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Analysis of natural convection flows of Jeffrey fluid with Prabhakar-like thermal transport

Zar Ali Khan [a,1], Nehad Ali Shah [b,1], Nadeem Haider [c], Essam R. El-Zahar [d,e], Se-Jin Yook [f,*]

[a] Department of Mathematics, Government Superior Science College Peshawar, Peshawar, 25000, Khyber Pakhtunkhwa, Pakistan
[b] Department of Mechanical Engineering, Sejong University, Seoul, 05006, South Korea
[c] Department of Mathematics, Government Degree College Badaber Peshawar, Peshawar, 25000, Khyber Pakhtunkhwa, Pakistan
[d] Department of Mathematics, College of Science and Humanities in Al-Kharj, Prince Sattam Bin Abdulaziz University, P.O. Box 83, Al-Kharj, 11942, Saudi Arabia
[e] Department of Basic Engineering Science, Faculty of Engineering, Menoufia University, Shebin El-Kom, 32511, Egypt
[f] School of Mechanical Engineering, Hanyang University, 222 Wangsimni-ro, Seongdong-gu, Seoul, 04763, Republic of Korea

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ABSTRACT

The free convection flow of Prabhakar fractional Jeffrey fluid on an oscillated vertical plate with homogenous heat flux is investigated. With the help of the Laplace transform and the Boussinesq's approximation, precise solutions for dimensionless momentum may be found. The temperature and velocity of Prabhakar fractional time free convection flows are compared to conventional thermal transport, as shown by Fourier's law. They met all of the requirements and recovered Newtonian and ordinary Jeffrey fluid solutions from fractional Jeffrey fluid. Finally, graphs show the effect of various physical parameters such as fractional parameters, Grashof number, Prandtl number and Jeffrey parameters on both temperature and velocity.

1. Introduction

Due to its various applications in engineering and environmental processes, natural convection flows of an incompressible and unsteady viscous fluid across a vertical infinite plate have been extensively investigated in the literature. Filtration processes, nuclear reactors, granular as well as fiber insulations, spacecraft design, geothermal systems, and other industrial technologies still pique their attention. Several scholars are looking at the unpredictability of natural convection flow through a moving vertical plate under varying conditions (thermal) at the boundary. Soundalgekar [1] was the first to provide a precise solution for natural convection which effects on an incompressible and viscous fluid flow across an endless vertical plate, which was started impulsively. Raptis and Singh [2] previously investigated the natural convection flow of a viscoelastic fluid through an accelerating vertical plate. Singh and Kumar [3] examined the effects of natural convection on the flow across an accelerated exponentially vertical plate. Soundalgekar [4] investigated natural free convection flow over a vertical, endless, porous plate with continuous suction. When an oscillating flow traverses a vertical plate, Mansour [5] looked at how natural convection interacts with heat radiation. Ishak [6,7] looked into the impact of heat radiation on the flow of the boundary layer on a horizontal plate. With the help of Laplace Haq et al. [8] found the analytical solutions

* Corresponding author.
E-mail addresses: zaralikhangmk@gmail.com (Z.A. Khan), nehadali199@sejong.ac.kr, nehadali199@yahoo.com (N.A. Shah), nadeemhaider98@gmail.com (N. Haider), er.elzahar@psau.edu.sa (E.R. El-Zahar), ysjnuri@hanyang.ac.kr (S.-J. Yook).
[1] Zar Ali Khan and Nehad Ali Shah are contributed equally to this work and co-first authors.

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for temperature field as well as for velocity of natural convection incompressible and viscous fluid passing from a plate which is vertical, infinite, and oscillating under the consideration of uniform heat flux and transverse magnetohydrodynamics.

Because conventional Newtonian fluids cannot completely describe the rheological characteristics of many fluids used in industrial and engineering applications, the theoretical study of non-Newtonian type fluids has acquired a lot of interest in recent years. Due to their malleable character, such fluids exhibit a typical non-linear relationship between shear strain and stress and show some worth intimation reality. Many important industrial materials, particularly multi-phase systems such as emulsions, Shear dependent viscosity, shear thickening or shear thinning, normal stress differences, stress relaxation, and other non-Newtonian characteristics can be found in slurries, foam, and polymer systems such as solution and melt. The Jeffrey model is the most basic model for rheological effects of an elastic fluid among non-Newtonian fluids. Ali et al. [9], investigated the impacts of a porous media and a transverse magnetic on a generalized Jeffrey nanofluid in a rotatory system using time fractional comparative analysis. Using the Caputo-Fabrizio fractional derivative, Abro et al. [10] lime lighted the impacts of heat radiation of Jeffery fluid with magnetic field. The formulation of mixed free convective Jeffrey nanoliquid stratified flow using magnetohydrodynamics were examined by Waqas et al. [11]. By using double diffusion with gold nanoparticles, Asha and Sunitha [12] investigated the effect of heat radiation on peristaltic blood flow in a Jeffrey fluid. Over an inclined permeable stretched cylinder, at the axisymmetric stagnation point, Ijaz and Ayub [13] studied the mixed convective flow of Jeffrey fluid. Yasmeen et al. [14] looked on the qualitative and quantitative techniques to analyzing Hartmann boundary layer type peristaltic flow of Jeffrey fluid. Krishna [15] looked into the effects of thermal radiation, ion-slip and Hall, on the unsteady MHD natural convective rotating type flow of Jeffrey's fluid across a plate which is vertical, infinite, and also porous with a ramping wall temperature. Hayat et al. [16] used OHAM method to find the numerical solution of Jeffrey nanofluid with activation energy, entropy generation with Joule heating, nonlinear thermal radiation, and viscous dissipation. With the help of RKF-45 method Gireesha et al. [17] calculated numerical solution of Jeffrey nanofluid under the impact of magnetohydrodynamic on a permeable stretching sheet. Saif et al. [18] investigated a curved stretchy sheet induces magnetohydrodynamic (MHD) Jeffrey nanomaterial fluid flow. Kumar et al. [19] investigated sort number impacts on Jeffrey MHD fluid via a plate which is vertical, permeable and moving, are investigated using finite element analysis. Variable liquid characteristics on the magnetohydrodynamics of peristaltic flow presented by Jeffrey fluid via a compliant-walled conduit were investigated by Divya et al. [20]. With the help of Adomian Decomposition Method (ADM), Nisar et al. [21] obtained analytical solution of natural convection Jeffry fluid flowing on a stretching surface under the impact of MHD. Gangavathi et al. [22] discussed the impact of hall and slip-on Jeffrey fluid passing from a porous medium in a channel of inclined type. The impact of velocity, concentration and Newtonian heating, on unsteady natural convective flow of Jeffrey type fluid on a plate which is long, vertical and an infinite ramped wall fixed in porous medium are discussed by Rehman et al. [23]. Hristov [24] discussed the conduction of steady-state heat in a medium with some spatial non-singular fading memory with Jeffrey's kernel and exact solutions, the Caputo-Fabrizio space non integer order derivative is derived from the Cattaneo concept in his book. From the definition of Cattaneo constitutive type equation with the Jeffrey's kernel to the Caputo-Fabrizio time non integer order derivative with transient heat diffusion with a non-singular fading memory discussed by Hristov [25]. Abdelsalam and Zaher [26] investigated a Rabinowitsch suspension fluid via elastic walls with heat transfer under the influence of electroosmotic forces using wavelength approximation and the creeping flow system (EOFs). Abdelsalam et al. [27] explored the electro-magneto-biomechanics of sperm swimming via the cervical canal in the female reproductive system. The heat transfer of radiative Williamson fluid on a stretchy curved surface via similarity transformation explored by Raza et al. [28]. Elkoumy and Abdelsalam [29] evaluated the combined thermal effect of peristaltically driven particle-fluid movement in a catheterized pipe in combination with slip circumstances. To analyze the peristaltically induced motion of Carreau fluid in a symmetrical channel under the effect of a generated and applied magnetic field, Bhatti and Abdelsalam [30] employed the perturbation technique. With the influence of a uniform magnetic field and an applied electric field across two coaxial tubes on a flow of DC and AC-operated micropumps, Elmaboud and Abdelsalam [31] discovered the closed form solution of a generalized Burgers' fluid. Using steady perturbation, Abumandour et al. [32] investigated the effects of thermal viscosity and magnetohydrodynamics on nanofluid peristalsis and produced an analytical solution for velocity and temperature. Elkoumy et al. [33] investigated the analytical solution for peristaltic flow of a Maxwell fluid passing from a porous medium with the impact of Hall and transverse magnetic field. The intra-uterine flow with the small, halted particles under the influence of heat transfer is investigated by Bhatti et al. [34]. The impacts of partial slip on magnetic dusty fluid caused by peristaltic wave through porous channel were explored by Bhatti and Abdelsalam [35], who looked at the entropy production and irreversibility process.

It has the capacity to describe the memory effects of many physical processes, fractional calculus has recently been popular in a variety of disciplines of research. Fractional calculus is now effectively used in a wide range of scientific areas, including electrochemistry, biophysics, mechanics, electrical engineering, rheology, viscoelasticity, biology and mechatronics [36]. Many mathematical operations, such as the Marchaud derivative, are given Liouville and Riemann-Liouville derivative, fractional derivatives, Leibnitz derivative, Hadamard derivative, Grünwald-Letnikov derivative, Caputo-Fabrizio derivative, Caputo derivative, Atangana-Baleanu derivative, Riesz derivative, Prabhakar derivative etc. Generalizations of classical derivatives [37,38] are examples of such operators. In the perspective of Caputo, the generalized Fourier Law has recently been used to study heat transport problems. The Caputo fractional derivatives are used to derive the fractionalization of the thermal equation [39–45]. The integral-balance technique was used by Hristov [46] to construct closed form numerical solutions to the non-linear heat (means mass) diffusion problem with power-law non-linearity of the thermal (means mass) diffusivity, bypassing the widely employed Kirchhoff transformation. The principal characteristics of the Prabhakar functions [47] and the Mittag-Leffler three-parameter functions given by Gara and Garrappa [48]. The relevance of Prabhakar's functions in understanding the splitting dielectric characteristics of disordered materials and heterogeneous systems that show non-linearity and non-locality at the same time is well known. A linearly viscoelastic type of model based on the Prabhakar fractional operators was studied by Giusti and Colombaro [49].

In this paper, we look into free convection flows of Prabhakar type fractional Jeffrey fluid. For heat flow, this mathematical model relies on extended fractional order constitutive equations. Based on generalized memory effects, we created time fractional Prabhakar derivatives in this work. The analytical solutions for the dimensionless velocity and temperature are derived with the help of integral transform. At the end, a comparison made among fractional Jeffrey, ordinary Jeffrey, fractional viscous and ordinary viscous fluids in terms of velocity and heat transfer. Finding a mathematical model using Prabhakar-like operators with certain fractional coefficients might be a suitable technique that is more consistent with experimental and theoretical data.

2. Mathematical formulation of the problem

Assume incompressible and unsteady Jeffrey fluid with free convection. At initial, time $t = 0$, the fluid as well as the plate are at rest at some constant temperature T_∞. But for the time $t = 0^+$, the plate started to move in their plane i.e. (y = 0) as stated in

$$v = u_o g(t) i, \ t > 0, \tag{1}$$

where u_0 represents constant velocity, i represents unit vector in flow direction. The fluid flowed slowly due to the shear, and its velocity had the following form

$$U = U(y,t) = v(y,t) i. \tag{2}$$

In the light of above premises and the conventional Boussinesq's approximation, the governing for fluid and thermal transport are [18,50].

$$\rho \frac{\partial v(y,t)}{\partial t} = \frac{\mu}{1+\lambda_1}\left[1+\lambda \frac{\partial}{\partial t}\right] \frac{\partial^2 v(y,t)}{\partial y^2} + g\rho\beta\,[\Theta(y,t) - \Theta_\infty], \text{(The momentum equation)} \tag{3}$$

$$\rho c_p \frac{\partial \Theta(y,t)}{\partial t} = -\frac{\partial q(y,t)}{\partial y}, \text{(The energy balance equation)} \tag{4}$$

$$q(y,t) = -k\frac{\partial \Theta(y,t)}{\partial y}, \text{(The Fourier's law of thermal flux)} \tag{5}$$

here, g and q are the gravitational acceleration and thermal flux of the fluid respectively.

For Eqs. (3)–(5), we assume the following initial and boundary conditions:

$$v(y,0) = 0, \ \Theta(y,0) = \Theta_\infty, \ y \geq 0, \tag{6}$$

$$v(0,t) = 0, \ \Theta(0,t) = \Theta_\infty + (\Theta_w - \Theta_\infty) g(t), \ t \geq 0, \tag{7}$$

$$v(y,t) \to 0, \ \Theta(y,t) \to \Theta_\infty, \ as\ y \to \infty \tag{8}$$

For the creation of non-dimensional equations, we define the following dimensionless quantities

$$\begin{aligned} &y^* = \frac{u_0 y}{\nu}, \ t^* = \frac{u_0^2 t}{\nu}, \ v^* = \frac{v}{u_0}, \ \Theta^* = \frac{\Theta - \Theta_\infty}{\Theta_w - \Theta_\infty}, \ q^* = \frac{q}{q_0}, q_0 = \frac{k(\Theta_w - \Theta_\infty) u_0}{\nu}, \\ &\Pr = \frac{\mu c_p}{k}, \ Gr = \frac{g\nu\beta(\Theta_w - \Theta_\infty)}{u_0^3}, \end{aligned} \tag{9}$$

then incorporate Eq. (9) in Eqs. (3)–(8), and neglect the star notations, we get the non-dimensional equations:

$$\frac{\partial v(y,t)}{\partial t} = \frac{\left(1+\lambda \frac{\partial}{\partial t}\right)}{1+\lambda_1} \frac{\partial^2 v(y,t)}{\partial y^2} + Gr\Theta(y,t), \tag{10}$$

$$\frac{\partial \Theta(y,t)}{\partial t} = -\frac{1}{\Pr}\frac{\partial q(y,t)}{\partial y}, \tag{11}$$

$$q(y,t) = -\frac{\partial \Theta(y,t)}{\partial y}, \tag{12}$$

with dimensionless conditions

$$v(y,0) = 0, \ \Theta(y,0) = 0, \quad \text{for } y \geq 0, \tag{13}$$

$$v(0,t) = 0, \ \Theta(0,t) = g(t), \ \text{for } t \geq 0, \tag{14}$$

$$v(y,t) \to 0, \ \Theta(y,t) \to 0, \ as\ y \to \infty \tag{15}$$

In the above relations, ν and $u_0 > 0$ represent kinematic and characteristic viscosity respectively, Pr and Gr represent Prandtl and

Grashof numbers respectively.

We offer a novel mathematical model in this work that accounts for generalized thermal memory effects. For achieving this goal, we have introduced a generalized Fourier's type law, which is based on the Prabhakar's non-integer derivative, namely:

$$q(y,t) = -{}^{C}D^{\gamma}_{\alpha,\ \beta,\ a}\frac{\partial \Theta(y,t)}{\partial y}. \tag{16}$$

where, the regularized Prabhakar fractional derivative is defined as [49,50].

$$\begin{aligned} {}^{C}D^{\gamma}_{\alpha,\beta,a}g(t) &= \varepsilon^{-\gamma}_{\alpha,m-\beta,a}g^{(m)}(t) = e^{-\gamma}_{\alpha,m-\beta}(a;t)*g^{(m)}(t) \\ &= \int_0^t (t-x)^{m-\beta-1}\varepsilon^{-\gamma}_{\alpha,m-\beta}(a(t-x)^{\alpha})g^{(m)}(x)dx, \end{aligned} \tag{17}$$

"*" use for convolution product, $g^{(m)}$ shows the mth order derivative of $g(t) \in BD^m(0,b)$, $BD^m(0,b)$ stands for the set of real-valued functions $g(t)$ whose derivatives are continuous up to $(m-1)$ order on the open interval $(0,b)$, where $g^{(m-1)}(t)$ continuous function, and $m = [\beta]$ represents the integer part of the parameter β.

The term used in Eq. (17), is $\varepsilon^{\gamma}_{\alpha,\ \beta,\ a}g(t) = \int_0^t (t-x)^{\beta-1}\varepsilon^{\gamma}_{\alpha,\ \beta}(a(t-x)^{\alpha})f(x)dx$, indicates the Prabhakar integral where, $\varepsilon^{\gamma}_{\alpha,\beta}(z) = \sum_{n=0}^{\infty}\frac{\Gamma(\gamma+n)z^n}{n!\Gamma(\gamma)\Gamma(\alpha n+\beta)},\ \ \alpha,\beta,\gamma,\ z\in\mathbb{C},\ \ \mathrm{Re}(\alpha)>0,$

Where the function used in above relation is $e^{\gamma}_{\alpha,\beta}(a;t) = t^{\beta-1}\varepsilon^{\gamma}_{\alpha,\beta}(at^{\alpha}),\ \ t\in\mathbb{R},\ \ \alpha,\ \beta,\ \gamma,\ a\in\mathbb{C},\ \ \mathrm{Re}(\alpha)>0$, known as the Prabhakar kernel.

Laplace transform of the regularized Prabhakar derivative is given by

$$\begin{aligned} L\left\{{}^{C}D^{\gamma}_{\alpha,\beta,a}g(t)\right\} &= L\left\{e^{-\gamma}_{\alpha,m-\beta}(a;t)*g^{(m)}(t)\right\} = L\left\{e^{-\gamma}_{\alpha,m-\beta}(a;t)\right\}L\left\{g^{(m)}(t)\right\} \\ &= s^{\beta-m}(1-as^{-\alpha})^{\gamma}L\left\{g^{(m)}(t)\right\}, \end{aligned} \tag{18}$$

where $h_P(\alpha,\beta,\gamma,a,t) = e^{-\gamma}_{\alpha,m-\beta}(a;t)$ (the Prabhakar kernel of the regularized Prabhakar fractional derivative). Thus, classical Fourier's law obtains by choosing $\beta = \gamma = 0$.

3. Problem formulation

3.1. Temperature formulation

By applying Laplace transform and incorporated it into Eq. (11), Eq. (16), Eq. $(14)_2$, and Eq. $(15)_2$, with condition given in Eq. $(13)_2$, we get the following transformed form for the temperature:

$$s\overline{\Theta}(y,s) = -\frac{1}{\mathrm{Pr}}\frac{\partial\overline{q}(y,s)}{\partial y}, \tag{19}$$

$$\overline{q}(y,s) = -s^{\beta}(1-as^{-\alpha})^{\gamma}\frac{\partial\overline{\Theta}(y,s)}{\partial y}, \tag{20}$$

$$\overline{\Theta}(0,s) = G(s),\ \ \lim_{y\to\infty}\overline{\Theta}(y,s) = 0, \tag{21}$$

moreover, $\overline{\Psi}(y,s) = \int_0^{\infty} e^{-st}\Psi(y,t)dt$, indicates the Laplace transform of $\Psi(y,t)$, with transform parameter is s.

Incorporate Eq. (20) in Eq. (19) and after rearranging, we get;

$$\frac{\partial^2\overline{\Theta}(y,s)}{\partial y^2} - \frac{\mathrm{Pr}s}{s^{\beta}(1-as^{-\alpha})^{\gamma}}\overline{\Theta}(y,s) = 0. \tag{22}$$

Eq. (22) gives the following form after solving the differential

$$\overline{\Theta}(y,s) = G(s)\exp\left(-y\left(\frac{\mathrm{Pr}s}{s^{\beta}(1-as^{-\alpha})^{\gamma}}\right)^{\frac{1}{2}}\right). \tag{23}$$

Eq. (23) takes the following form after using the exponential function series formula:

$$\overline{\Theta}(y,s) = G(s) + G(s)\sum_{k=1}^{\infty}\frac{(-y)^k(\mathrm{Pr})^{\frac{k}{2}}}{k!}s^{\frac{(1-\beta)k}{2}}(1-as^{-\alpha})^{-k\gamma/2}. \tag{24}$$

The inverse Laplace formula of the transformed temperature given in Eq. (24) is

$$\begin{aligned}\Theta(y,t) &= g(t) + g(t)*\sum_{k=1}^{\infty}\frac{(-y)^k(\Pr)^{\frac{k}{2}}}{k!}\, e_{\alpha,\ \frac{(\beta-1)k}{2}}^{\ \frac{\gamma k}{2}}(a;t)\\ &= g(t) + g(t)*\sum_{k=1}^{\infty}\frac{(-y)^k(\Pr)^{\frac{k}{2}}}{k!}\, t^{\frac{(\beta-1)k}{2}-1}\varepsilon_{\alpha,\ \frac{(\beta-1)k}{2}}^{\ \frac{\gamma k}{2}}(a;t).\end{aligned} \tag{25}$$

3.1.1. Case of classical thermal transport ($\beta = \gamma = 0$)

With help of Eq. (18), we can see that:

$$L\left\{e_{\alpha,\beta}^{-\gamma}(a\,;t)\right\} = L\left\{t^{\beta-1}\varepsilon_{\alpha,\beta}^{-\gamma}(at^{\alpha})\right\} = s^{\beta}(1-as^{-\alpha})^{\gamma}. \tag{26}$$

Eq. (26), becomes by putting $\beta = \gamma = 0$

$$L\left\{e_{\alpha,0}^{0}(a\,;t)\right\} = 1 = L\{\delta(t)\}, \tag{27}$$

where $\delta(\cdot)$ is Dirac's distribution. Specifically, in this instance, the generalized Fourier's law given above in Eq. (16), enhance the classical Fourier's law, as a result, ordinary type thermal transport has been recovered. Eq. (23), becomes as in this particular case:

$$\overline{\Theta}(y,s) = sG(s)\frac{e^{-y\sqrt{\Pr s}}}{s}. \tag{28}$$

Eq. (28), takes the following form after using the inverse Laplace

$$\Theta(y,t) = f'(t)*erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}\right). \tag{29}$$

here, $g'(t) = \frac{dg(t)}{dt}$ and $erfc(\bullet)$, represents the complementary Gauss error function.

3.2. Velocity formulation

Applying the Laplace transform on Eq. (10), Eq. $(14)_1$, Eq. $(15)_1$, and condition given in Eq. $(13)_1$, we get the following form for velocity field:

$$s\overline{v}(y,s) = \frac{1+\lambda s}{1+\lambda_1}\frac{\partial^2\overline{v}(y,s)}{\partial y^2} + Gr\overline{\Theta}(y,s), \tag{30}$$

$$\overline{v}(0,s) = 0,\quad \lim_{y\to\infty}\overline{v}(y,s) = 0. \tag{31}$$

By incorporating Eq. (23), into Eq. (30), we obtain

$$\frac{\partial^2\overline{v}(y,s)}{\partial y^2} - \frac{1+\lambda_1}{1+\lambda s}s\overline{v}(y,s) = -Gr\frac{1+\lambda_1}{1+\lambda s}G(s)e^{-y\sqrt{\frac{\Pr s^{1-\beta}}{(1-as^{-\alpha})^{\gamma}}}}. \tag{32}$$

The solution of Eq. (32) with the help of conditions given in Eq. (31) is

$$\overline{v}(y,s) = \frac{GrsG(s)}{\left(\frac{1+\lambda_1}{1+\lambda s}s - \frac{\Pr s}{s^{\beta}(1-as^{-\alpha})^{\gamma}}\right)}\left[\frac{e^{-y\sqrt{\frac{\Pr s^{1-\beta}}{(1-as^{-\alpha})^{\gamma}}}}}{s} - \frac{e^{-y\sqrt{\frac{1+\lambda_1}{1+\lambda s}s}}}{s}\right]. \tag{33}$$

$$\overline{v}(y,s) = GrG(s)\left(1 - \frac{\Pr s}{s^{\beta}(1-as^{-\alpha})^{\gamma}}\right)^{-1}\left[\frac{e^{-y\sqrt{\frac{\Pr s^{1-\beta}}{(1-as^{-\alpha})^{\gamma}}}}}{s} - \frac{e^{-y\sqrt{\frac{1+\lambda_1}{1+\lambda s}s}}}{s}\right]. \tag{34}$$

After simplification of Eq. (33), we get the following form:

$$\begin{aligned}\overline{v}(y,s) &= GrG(s)\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\binom{k}{m}\frac{(\Pr)^k\lambda^m}{(1+\lambda_1)^k}s^{m-\beta k}(1-as^{-\alpha})^{-\gamma k}\\ &\times\left[\sum_{j=0}^{\infty}\frac{(-y)^j(\Pr)^{\frac{j}{2}}}{j!}s^{\frac{(1-\beta)j}{2}-1}(1-as^{-\alpha})^{-\frac{\gamma j}{2}} - \frac{e^{-y\sqrt{\frac{(1+\lambda_1)}{1+\lambda s}s}}}{s}\right].\end{aligned} \tag{35}$$

The solution of velocity can be calculated by applying inverse Laplace transform on Eq. (35), we get

$$v(y,t) = Grg(t)^{*}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\binom{k}{m}\frac{(\mathrm{Pr})^{k}\lambda^{m}}{(1+\lambda_1)^{k}}e^{\gamma k}_{\alpha,\beta k-m}(a;t)$$
$$*\left[\sum_{j=0}^{\infty}\frac{(\mathrm{Pr})^{\frac{j}{2}}(-y)^{j}}{j!}e_{\alpha,\frac{j}{2}(\beta-1)+1}^{\frac{\gamma j}{2}}(a;t)-1+\frac{2(1+\lambda_1)}{\pi}\int_0^{\infty}\frac{sin\left(\frac{-yx}{\sqrt{\lambda}}\right)}{x(x^2+1+\lambda_1)}\exp\left(\frac{-x^2t}{\sqrt{\lambda}(x^2+1+\lambda_1)}\right)dx\right]. \tag{36}$$

3.2.1. *Case for ordinary Jeffrey fluid with the classical Fourier law*

The classical case has been recovered by choosing the fractional parameters $\beta = \gamma = 0$, and therefore, obtained the following transformed form for velocity field;

$$\bar{v}(y,s)=\frac{Gr(1+\lambda_1)sG(s)}{\left[\frac{(1+\lambda_1)s}{(1+\lambda s)}-\mathrm{Pr}s\right](1+\lambda s)}\left[\frac{e^{-y\sqrt{\mathrm{Pr}s}}}{s}-\frac{e^{-y\sqrt{\frac{(1+\lambda_1)s}{(1+\lambda s)}}}}{s}\right], \tag{37}$$

with the inverse Laplace transform

$$v(y,t) = Grg^{/}(t)^{*}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\binom{k}{m}(a_0)^{k}(\mathrm{Pr})^{k}\lambda^{m}\frac{1}{t^{m}\Gamma(1-m)}$$
$$*\left[erfc\left(\frac{y\sqrt{\mathrm{Pr}}}{2\sqrt{t}}\right)-1+\frac{2(1+\lambda_1)}{\pi}\int_0^{\infty}\frac{sin\left(\frac{-yx}{\sqrt{\lambda}}\right)}{x(x^2+1+\lambda_1)}\exp\left(\frac{-x^2t}{\sqrt{\lambda}(x^2+1+\lambda_1)}\right)dx\right]. \tag{38}$$

3.2.2. *Case for fractional viscous fluid*

Fractional viscous fluid has recovered by choosing the Jeffery parameters $\lambda_1,\lambda \to 0$, and then Eq. (38) becomes

$$\bar{v}(y,s)=\frac{GrsG(s)}{\left(s-\frac{\mathrm{Pr}s}{s^{\beta}(1-as^{-\alpha})^{\gamma}}\right)}\left[\frac{e^{-y\sqrt{\frac{\mathrm{Pr}s^{1-\beta}}{(1-as^{-\alpha})^{\gamma}}}}}{s}-\frac{e^{-y\sqrt{s}}}{s}\right]. \tag{39}$$

With the inverse Laplace

$$v(y,t)=Grg(t)^{*}\sum_{k=0}^{\infty}(\mathrm{Pr})^{k}e^{\gamma k}_{\alpha,\beta k}(a;t)*\left[\sum_{j=0}^{\infty}\frac{(-y)^{j}(\mathrm{Pr})^{\frac{j}{2}}}{j!}e_{\alpha,\frac{j}{2}(\beta-1)+1}^{\frac{\gamma j}{2}}(a;t)-erfc\left(\frac{y}{2\sqrt{t}}\right)\right]. \tag{40}$$

3.2.3. *Case for classical viscous fluid*

Classical viscous fluid has been recovered by choosing the Jeffrey parameters $\lambda_1,\lambda \to 0$ and fractional parameters $\beta,\gamma \to 0$, and then Eq. (40) becomes

$$\bar{v}(y,s)=\frac{GrG(s)}{(1-\mathrm{Pr})}\left[\frac{e^{-y\sqrt{\mathrm{Pr}s}}}{s}-\frac{e^{-y\sqrt{s}}}{s}\right], \mathrm{Pr}\neq 1. \tag{41}$$

Eq. (41), becomes after taking Laplace inversion:

$$v(y,t)=\frac{Grg(t)}{1-\mathrm{Pr}}*\left[erfc\left(\frac{y\sqrt{\mathrm{Pr}}}{2\sqrt{t}}\right)-erfc\left(\frac{y}{2\sqrt{t}}\right)\right]. \tag{42}$$

4. Numerical inversions formula

For the validation of our work, we apply the Stehfest's formula [51] for numerical algorithm of inverse Laplace transform method and the compressions are presented in Figs. 15 and 16 and Tables 1 and 2. The Stehfest's formula is defined as

$$\Theta_S\left(\xi,t\right)=\frac{\ln(2)}{t}\sum_{j=1}^{2m}d_j\overline{\Theta}\left(\xi,j\frac{\ln(2)}{t}\right), \tag{43}$$

$$v_S\left(y,t\right)=\frac{\ln(2)}{t}\sum_{j=1}^{2m}d_j\bar{v}\left(r,j\frac{\ln(2)}{t}\right), \tag{44}$$

where $d_j = (-1)^{j+m} \sum_{i=\left[\frac{j+1}{2}\right]}^{\min(j,m)} \frac{i^m (2n)!}{(m-i)! i! (i-1)! (j-i)! (2i-j)!}$, m is a positive integer and $[r]$ denotes the integer value function or bracket function.

5. Results and discussions

The study of non-compressible free convection it is well-known that Jeffrey fluids flow over an infinite vertical plate with a homogeneous heat flux. Prabhakar fractional derivative has been utilized in the constitutive equations to assume generalized memory effects. With the help of Laplace transform the non-dimensional partial differential equations have been solved. The analytical solution solutions for temperature and velocity are obtained in terms of exponential function. Which these solutions satisfy all the conditions which is initial as well as boundary conditions, and they reduce to a known result from the literature in special cases. Numerical computations for temperature and velocity for suitable parameters such as fractional parameters α, β, γ, Jeffrey parameter λ and λ_1, Prandtl number Pr, Grashof number Gr and time t are done to get some physical aspects and comprehend the impacts of various parameters in the situation.

From Figs. 1–3 give the influence of the fractional parameters $(\alpha, \beta$ and $\gamma)$ on temperature, respectively. Then, temperature variation is expressed for every figure and concluded that, at $t = 4.5$, temperature decreases with increased of fractional parameters α, β and γ. Fig. 4 sketch between time t versus special variable y on temperature profile an observed that temperature is an increasing function of time t. In Fig. 5 we discussed the impact the influence of Prandtl number Pr on temperature and concluded from Fig. 5 that temperature is decreasing by increasing value of Pr.

Figs. 6–8 are drawn to explained the influence of fractional parameters $(\alpha, \beta$ and $\gamma)$ at $t = 1$ on velocity profile of Jeffery fluid, respectively. It is conclude from the said figures that velocity decreases by increasing values of α and β, while an opposite behavior shown for fractional parameter γ. Fig. 9 and Fig. 10 are drawn against Jeffrey parameters λ and λ_1 versus special variable y at $t = 1$. As may be seen from the graphs, that velocity is decreasing by increasing Jeffrey parameter λ, while velocity is increasing by increasing values of Jeffery parameter λ_1.Fig. 11 demonstrated the impact of time parameter t on fluid motion and observed from this figure that the motion of the fluid enhanced with time t. Fig. 12 displayed the impact of Prandtl number Pr on velocity profile and note from this figure that the motion of the fluid decreased by increasing Pr and the motion of the fluid is almost stop for large value of Pr. Fig. 13 displays velocity profiles versus special variable y for various values of Grashof number Gr. It is notified from this figure that velocity is increasing by increasing values of Gr. It is also indicated that for large value of Gr velocity over-shoots across the plate. Inside Fig. 14 a comparison between fractional Jeffrey and viscous fluids, ordinary Jeffrey and viscous fluids respectively, has been examined. When compared to fractional Jeffrey and fractional viscous fluids, it was discovered that fractional viscous and ordinary viscous fluids move quicker. Furthermore, fractional fluid moves more slowly than, ordinary fluid. These fluids, on the other hand, have a distinct character around the plate area. Furthermore, the downward direction of the arrow shows decreasing behavior, while upward direction of the arrow shows increasing behavior. For validation of our results, we compare with Stehfest's formula [51] for numerical algorithm of inverse Laplace transform method and presented in Figs. (15)-(16) and Tables 1 and 2. It is pointed out our results has a good agreement with inversion method results. In Fig. 17 we presented a compare of our results with published work.

6. Conclusions

The effects of uniform heat flux on natural convection flows of Prabhakar fractional Jeffrey fluid on a perpendicular, infinite, and isothermal plate have been studied. The generalized fractional constitutive equations for shear stress and heat flow are used in the mathematical model. The time-fractional Prabhakar derivatives are used to characterize generalized memory effects. The Laplace transform is used to find analytical solutions for dimensionless velocity and temperature.

Prabhakar-like fractional Jeffrey fluid with generalized thermal transport are compared to ordinary Jeffrey fluid with generalized thermal transport and ordinary viscoelastic fluids with classical Fourier thermal flux in terms of velocity and heat transfer. Using Prabhakar fractional operator with certain fractional coefficient values might be a handy method to select an appropriate mathematical model with excellent agreement among theoretical and experimental data. The following are the major conclusions based on the current analysis:

(i). Temperature is decreasing with fractional parameters α, β, γ while, an opposite behavior shows for velocity.

Table 1
Comparison of our result with Stehfest's formula for temperature distribution.

Y	(Eq. (25))	(Eq. (43))	(Eq. (25)-Eq. (43))
0	1	1	0
0.2	0.6766685474	0.6742750453	0.0023935021
0.4	0.4430811622	0.4435520025	0.0004708403
0.6	0.2846138736	0.2847541134	0.0001402398
0.8	0.1785070843	0.1784860775	0.0000210068
1	0.109300007	0.1092854662	0.0000145407
1.2	0.0653999376	0.0653988734	0.0000010642
1.4	0.0382687253	0.0382698748	0.0000011495
1.6	0.0219099657	0.0219103767	0.000000411
1.8	0.0122792387	0.0122792181	0.0000000205
2	0.0067396696	0.006739607	0.0000000626

Table 2
Comparison of our result with Stehfest's formula for velocity field.

Y	(Eq. (36))	(Eq. (44))	(Eq. (36)-Eq. (44))
0	0	0	0
0.2	1.0322363565	1.0323217533	0.0000853968
0.4	1.4498943271	1.4499270411	0.0000327141
0.6	1.5655818664	1.5655689928	0.0000128737
0.8	1.535590485	1.5355625359	0.0000279491
1	1.4388434636	1.4388135873	0.0000298763
1.2	1.3152790055	1.3152525407	0.0000264648
1.4	1.1849576601	1.1849373121	0.0000203479
1.6	1.0577351383	1.0577220999	0.0000130384
1.8	0.9382168608	0.93821128	0.0000055808
2	0.8283127377	0.8283141475	0.0000014097

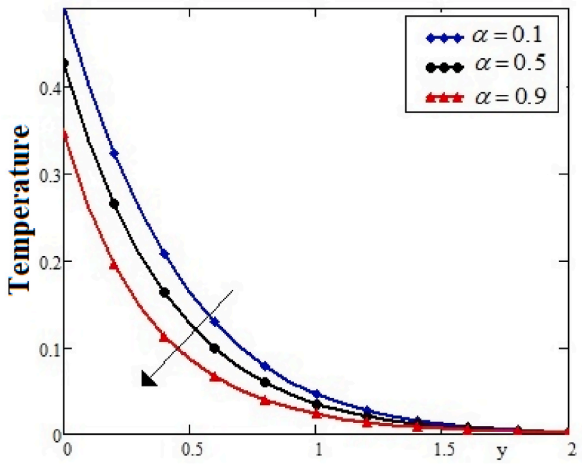

Fig. 1. Temperature profiles $\Theta(y,t)$ for Jeffrey fluid at $\Pr = 11, \beta = 0.3, \gamma = 0.7$, $a = 0.3, t = 4.4$ for various α.

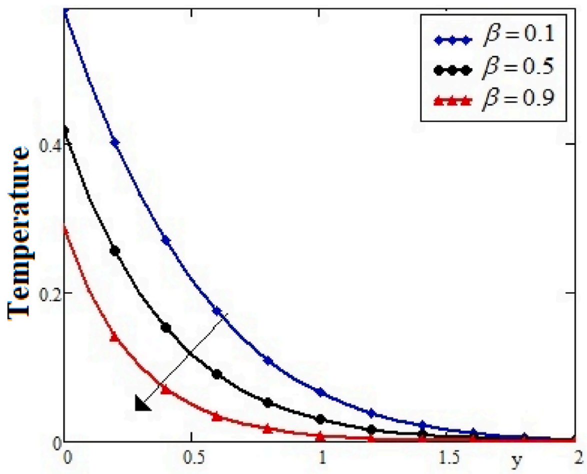

Fig. 2. Temperature profiles $\Theta(y,t)$ for Jeffrey fluid at $\Pr = 11, \alpha = 0.1, \gamma = 0.7$, $a = 0.3, t = 4.4$ for various β.

(ii). The Prandtl number Pr retards the temperature and motion of the fluid.
(iii). The fluid's velocity is accelerated by the Grashof number *Gr*.
(iv). With an increase in *t*, the fluid temperature and velocity increase.
(v). Increasing Jeffrey parameter λ decreases fluid velocity. A reversed effect is observed for λ_1.

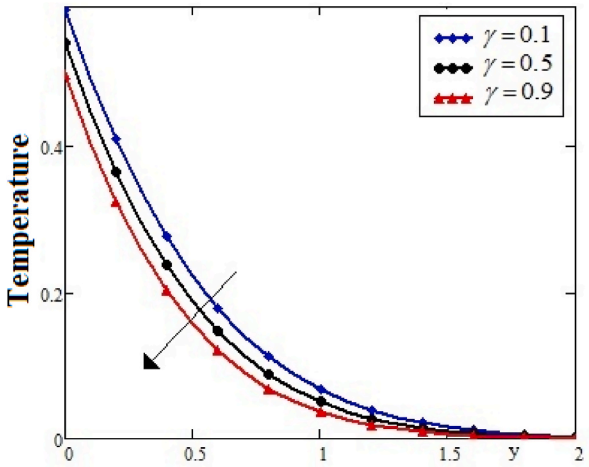

Fig. 3. Temperature profiles $\Theta(y,t)$ for Jeffrey fluid at $\mathrm{Pr}=11, \alpha=0.1, \beta=0.1,\ a=\ 0.3, t=4.4$ for various γ.

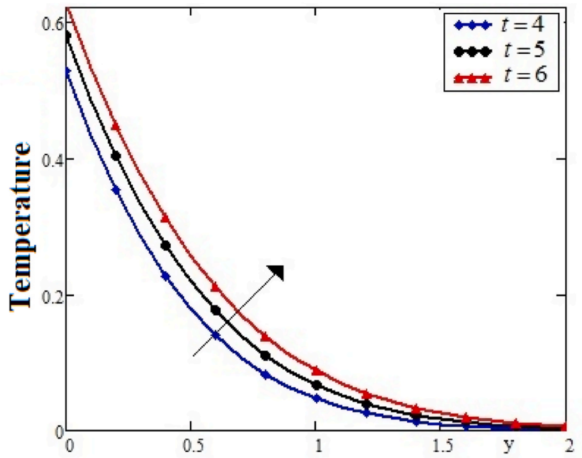

Fig. 4. Temperature profiles $\Theta(y,t)$ for Jeffrey fluid at $\alpha=0.1, \beta=0.1,\ \gamma=0.9, a=\ 0.3, \mathrm{Pr}=11$ for various t.

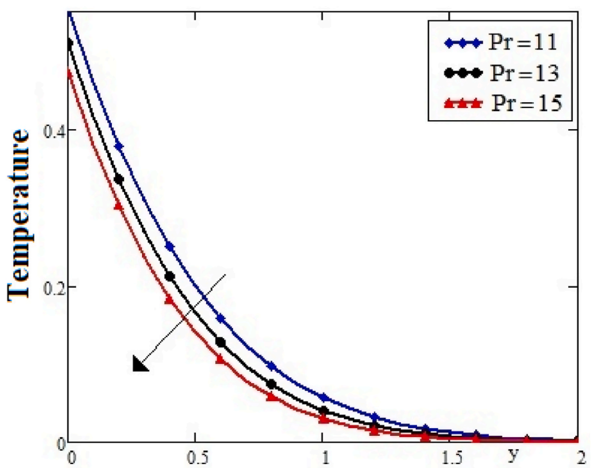

Fig. 5. Temperature profiles $\Theta(y,t)$ for Jeffrey fluid at $\alpha=0.1, \beta=0.1,\ \gamma=0.9, a=\ 0.3, t=4.5$ for various Pr.

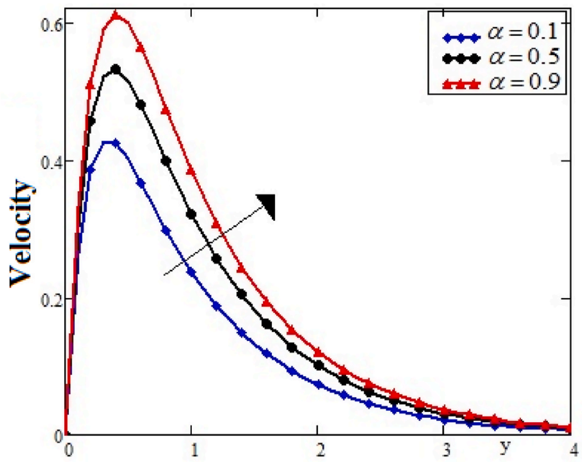

Fig. 6. Velocity profiles $v(y,t)$ for Jeffrey fluid at $a = 0.6, \mathrm{Gr} = 20, \lambda = 0.7, \lambda_1 = 0.5$, $t = 1, \beta = 0.1, \gamma = 0.9, \mathrm{Pr} = 10$ for various values of α.

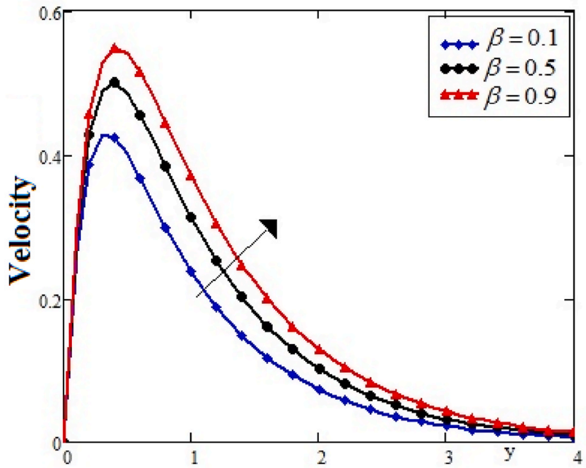

Fig. 7. Velocity profiles $v(y,t)$ for Jeffrey fluid at $a = 0.6, \mathrm{Gr} = 20, \lambda = 0.7, \lambda_1 = 0.5$, $t = 1, \alpha = 0.1, \gamma = 0.9, \mathrm{Pr} = 10$ for various values of β.

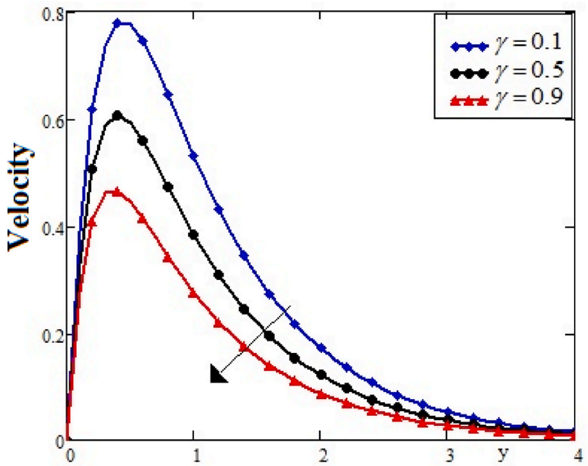

Fig. 8. Velocity profiles $v(y,t)$ for Jeffrey fluid at $a = 0.6, \mathrm{Gr} = 20, \lambda = 0.7, \lambda_1 = 0.5$, $t = 1, \alpha = 0.1, \beta = 0.3, \mathrm{Pr} = 10$ for various γ.

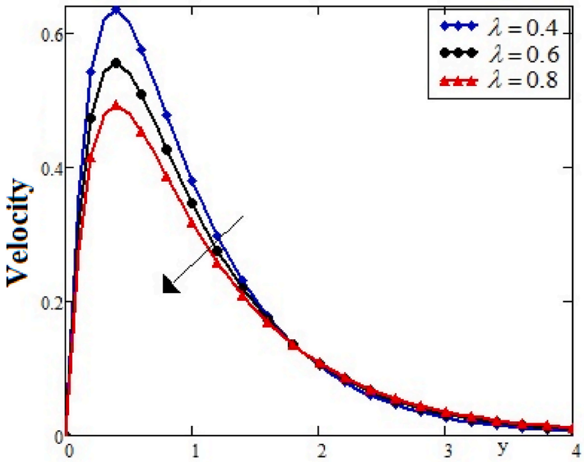

Fig. 9. Velocity profiles $\upsilon(y,t)$ for Jeffrey fluid at $a = 0.7, \mathrm{Gr} = 20, \lambda_1 = 0.5, t = 1, \alpha = 0.5, \beta = 0.3, \gamma = 0.9, \mathrm{Pr} = 10$ for various values of λ.

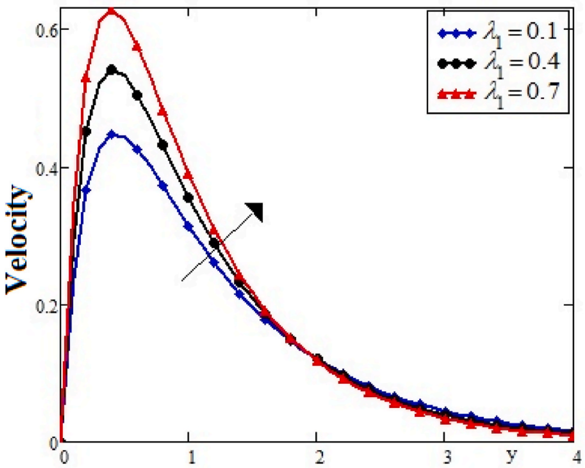

Fig. 10. Velocity profiles $\upsilon(y,t)$ for Jeffrey fluid at $a = 0.6, \mathrm{Gr} = 20, \lambda = 0.7, t = 1, \alpha = 0.5, \beta = 0.3, \gamma = 0.9, \mathrm{Pr} = 10$ for various values of λ_1.

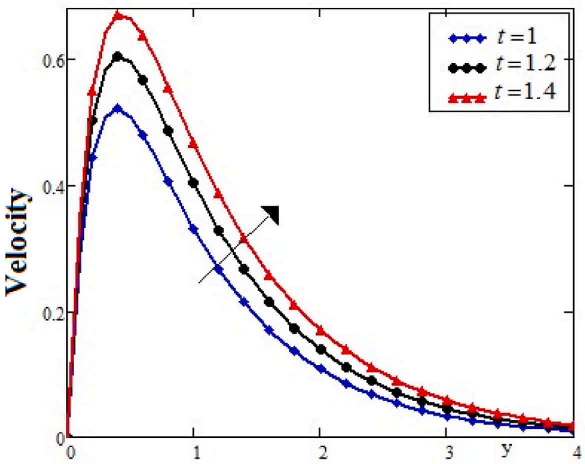

Fig. 11. Velocity profiles $\upsilon(y,t)$ for Jeffrey fluid at $a = 0.7, \mathrm{Gr} = 20, \lambda = 0.7, \lambda_1 = 0.5, \alpha = 0.5, \beta = 0.3, \gamma = 0.9, \mathrm{Pr} = 11$ for various values of t.

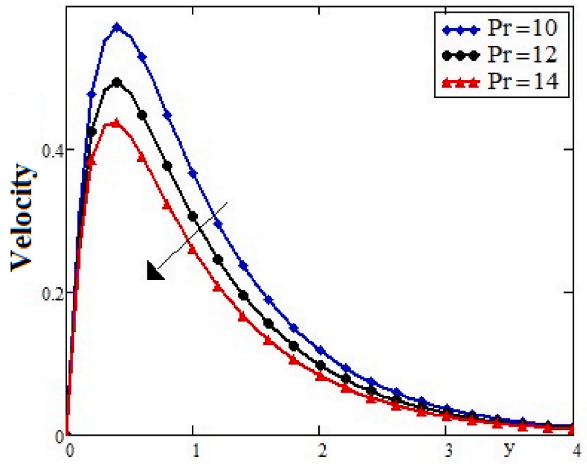

Fig. 12. Velocity profiles $\upsilon(y,t)$ for Jeffrey fluid at $a = 0.6, \mathrm{Gr} = 20, \lambda = 0.7, \lambda_1 = 0.5, t = 1, \alpha = 0.5, \beta = 0.3, \gamma = 0.9$ for various values of Pr.

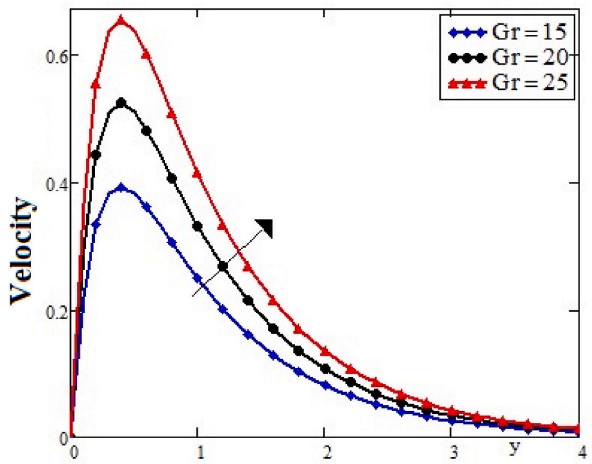

Fig. 13. Velocity profiles $\upsilon(y,t)$ for Jeffrey fluid at $a = 0.7, \lambda = 0.7, \lambda_1 = 0.5, t = 1, \alpha = 0.5, \beta = 0.3, \gamma = 0.9, \mathrm{Pr} = 10$ for various values of Gr.

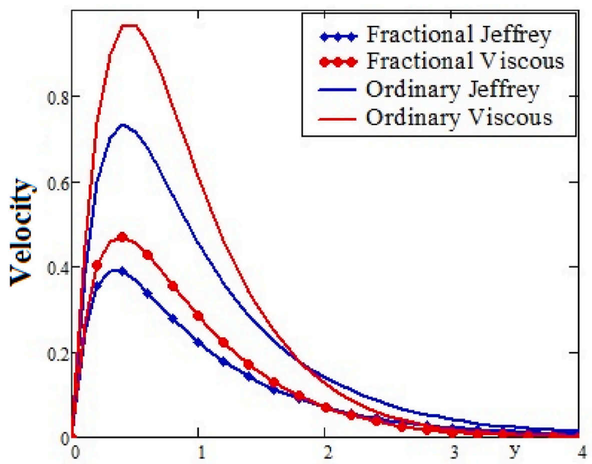

Fig. 14. Profiles for velocity comparison among fractional Jeffrey and viscous fluids, ordinary Jeffrey and viscous fluids at $\lambda_1 = 0.5, a = 0.7, \alpha = 0.1, \mathrm{Gr} = 20, \lambda = 0.7, t = 1, \beta = 0.3, \gamma = 0.9$ and $\mathrm{Pr} = 10$.

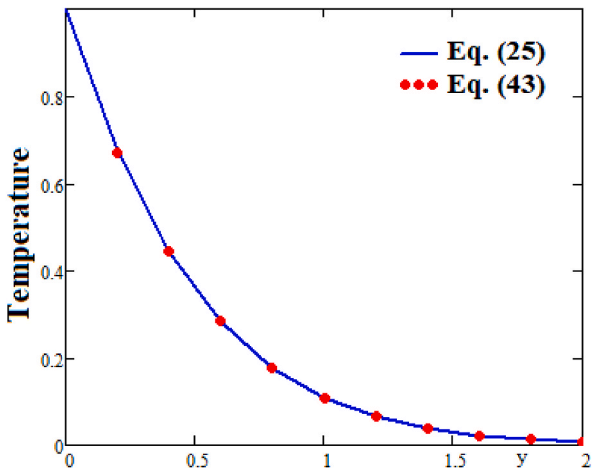

Fig. 15. Comparison of our result with Stehfest's formula for temperature distribution.

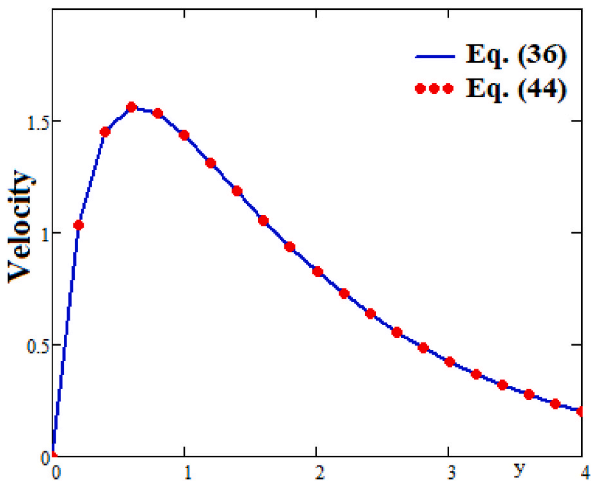

Fig. 16. Comparison of our result with Stehfest's formula for velocity profile.

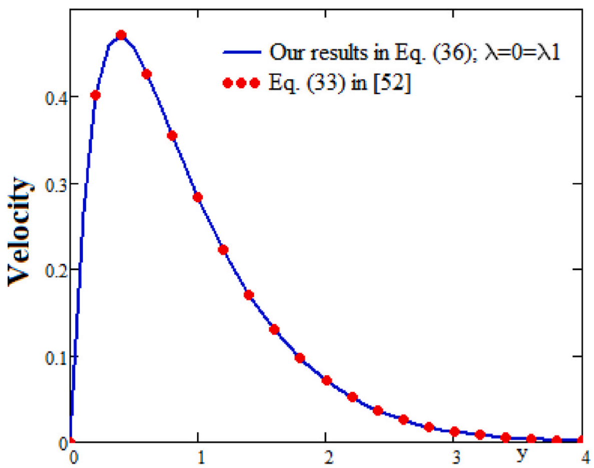

Fig. 17. Comparison of our result with (Eq. (22), [52]).

Authors contributions

Zar Ali Khan and Nehad Ali Shah contributed equally to this work.

NAS: Conceptualization; NH: Data curation; ZAK and NAS: Formal analysis; Roles/Writing - original draft; Methodology; ERE: Investigation; NAS and YJS: Resources; Writing - review & editing; SJY: Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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