q : X \to Y$ satisfies (1.4) for all $x, y \in X$. Since $f : X \to Y$ is even, $Q : X \to Y$ is even. By [[44], Lemma 2.1], the mapping $Q : X \to Y$ is quartic. Letting the limit as $n \to \infty$ in (3.11), we get (3.4).

Finally, we prove the uniqueness of the quartic mapping $Q : X \to Y$ subject to (3.4). Let us assume that there exists another quartic mapping $L : X \to Y$, which satisfies (3.4). Since $Q(2^n x) = 16^n Q(x)$, $L(2^n x) = 16^n L(x)$ for all $x \in X$ and all $n \in \mathbb{N}$, from (3.4), it follows that

$$\begin{aligned}\mu_{Q(x)-L(x)}(2t) &= \mu_{Q(2^n x)-L(2^n x)}(2\cdot 16^n t)\\ &\geq T(\mu_{Q(2^n x)-h(2^n x)}(16^n t), \mu_{h(2^n x)-L(2^n x)}(16^n t))\\ &\geq T(T_{k=1}^{\infty}(T(\rho_{2^{n+k-1}x, 2^{n+k-1}x}(2\cdot 16^{n-1}t), \rho_{2^{n+k}x, 2^{n+k-1}x}(8\cdot 16^{n-1}t))),\\ &\quad T_{k=1}^{\infty}(T(\rho_{2^{n+k-1}x, 2^{n+k-1}x}(2\cdot 16^{n-1}t), \rho_{2^{n+k}x, 2^{n+k-1}x}(8\cdot 16^{n-1}t)))\\ &\geq T(T_{k=1}^{\infty}(T(\rho_{2^{n+k-1}x, 2^{n+k-1}x}(2\cdot 4^{n-2}t), \rho_{2^{n+k}x, 2^{n+k-1}x}(2\cdot 4^{n-1}t))),\\ &\quad T_{k=1}^{\infty}(T(\rho_{2^{n+k-1}x, 2^{n+k-1}x}(2\cdot 4^{n-2}t), \rho_{2^{n+k}x, 2^{n+k-1}x}(2\cdot 4^{n-1}t))))\end{aligned} \tag{3.13}$$

for all $x \in X$ and all $t > 0$. Letting $n \to \infty$ in (3.13), we conclude that $Q = L$, as desired. □

Similarly, one can obtain the following result.

Theorem 3.2. *Let $f : X \to Y$ be an even mapping for which there is a $\rho : X^2 \to D^+$ (ρ (x, y) is denoted by $\rho\ x, y$) satisfying $f(0) = 0$ and* (2.1). *If*

$$\lim_{n\to\infty} T_{k=1}^{\infty}\left(T\left(\rho_{\frac{x}{2^{k+n}}, \frac{x}{2^{k+n}}}\left(\frac{2t}{16^{n+2k}}\right), \rho_{\frac{x}{2^{k+n-1}}, \frac{x}{2^{k+n}}}\left(\frac{8t}{16^{n+2k}}\right)\right)\right) = 1$$

and

$$\lim_{n\to\infty} \rho_{\frac{x}{2^n}, \frac{y}{2^n}}\left(\frac{t}{16^n}\right) = 1$$

for all $x, y \in X$ and all $t > 0$, then there exist a unique quadratic mapping $P : X \to Y$ and a unique quartic mapping $Q : X \to Y$ such that

$$\mu_{f(2x)-16f(x)-P(x)}(t) \geq T_{k=1}^{\infty}\left(T\left(\rho_{\frac{x}{2^k}, \frac{x}{2^k}}\left(\frac{2t}{4^{2k+1}}\right), \rho_{\frac{x}{2^{k-1}}, \frac{x}{2^k}}\left(\frac{2t}{4^{2k}}\right)\right)\right),$$

$$\mu_{f(2x)-4f(x)-Q(x)}(t) \geq T_{k=1}^{\infty}\left(T\left(\rho_{\frac{x}{2^k}, \frac{x}{2^k}}\left(\frac{2t}{16^{2k}}\right), \rho_{\frac{x}{2^{k-1}}, \frac{x}{2^k}}\left(\frac{8t}{16^{2k}}\right)\right)\right)$$

for all $\times \in X$ and all $t > 0$.

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Authors' contributions
All authors conceived of the study, participated in its design and coordination, drafted the manuscript, participated in the sequence alignment, and read and approved the final manuscript.

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