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Dynamics of optimal harvesting in an Ammensal-Adversarial interaction

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Abstract This article discusses the stability of a two-species ecosystem composed of an ammensal (x) and an adversarial (y) species that are continuously harvested. A mathematical model is defined by a system of two nonlinear ordinary differential equations of first order. The considered system's boundedness is investigated. The local stability of the system is described using a variational matrix, while the global stability is examined using Lyapunov's function. The prerequisite for the system to exist in bionomic equilibrium has been identified. The ideal harvesting technique is determined using the maximal principle proposed by Pontryagin. In MATLAB simulations, the stability of the deterministic system is demonstrated for the specified set of parameters.

1. Introduction

Numerous ecological and biological processes are driven by the impulsive energetic relationships between species and their complex possessions. Ammensalism is one such affiliation. It is a dynamic relationship between an ammensal and an adversarial species. Ammensalism occurs when one species is harmed by another that is unaffected. One is referred to as an ammensal species, while the other is referred to as an adversary

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species. Ammensalism occurs naturally when cattle tramp the grass; the grass is crushed but the cattle are not harmed or benefited by this action. Due to its universality and significance, this relationship has long been and will continue to be a dominant theme in both ecology and mathematical ecology. A. Lotka [1] and V. Volterra [2] independently developed the first model to describe the size dynamics of two populations interacting as a predator–prey system. Due to the inherent limitations of the classical Lotka–Volterra models in accurately describing a large number of realistic biological phenomena, they should occasionally give way to more sophisticated models from both a mathematical and biological perspective. All of these interactions were facilitated by bionomics of natural resources. Harvesting has a significant effect on population dynamics, as evidenced by the exploitation of biological resources and harvesting of population species in fishery, forestry, agriculture, and wild life management.

Harvesting has a significant impact on a population's dynamic evolution. We know that the population's long-run stationary density can be much lower than the population's long-run stationary density in the absence of harvesting [3], depending on the nature of the harvesting strategy used. In general, a bionomic model is composed of two components: a biology type that represents the behavior of a living system and an economic model that connects the biological system to market pricing and resources according to institutional limitations. Bio economic models are mathematical equations that are used to depict biological processes. In fishery and forestry management, the logistic equation is the most frequently employed function for capturing the essential characteristics of population densities. However, there is a growing trend toward the development of simulation models by biologists and agricultural scientists. These models also approximate the dynamical behavior of real-world systems, and their complexity may limit their direct usage as components of optimal control models.

Numerous researchers have examined the challenges associated with an ammensalism system that includes harvesting, with a particular emphasis on economic harvesting practises. The majority of authors have concentrated their efforts on optimal exploitation, which is entirely motivated by harvesting profits. Clark [4] provided an in-depth review of the difficulties and approaches involved in bionomic exploitation of natural resources. Brauer and Soudack conducted a research on these models under conditions of consistent harvesting rates for both species [5,6]. Chaudhuri [7,8] examined multi-species harvesting models in depth, whereas Mesterton-Gibbons [9] examined an optimal policy for maximising the present value of a combined harvest of two biologically distinct species that would coexist as predator and prey in the absence of harvesting. We evaluated models for the combined harvesting of a two-species prey-predator fishery. According to Ragozin and Brown [10], as well as Chaudhuri and Ray [11]. Chattopadhyay examined the persistence and global stability of a resource-based competitive system with three species [12]. Dai and Tang [13] discussed a prey-predator model with harvesting. Kar [14] examined harvesting for predators and prey in a prey-dependent model with prey refuge, as well as an optimal harvesting policy. Shiva Reddy [15] presented a mathematical model for a three-species ecosystem with two predators competing for the same prey and studied the concept of stability using a variety of mathematical methodologies. Numerous authors have recently investigated the fractional order Predator–Prey Model using the Harvesting and covid models [16–29].

This article presents a novel approach to the joint harvesting of two species that have an ammensalism relationship. The following sections comprise the current study paper: Section 2 contains the mathematical model of the ecosystem. The system's boundedness is specified in Section 3. Section 4 illustrates the presence of equilibrium points. Sections 5 and 6 determine the system's local and global stability at the interior steady state. Section 7 discusses the requirements for the existence of a bionomic equilibrium. Section 8 presents the ideal harvesting policy and numerical simulations. Section 9 emphasizes the concluding remarks.

2. Construction of Ammensal-Adversarial model

The nonlinear ordinary differential equations for this Ammensal Model are formed by.

$$\frac{dx}{dt} = a_1x - \alpha_{11}x^2 - \alpha_{12}xy - q_1E_1x \tag{2.1}$$

$$\frac{dy}{dt} = a_2y - a_{22}y^2 - q_2E_2y \tag{2.2}$$

Here $x(t)$ represents the biomass density of Ammensal and y(t) represents the biomass density of Adversarial. For species, leta_1,a_2 be the natural growth rates of the species.α_{12} is the rate of decrease of the Ammensal species owing to predation, α_{11}, are the rates of decline of two species due to natural resource constraints, andq_1, q_2 the coefficients of catchability of the Ammensal and Adversarial species.E_1 and E_2 represent the potential harvesting attempts for both species.

3. Boundedness of the system

In this section, we shall provide some appropriate conditions for the system's boundedness.

Theorem 3.1. *With initial conditions in the positive quadrant, the system* (2.1)–(2.2) *has bounded solutions.*

Proof: Considering,$w(t) = x(t) + y(t)$

Clearly, $\frac{d\ w}{dt} = \frac{dx}{dt} + \frac{dy}{dt}$

$$\frac{d\ w}{dt} = a_1x - \alpha_{11}x^2 - \alpha_{12}xy - q_1E_1 + a_2y - a_{22}y^2 - q_2E_2y$$

$$\frac{dw}{dt} + nw = (n + a_1 - q_1E_1)x - \alpha_{11}x^2 + (a_2 - q_2E_2)y - a_{22}y^2 - \alpha_{12}xy$$

$$\leqslant (n + a_1 - q_1E_1)x - \left(\alpha_{11} + \frac{\alpha_{11}}{2}\right)x^2 + (n + a_2 - q_2E_2)y - \left(a_{22}\frac{\alpha_{11}}{2}\right)y^2$$

$$\leqslant \frac{(n + a_1 - q_1E_1)^2}{2(2\alpha_{11} + \alpha_{12})} + \frac{(n + a_2 - q_2E_2)^2}{2(2\alpha_{22} + \alpha_{12})} = \mu$$

$$0 < w\ \ [x(t), y(t)] \quad \leqslant \quad \frac{\mu}{n} \quad (1 - e^{-nt}) \ \ + w\ [x(0), y(0)]\ \ e^{-nt}$$

As $t \to \infty$ the above inequality becomes,$0 < w(t) \leqslant \frac{\mu}{n}$ with$n + a_1 > q_1E_1$ and $n + a_2 > q_2E_2$.

Hence solutions of the system (2.1)–(2.2) are $n + a_1 > q_1E_1$ and$n + a_2 > q_2E_2$ bounded and the set.

$$\Omega = \left\{ (x, y) \in \mathbb{R}_2^+ : \; 0 < w = x + y \leqslant \frac{\mu}{n} \right\}$$

is referred to be a promoter of alternative solutions based on positive starting circumstances.

4. Interior equilibria

It is a known fact that $E_1(0, 0)$ exists at any time, and another equilibrium point that exists interior,$E_2(x^*, y^*)$ is given by.

$$x^* = \frac{1}{\alpha_{11}}\left[a_1 - \alpha_{12}\left(\frac{a_2 - q_2E_2}{\alpha_{22}}\right) - q_1E_1\right] y^* = \frac{a_2 - q_2E_2}{\alpha_{22}}$$

x^* and y^* are positive if $\alpha_{22}(a_1 - q_1E_1) > \alpha_{12}(a_2 - q_2E_2)$ and $a_2 > q_2E_2$ respectively.

5. Local stability analysis

The purpose of this work was to investigate local stability at available interior equilibrium $E_2(x^*, y^*)$ conditions when a variational matrix was applied.

The varitional matrix at the interior equilibrium point is.

$$J(x, y) = \begin{bmatrix} -\alpha_{11}x^* & -\alpha_{12}x^* \\ 0 & -\alpha_{22}y^* \end{bmatrix}$$

The characteristic equation is $(\alpha_{11}x^* + \lambda)(\alpha_{22}y^* + \lambda) = 0$ and the roots of equations are $\lambda_1 = -\alpha_{11}x^*$ and$\lambda_2 = -\alpha_{22}y^*$,

Hence the proposed system is locally asymptotically stable occurs at the steady state$E_2(x^*, y^*)$.

6. An examination of global stability

By developing a relevant Lyapunov function, the global stability, is to be discussed, of the considered Ammensal model in (2.1)–(2.2).

Theorem (*(6.1):*). *The proposed system globally asymptotic stable at the equilibrium point $E_2(x^*, y^*)$ for l is any positive constant.*

Proof: By the concept of Lyapunov function, consider.

$$V(x, y) = x - x^* - x^* \ln\left(\frac{x}{x^*}\right) + l\left(y - y^* - y\ln\left(\frac{y}{y^*}\right)\right) l > 0$$

Now$\frac{dv}{dt} = \frac{x - x^*}{x}\frac{dx}{dt} + l\frac{y - y^*}{y}\frac{dy}{dt}$.

$$\frac{dv}{dt} = (x - x^*)[\alpha_{11}x^* + \alpha_{12}y^* - \alpha_{11}x - \alpha_{12}y] + l(y - y^*)[\alpha_{22}y^* - \alpha_{22}y]$$

Choose l as any positive constant.

$$\frac{dv}{dt} \leqslant -\left(\alpha_{11} + \frac{\alpha_{12}}{2}\right)(x - x^*)^2 - \left(l\alpha_{22} + \frac{\alpha_{12}}{2}\right)(y - y^*)^2 < 0$$

Hence $E_2(x^*, y^*)$ is globally asymptotically stable.

7. Bionomic equilibria

Bionomic equilibrium is a concept that integrates the ideas of biological and economic equilibrium. When the total income received from selling gathered biomass equals the total cost of harvesting effort, the economic equilibrium is considered to have been established. Let c_1, c_2 the fishing costs per unit effort for both ammensal and adversarial species respectively. Let p_1, p_2 be the ammensal and adversarial species' prices per unit biomass, respectively. As a result, at any time t, the net income or financial rent is$M = M_1 + M_2$.

Where $M_1 = (p_1q_1x - c_1)E_1$ is the net economic revenue for ammensal species and $M_2 = (p_2q_2y - c_2)E_2$ is the net economic revenue for adversarial species at time t.

The bionomic equilibrium is calculated using the following equations:$a_1x - \alpha_{11}x^2 - \alpha_{12}xy - q_1E_1x = 0$,

$$a_2y - a_{22}y^2 - q_2E_2y = 0$$

$$R = (\rho_1q_1x - c_1)E_1 + (\rho_2q_2y - c_2)E_2 = 0$$

We investigate the following cases to determine their bionomic equilibrium.

Case (i): If,$c_1 > \rho_1q_1x$ and $c_2 > \rho_2q_2y$ then the cost exceeds the revenue for both species, the entire system will be terminated.

Case (ii): If the cost of getting the adversarial species exceeds the revenue$c_2 > \rho_2q_2y$, then availability for the adversarial species is not practicable. Ammensal species population remains operative, i.e. $(c_1 > \rho_1q_1x)$ Thus, when $E_2 = 0$ and $c_1 < \rho_1q_1x$ then we have,

$$(x)_\infty = \frac{c_1}{p_1q_1}, \; (y)_\infty = \frac{1}{\alpha_{12}}\left[a_1 - \alpha_{11}\frac{c_1}{p_1q_1} - q_1E_1\right]$$

Case (iii): If the cost of ammensal species fishing exceeds the revenue, the ammensal species fishing is not available (i.e. closed) i.e., $E_1 = 0$ and only adversarial species fishing will be allowed to continue. Therefore,

$$(x)_\infty = 0, \; (y)_\infty = \frac{c_2}{p_2q_2}$$

Case (iv): If $c_1 < \rho_1q_1x$ and$c_2 < \rho_2q_2y$. The revenue for the two species will then be positive, and the entire fishery gets operational, then we have.

$$(x)_\infty = \frac{c_1}{p_1q_1} \; (y)_\infty = \frac{c_2}{p_2q_2}$$

$$(E_1)_\delta = \frac{1}{q_1}\left[a_1 - \alpha_{11}\frac{c_1}{p_1q_1} - \alpha_{12}\frac{c_2}{p_2q_2}\right](E_2)_\delta = \frac{1}{q_2}\left[a_2 - \alpha_{22}\frac{c_2}{p_2q_2}\right]$$

Clearly $(E_1)_\delta > 0$ and $(E_2)_\delta > 0$ occurs for the following inequalities:

$a_1 > \frac{\alpha_{11}c_1}{p_1q_1} + \frac{\alpha_{12}c_2}{p_2q_2}$ and$a_2 > \frac{\alpha_{22}c_2}{p_2d_2}$.

So that, the non-trivial point of bionomic equilibrium $((x)_\infty, (y)_\infty, (E_1)_\infty, (E_2)_\infty)$ exit.

8. Qualitative study of the best harvesting policy

The essential issue in establishing an optimal harvest programme in a commercial fishery is establishing the ideal tradeoff between current and future harvests [30–36]. As Clark [30] points out, this challenge, which lies at the heart of resource conservation, is incredibly tough, perhaps not mathematically, but certainly politically and philosophically. Time discounting is the primary technique for resolving concerns of intertemporal economic gains. Although much debate exists regarding the concept's societal justification [37], time discounting is a common technique in company management. To calculate the ideal harvesting strategy, we consider the present value J of a continuous time-stream of revenues denoted by.

$$J = \int_0^{\infty} e^{-\delta t}((p_1 q_1 x - c_1)E_1(t) + (p_2 q_2 - c_2)E_2(t))dt \tag{8.1}$$

where δ is the current annual discount rate. From Pontryagin's maximum principle, maximize (8.1) while keeping the state equations (2.1)–(2.2) in mind. The control variable E (t) is subjected to the constrains, $0 \leq E_i(t) \leq (E_i)_{max}$

The Hamiltonian is provided by.

$$H = e^{-\delta t}\{(p_1 d_1 x - c_1)E_1(+) + (p_2 d_2 - c_2)E_2(+)\} \\ +\lambda_1\{a_1 x - \alpha_{11}x^2 - \alpha_{12}xy - q_1 E_1 x\} + \lambda_2\{a_2 y - \alpha_{22}y^2 - q_2 E_2 y\} \tag{8.2}$$

where λ_1 and λ_2 are the adjoint variables. In the H i.e. Hamiltonian function, the regulator variable $E(t)$ occurs linearly. The control restrictions are assumed to be non-binding, which is implying that there is no best solution at $(E_i)_{max}$. At $(E_i)_{max}$, there must have complete control. According to the maximal principle of Pontryagin's, we have.

$$\frac{\partial H}{\partial E_1} = 0, \frac{\partial H}{\partial E_2} = 0, \frac{d\lambda_1}{dt} = -\frac{\partial H}{\partial x}, \frac{d\lambda_2}{dt} = -\frac{\partial H}{\partial y}$$

Now, $$\lambda_1 = e^{-\delta t}\left(p_1 - \frac{c_1}{q_1 x^*}\right) \tag{8.3}$$

$$\lambda_2 = e^{-\delta t}\left(P_2 - \frac{c_2}{q_2 y^*}\right) \tag{8.4}$$

$$\frac{d\lambda_1}{dt} - \alpha_{11}x^*\lambda_1 = -e^{-\delta t}E_1 p_1 q_1 \tag{8.5}$$

$$\frac{d\lambda_2}{dt} - \alpha_{22}y^*\lambda_2 - \alpha_{12}x^*\lambda_1 = -e^{-\delta t}p_2 q_2 E_2 \tag{8.6}$$

Solve (8.5) we get.

$$\lambda_1 = B_1 e^{\alpha_{11}x^* t} + \frac{P_1 d_1 E_1}{\delta + \alpha_{11}x^*}e^{-\delta t} \tag{8.7}$$

Solve (8.6) we get.

$$\lambda_2 = B_2 e^{\alpha_{22}y^* t} + \frac{1}{(\delta - \alpha_{22}y^*)}\left[\frac{P_1 d_1 E_1}{\delta + \alpha_{11}x^*} - p_2 d_2 E_2\right] \\ + \frac{1}{\alpha_{11}x^* + \alpha_{22}y^*}B_1\alpha_{12}x^* e^{\alpha_{11}x^* t} \tag{8.8}$$

where B_1 and B_2 are arbitrary constants.

From (8.3) and (8.7),The singular path as.

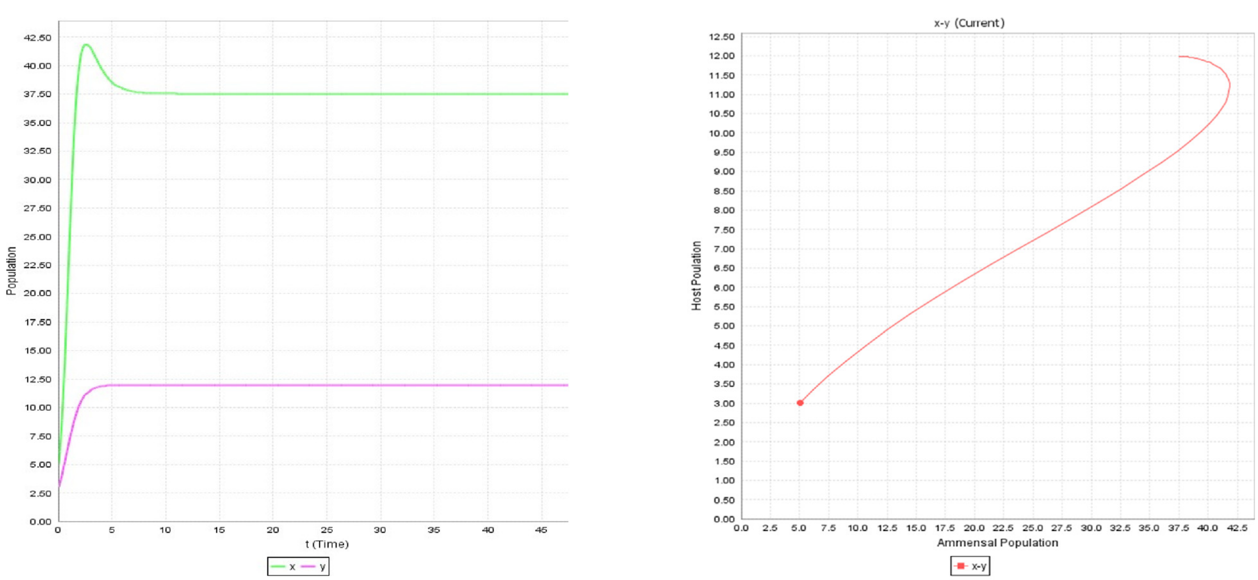
$$F_1(x) = \left(p_1 - \frac{c_1}{q_1 x^*}\right) + B_1 e^{(\alpha_{11}x^* + \delta)t} + \frac{P_1 d_1 E_1}{\delta + \alpha_{11}x^*} = 0$$

From (8.4) and (8.8),The singular path as.

$$F_2(x) = \left(p_2 - \frac{c_2}{q_2 y^*}\right) + B_2 e^{(\alpha_{22}y^* + \delta)t} \\ + \frac{1}{(\delta - \alpha_{22}y^*)}\left[\frac{P_1 d_1 E_1}{\delta + \alpha_{11}x^*} - p_2 d_2 E_2\right]e^{\delta t} + \frac{1}{\alpha_{11}x^* + \alpha_{22}y^*}B_1\alpha_{12}x^* e^{(\alpha_{11}x^* + \delta)t} = 0$$

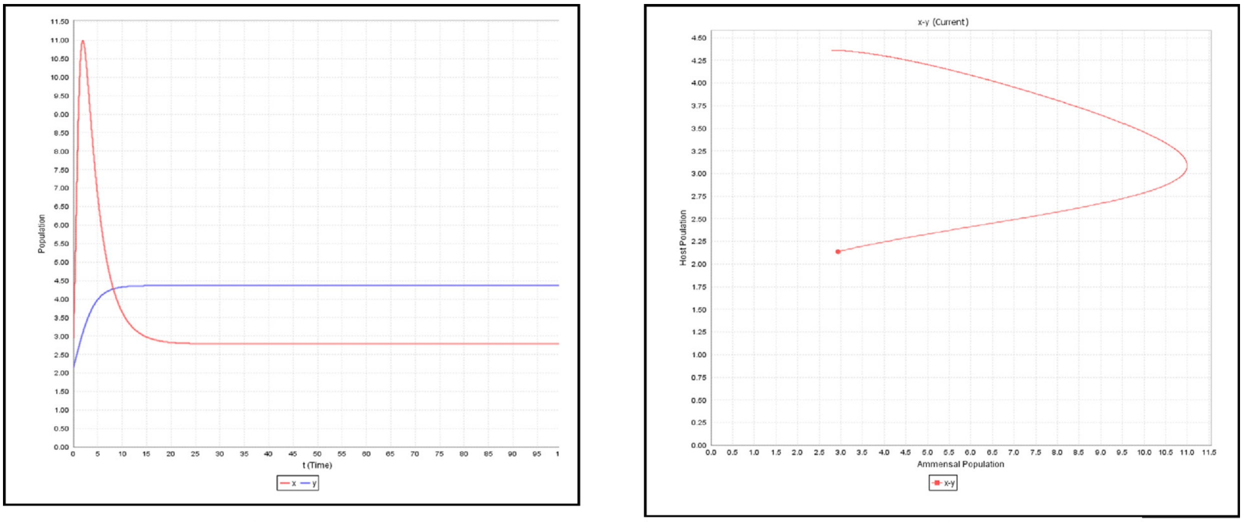
There is a distinct positive root that exists.

$y^* = (y)_\delta, x^* = (x)_\delta$ if $F(y^*) = 0, F(x^*) = 0$ In the interval $0 < y^* < K, 0 < x^* < K$



(a) (b)

Fig. 1 a. The population's variation versus time, with $x = 5; y = 3$ and deviation between ammensal and adversarial populations, for the parameters $a_1 = 3.15; \alpha_{11} = 0.02; \alpha_{12} = 0.02; q_1 = 0.15; E_1 = 5.84; a_2 = 2.5; \alpha_{22} = 0.12; q_2 = 0.2; E_2 = 5.31$. Fig. 1b. The phase portrait between the ammensal and adversial population for the parameters $a_1 = 3.15; \alpha_{11} = 0.02; \alpha_{12} = 0.02; q_1 = 0.15; E_1 = 5.84; a_2 = 2.5; \alpha_{22} = 0.12; q_2 = 0.2; E_2 = 5.31$.



(a) (b)

Fig. 2 a. The population's variation versus time, with $x = 3; y = 2$ and the deviation between ammensal and adversarial populations, for the parameters $a_1 = 4.18$; $\alpha_{11} = 0.11$; $\alpha_{12} = 0.7459$; $q_1 = 0.064$; $E_1 = 9.4$; $a_2 = 1.35$; $\alpha_{22} = 0.11$; $q_2 = 0.116$; $E_2 = 7.5$. Fig. 2b. he phase portrait between the ammensal and adversial population for the parameters $a_1 = 4.18$; $\alpha_{11} = 0.11$; $\alpha_{12} = 0.7459$; $q_1 = 0.064$; $E_1 = 9.4$; $a_2 = 1.35$; $\alpha_{22} = 0.11$; $q_2 = 0.116$; $E_2 = 7.5$.

If the inequality hold,$F(K) > 0$,$F(0) < 0$,$F'(x^*) > 0$, $F'(y^*) > 0$ for$x^* = (x)_\delta$, $y^* = (y)_\delta$ then$(E_1)_\delta = \frac{1}{q_1}\ [a_1 - \alpha_{11} \frac{c_1}{p_1 q_1} - \alpha_{12} \frac{c_2}{p_2 q_2}]$,and $(E_2)_\delta = \frac{1}{q_2}\left[a_2 - \alpha_{22}\frac{c_2}{p_2 q_2}\right]$

Hence, when the optimal equilibrium.

$\left[(x)_\delta, (y)_\delta\right]$ is determined, the optimal harvesting effect $(E_1)_\delta$ and $(E_2)_\delta$ also be determined easily. It is noted that $\lambda_i(t)\ e^{\delta t}$ where $i = 1, 2$ in an optimum equilibrium independent of time, Hence as$t \to \infty$, they remain the same.

9. Conclusions

This study resulted in the development of a particular model of two distinct species (Ammensal and Adversarial) that is effective in that it contains a syneco-system in which both species are harvested. With the help of the Routh–Hurwitz criteria and the Lyapunov function, the local stability and global stability of a system are thoroughly addressed and studied. With the use of Pontryagin's maximal principle, the concept of bionomic equilibrium, as well as the harvesting strategy, can be calculated for this model. The numerical results obtained are consistent with the analytical computations of an eco-system model consisting of two species, and as a result, this deterministic model is considered stable. Model stability is illustrated in Figs. 1 and 2, where the parameters of the system trajectory converge to a point inside the system's interior equilibrium point.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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