Design Consideration of Back-EMF Constant for 3-D.O.F. Spherical PM Motor

Sung-chul Go[1], Dong-woo Kang[1], Jong-bin Im[1], Ju Lee[1]*, Sung-hong Won[2], and Seung-Bin Lim[3]

[1]Department of Electrical Engineering, Hanyang University, Seoul 133-791, Korea
[2]Department of Electric System, Dongyang Technical College, Seoul 152-080, Korea
[3]Korea Institute of Energy Technology Evaluation and Planning, Seoul 135-280, Korea

A 3-D.O.F. spherical PM motor has 3 degrees of freedom in its motion by tilting and rotating of a shaft, which can be applied in a range of fields. The back-EMF is proportional to the field flux and angular velocity. The back-EMF constant in conventional rotating machine has a uniform value. However, in a spherical PM motor, the back-EMF constant of the coils varies according to the tilting conditions regardless of whether the angular speed is constant. Consideration of the back-EMF constant is useful for designing 3-D.O.F. spherical PM motors. In this study, the back-EMF constant of the spherical PM motor was considered carefully.

Keywords : design, spherical motor, back-EMF, 3 degree of freedom

1. Introduction

The conventional classification of motors is divided into linear and rotary motors. Therefore, the system for multi-D.O.F. is a combination of motors. The PM motors need to be compact and have high power characteristics. This system employs a multi-frame structure with rotary motors for objected motion. However, a combination of many motors requires high power, volume and mass.

A spherical motor has been studied to overcome of the abovementioned shortcomings. The motor can be operated with 3-D.O.F. Therefore, the operation characteristics of the motor is different from conventional motors. However, classification by torque generation mechanism of a spherical motor is similar to the conventional motors [1-4].

The back-EMF is proportional to the linkage flux and angular velocity. The back-EMF constant is a useful indication of the operating characteristics in the motor design and control. It is possible that the back-EMF in a specified angular velocity and rated speed can determine the voltage, current and power. A back-EMF forward compensation algorithm can be used to control the motor, but the concept is not directly applicable to a spherical PM motor.

The magnitude of back-EMF has the same value for

each phase coil in conventional PM motors because of the static rotating plane. All back-EMF vectors have the same direction vector and the direction vector can be ignored. The shaft of the spherical PM motor can move through a spherical surface and rotate in a specified shaft position. Therefore, the magnet poles rotate in different rotating planes under tilt conditions. The coils have the same back-EMF only when the shaft is positioned in the vertical direction. The magnitude of the back-EMF is not balanced in each phase coil if the shaft does not exist in that direction. This is due to the discordance coordinates between the coils and magnetic poles. Therefore, it is important to determine the back-EMF constant in a spherical PM motor using the back-EMF vector.

2. Principle of Spherical PM Motor Classification

2.1. Basic concept of spherical motor

It is difficult to explain the operating principle of a spherical PM motor because the shaft has movement in a spherical surface. Fig. 1(a) shows that the direction of torque in conventional PM motor is in the x-y plane. Fig. 1(b) shows that rotor of a spherical PM motor can move in 3-dimensions. The proper disposition of coils is essential for achieving correct positioning in the x-y-z axes. At least 6 coils are necessary for the operation of a spherical PM motor. Hence, the spherical motor has rotating and

*Corresponding author: Tel: +82-2-2220-0342
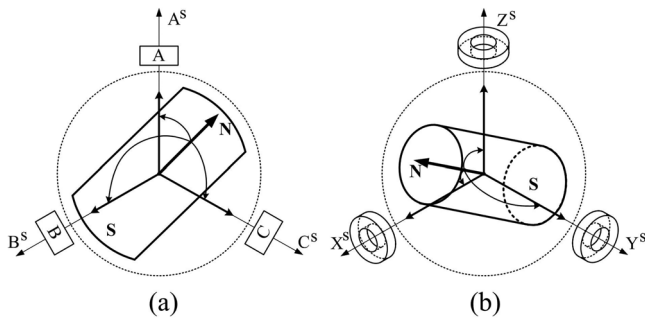Fax: +82-2-2295-7111, e-mail: julee@hanyang.ac.kr

Fig. 1. Movement region of the rotor in conventional and spherical PM motors.

tilting characteristics.

2.2. Structure Basic concept of spherical motor

Two poles and 6 coils are needed to satisfy the spherical operation. Fig. 2(a) shows a stator with 12 coils in 2 layers. Fig. 2(b) shows a rotor composed 2 pole pairs. In addition, the shaft can only move in the open surface of the stator. Some frame structures are essential to sup-
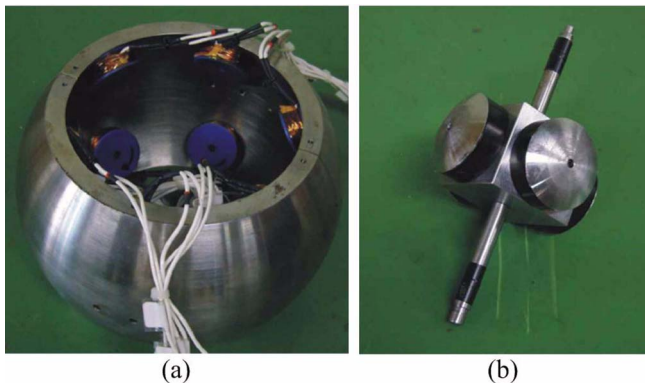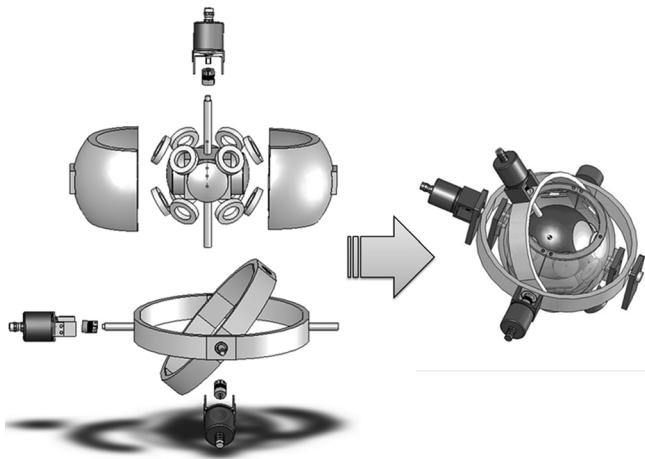

Fig. 2. Structure of a spherical PM motor.

Fig. 3. Composition of a 4 pole spherical PM motor.
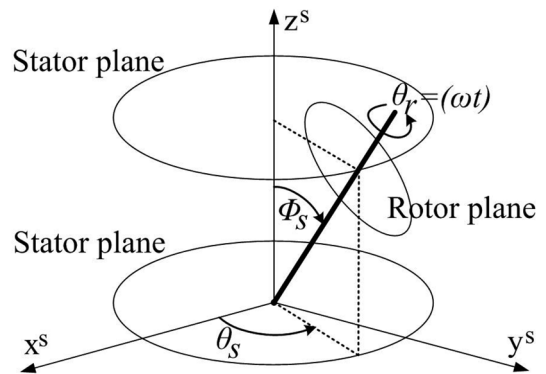

Fig. 4. Relationship between the shaft and stator plane of the spherical PM motor.

port rotor centering. Fig. 3 shows the precise composition of the spherical PM motor. No change in shaft position makes a stand-still of two frames. The shaft position is defined by the angles of Φ_s, θ_s, and $\theta_r(\omega t)$ in Fig. 4. In this case, the shaft position is defined as angle Φ_s from the z-axis first. The shaft position in the x-y plane is defined by θ_s. The rotation of the shaft is defined by $\theta_r(\omega t)$. The parameter ω is the angular velocity. This angle θ_r is related to the position of the magnet pole.

2.3. Coordination transformation

The position of the flux vector is the main element for obtaining the back-EMF. The coils are in the stator and the magnetic poles are in the rotor. A Z-Y-Z Euler transformation using Z-Y-Z Euler angles is a useful relative transformation [5]. The Z-Y-Z Euler transformation is shown in equation (1). Fig. 5 shows the α, β, γ rotation angles used in equation (1). The Z-Y-Z Euler transformation is a relative transformation. The vector position can be determined in different axes frames.

$$[\Gamma] = \begin{bmatrix} \cos(\alpha)\cos(\beta)\cos(\gamma) - \sin(\alpha)\sin(\gamma) \\ \sin(\alpha)\cos(\beta)\cos(\gamma) + \cos(\alpha)\sin(\gamma) \\ -\sin(\beta)\cos(\gamma) \end{bmatrix}$$
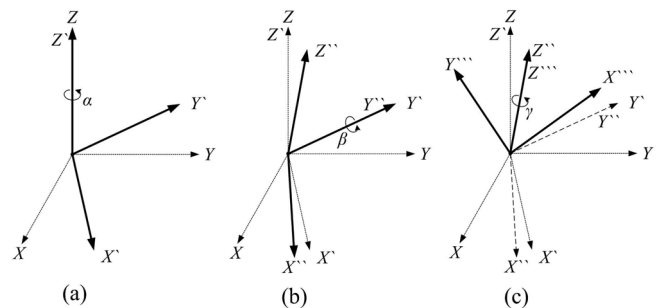

Fig. 5. Transformation by the Z-Y-Z transfer function.

$$\left.\begin{array}{cc} -\cos(\alpha)\cos(\beta)\sin(\gamma)-\sin(\alpha)\sin(\gamma) & \cos(\alpha)\sin(\beta) \\ -\sin(\alpha)\cos(\beta)\sin(\gamma)+\cos(\alpha)\cos(\gamma) & \sin(\alpha)\sin(\beta) \\ \sin(\beta)\sin(\gamma) & \cos(\beta) \end{array}\right] \quad (1)$$

3. Back-EMF Vector

The magnitudes of the back-EMF in stator coils are the affected position of the shaft in a spherical PM motor. Therefore, the concept of the back-EMF constant is necessary in a spherical PM motor. The back-EMF vector calculation is necessary for the back-EMF constant. The back-EMF is related to the linkage flux and velocity. Accordingly, the position vector of the coils, the vector of the airgap magnetic density, and the line velocity in the coils are essential for calculating the back-EMF vector.

3.1. Definition of position vector of coils

The definition of the coil position is essential for obtaining the linkage flux and line velocity in coils. The spherical PM motor has a total of 12 coils. The spherical coordinate is useful for defining the position vector of the coils in the stator. Therefore, the vector is defined in equation (2).

$$s=R_s[\sin(\phi)\cos(\theta) \quad \sin(\phi)\sin(\theta) \quad \cos(\phi)]^T \quad (2)$$

where R_s denotes the semi diameter between the diagonal coils. It was assumed that m numbers of coils are dispatched with the δ_c angle. The position vector of the *j* coil is expressed in equation (3).

$$s_j^s=R_s[\sin(\phi)\cos((j-1)\delta_c) \quad \sin(\phi)\cos((j-1)\delta_c) \quad \cos(\phi)]^T \quad (3)$$

$$(j = 1, 2, …, m)$$

3.2. Definition of magnetic density vector

The magnetic density is related to the position of the magnetic pole and rotor [6]. Therefore, the position vector of the magnetic pole is important. It is assumed that n numbers of magnetic poles are dispatched with a δ_m angle around the shaft. The position vector of the k magnetic poles r_k^r is expressed as equation (4).

$$r_k^r=R_r[\sin(\phi)\cos((k-1)\delta_m) \quad \sin(\phi)\cos((k-1)\delta_m) \quad \cos(\phi)]^T \quad (4)$$
$$(k = 1, 2, …, n)$$

The position vector of the magnetic poles r_k^r needs to transform to the stator plane because the induced EMF is calculated in the stator coils. Therefore, the position vector of the magnetic poles is transformed to orthogonal coordinates by the Z-Y-Z Euler angles (5).

$$r_k^s=[\Gamma]R_r[\sin(\phi)\cos((k-1)\delta_m) \quad \sin(\phi)\cos((k-1)\delta_m) \quad \cos(\phi)]^T \quad (5)$$

The vector of the magnetic flux density for back-EMF was calculated in the stator plane from r_k^r and the magnitude of the magnetic flux density using equation (6).

$$B_k^s=B_{max}r_k^s=B_{max}[\Gamma]r_k^r \quad (6)$$

The linkage flux in the *j* coil is obtained by the inner product of the magnetic flux density vector and position vector of the coils (7).

$$\Psi_j^s=k_\phi\sum_{k=1}^{n} B_k^s \cdot r_j^s \quad (7)$$

($j = 1, 2, …, m$, k_Φ is constant of linkage flux)

3.3. Definition of back-EMF Vector

The back-EMF vector was calculated by the outer product of the magnetic density vector and line velocity in coils. The line velocity was obtained first, as shown in Fig. 6. The unit direction vector of the rotor shaft is u_A, the position vector of the coil is u_R, and the vector of the line velocity is u_v. The relationship can be expressed as follows (8).

$$u_A=u_R \times u_v \quad (8)$$

Hence, u_v is expressed as equation (9).

$$u_R \times u_A=u_R(u_v \cdot u_R)-u_v(u_R \cdot u_R)=-u_v \quad (9)$$

The unit vector of the line velocity of the j coil considers the angular velocity as follows (10): ($j = 1, 2, …, m$)

$$v_j^s=\omega(u_A \times s_j^s) \quad (10)$$

The back-EMF vector in the coils was calculated as follows (11):

$$e_j^s=\Psi_j^s(u_R \times v_j^s) \quad (11)$$

Therefore, the back-EMF vector has a direction orthogonal to the direction of the position vector of the coils. The direction of rotation is constant. In addition, the upper coils and lower coils have the same direction of the
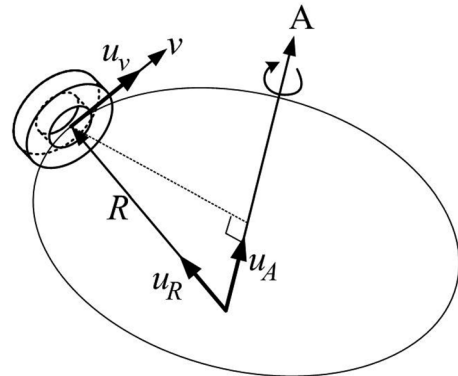

Fig. 6. Vector of the line velocity in a coil.

z-axis component. However, the upper coils have a central x and y axis direction. On the other hand, the lower coils are in the outer x and y axis direction of the back-EMF vector because the back-EMF vectors are orthogonal to the position vector of the coils.

3.4. Consideration of back-EMF constant

In a conventional motor, the relationship between the coils and poles are not changed in the steady state [7]. The back-EMF vector in the stator coils is affected by the shaft position because of the magnetic pole. In a conventional rotating motor, each phase coils has the same direction of the back-EMF vector. Therefore, the direction has been ignored. However, only the magnitude of the back-EMF cannot represent the spherical motor characteristic. The direction of the back-EMF vector is related to the position vector of the coil and rotating direction.

In different shaft tilt conditions, the magnitude of the back-EMF in each coil has a different value. Therefore, the same speed condition does not mean the same back-EMF measurement in each coil. Therefore, the back-EMF vector must be considered carefully. The three direction element of the back-EMF vector exists in the stator reference frame. In addition, the component in the shaft tilt condition was varied. The back-EMF is produced by coils and a magnet. The magnet direction affects the back-EMF. In addition, the position of the magnet is fixed in the rotor. Therefore, the back-EMF was considered in the rotor reference frame. In this case, the z-axis of the rotor is the direction of the shaft. In addition, the back-EMF components of the x and y axes are almost zero due to the back-EMF vector direction. Consequently, the component of the z-axis is the effective back-EMF vector in the rotor plane. The vector is transformed to the rotor reference frame. The Z-Y-Z Euler angles are as follows (12):

$$\alpha = \theta_s$$
$$\beta = \phi_s \qquad (12)$$
$$\gamma = \theta_r - \theta_s \cos \phi_s$$

The element of the z-axis in the rotor reference frame is expressed as (13).

$$e_Z^s = \left(\sum_{j=1}^{m} (u_A \cdot [\Gamma] e_j^s) \right) u_A \qquad (13)$$

The back-EMF constant was calculated by the back-EMF in the rotor shaft axis over the angular velocity (14).

$$k_e^r = \frac{e_z^r \cdot u_A}{\omega} \qquad (14)$$

In this case, the back-EMF constant is unaffected by the change in shaft position. Therefore, this concept is considered appropriate in a spherical PM motor.

In a spherical PM motor, the back-EMF constant means the maximum back-EMF in the range of all operations. In the vertical shaft position, rotation produces a lower back-EMF than that back-EMF calculated from the proposed back-EMF constant. However, some coils have a maximum back-EMF in the hard tilting operation condition.

4. Measurement of Back-EMF Constant

The back-EMF constant was verified by measuring the back-EMF. The back-EMF measurement system is composed of a spherical PM motor, load motor, and PXI instrument, as shown in Fig. 7. The PXI instrument is operated using Labview language and a FPGA module for data acquisition. Table 1 shows the specification of the test bed. The 4 poles spherical PM motor has basically 12 coils in two layers. Therefore, each coil has a different effect of magnet of rotor rotation. Therefore, it is impossible to consider all cases. Three cases were selected to consider the back-EMF in different shaft positions. Fig. 8 shows the cases. The 12 coils were named with AU, BU, CU, DU, EU, FU, AD, BD, CD, DD, ED, and FD in 60(°)
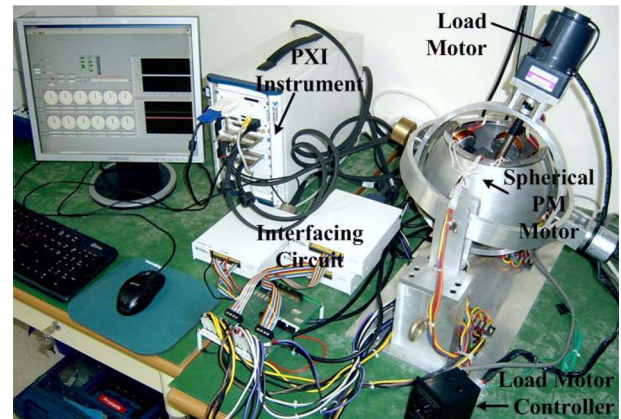

Fig. 7. Back-EMF measurement system of a spherical PM motor.

Table 1. Specification of the test bed.

Test system	Load motor	BLDC Motor (700W)+Controller
	Torque transducer	None
	Back-EMF measurement	NI PXI Instrument+Labview 8.5
Test target	Spherical PM Motor	4 poles, 12 coils
Test points	Angular speed	132 [rpm]
	Case I	no tilt
	Case II	Tilt to negative x-axis
	Case III	Tilt to negative y-axis

Table 2. Magnitude of Back-EMF in upper coils (132[rpm]).

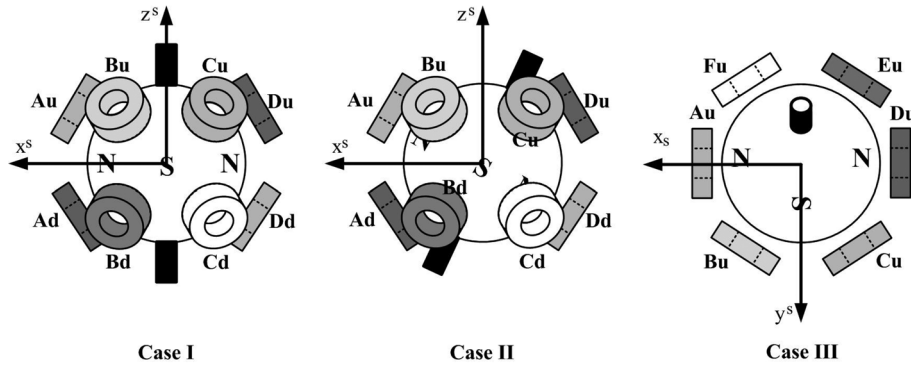| Case	AU Coil [V]	BU Coil [V]	CU Coil [V]	DU Coil [V]	EU Coil [V]	FU Coil [V]
Case I	0.4585	0.4323	0.4768	0.4503	0.3945	0.4441
Case II	0.6041	0.5256	0.3650	0.2005	0.3349	0.5119
Case III	0.4951	0.5584	0.5524	0.4829	0.2107	0.2111

Fig. 8. Determination of 3 cases for a back-EMF consideration.

intervals and 2 layers. The notation, AU, means the A phase and upper coils. The notation, FD, means the F phase and lower coils. The cases were determined to consider the various back-EMF magnitudes of 12 coils. Case I is the no tilt condition of the shaft, $\Phi_s=0(°)$ $\theta_s=0(°)$. Therefore, the magnet pole affects the same magnitude of the back-EMF to each coil. The condition of case II was tilted by the y-axis negative rotation in the rotor reference frame $\Phi_s=18(°)$ $\theta_s=180(°)$. Hence, the magnet pole is closest to the AU and DD coils, and magnitude of the back-EMF is highest in those coils. Case III is the tilt to the negative y-axis, $\Phi_s=18(°)$ $\theta_s=270(°)$. Therefore, the magnet pole is closest to the middle point of the BU and CU coils and the back-EMF magnitude reaches a peak in the BU, CU, ED and FD coils.

The system receives an encoder signal and back-EMF signal from a motor with interfacing. The configuration for the measurement was divided into 3 cases. Fig. 9 shows the back-EMF waveform of 3 cases. Fig. 9(a) and (b) presents the results of case I. Fig. 9(c) and (d) is shows case II and Fig. 9(e) and (f) shows case III. Table 2 shows the effective value of the back-EMF in the upper coils. The back-EMF was highest in the AU coil in the case II measurement.

The back-EMF constant was obtained by the back-EMF over speed. In addition, the constant did not change under the linear magnetic saturation region in a conventional motor. However, the back-EMF magnitudes of the coils are different in cases II and III. Therefore, the back-EMF was calculated in the in the rotor reference frame. In the

case I, the α and β Euler angles are both 0°. In the case II, the α the β Euler angle is 18(°) and 180(°), respectively. In the case III, the α and β Euler angle is 18(°) and 270(°), respectively. The γ Euler angle was determined by the position angle between the magnet and each coil. The back-EMF vector was calculated using equation (1) with the angles. Table 3 lists the back-EMF constant in the rotor x-y-z directions and the units are voltage per rad/s. The Euler angles were determined by the shaft position. The back-EMF constant of the z-axis was almost constant in the three cases. The difference in the z-axis back-EMF constant in Fig. 9 was considered to be measurement error. The back-EMF constant in the rotor reference frame was transferred from the value in the stator reference frame by Euler's transformation.

5. Conclusion

The magnitude of the back-EMF in a spherical PM motor varies according to the shaft position. It is not possible have a constant back-EMF in the rated speed of a spherical PM motor due to shaft movement. The advance of a spherical PM motor varies according to the operating condition in same speed rotation. However, the maximum back-EMF is important under the condition of rated rotation. A proper back-EMF constant consideration is a fixed back-EMF constant. In addition, the back-EMF is determined by magnet poles rotating frame. In a conventional rotating motor, the direction of the back-EMF was ignored because of the same direction. However, in a
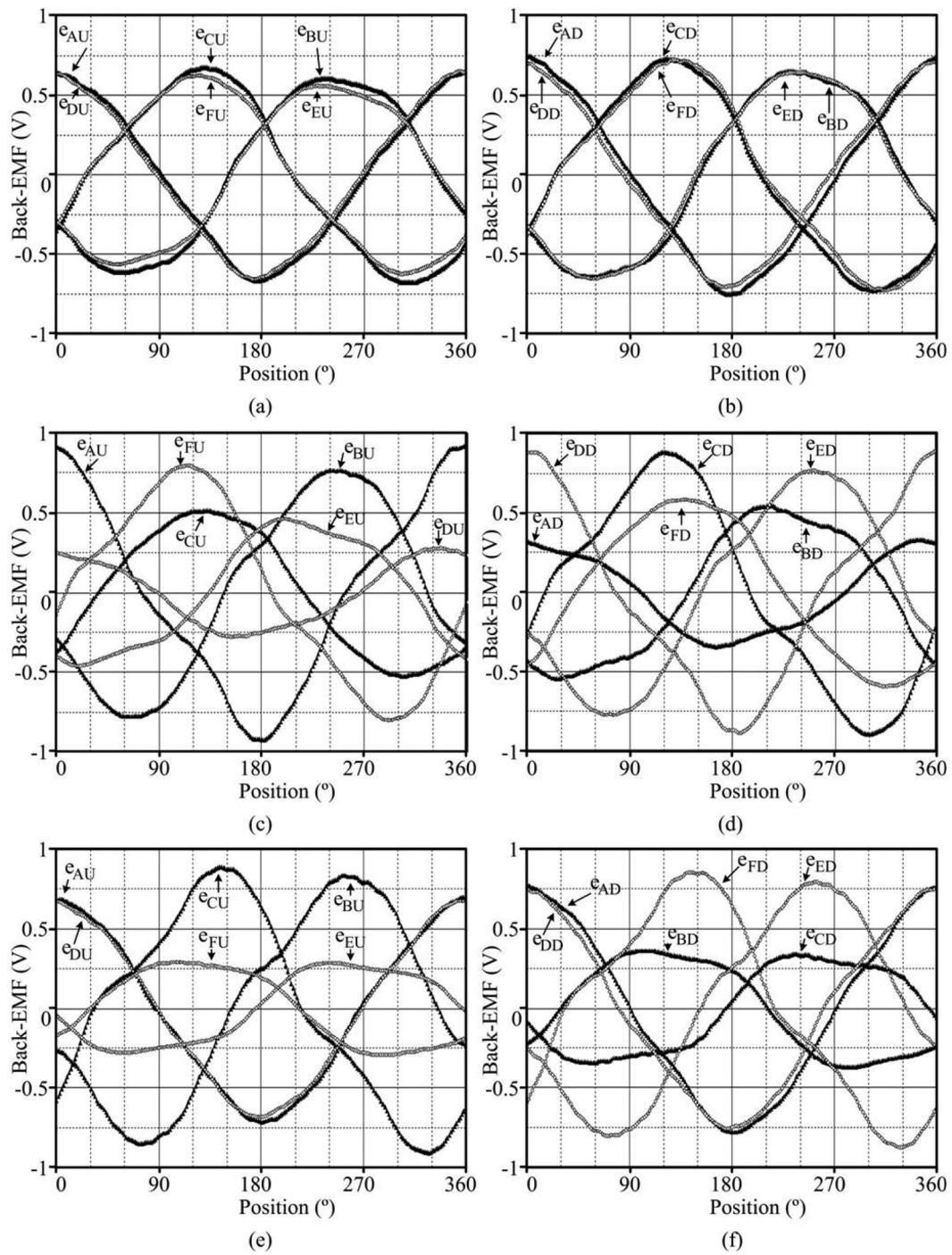
Fig. 9. Back-EMF waveform in three cases.

Table 3. Back-EMF constant in rotor reference frame.

Case	Euler angles (Degree)		Back-EMF constant (V/(rad/s)) In rotor reference frame		
	α	β	X-axis	Y-axis	Z-axis
Case I	0	0	0.0000	0.0000	0.0154
Case II	180	18	0.0042	0.0008	0.0147
Case III	270	18	0.0000	0.0037	0.0148

spherical PM motor, the direction of the back-EMF in each coil must be considered. The shaft direction varies according to the operation. The back-EMF is affected by the shaft position. In this case, the spherical PM motor designer must consider the limitation of the maximum operation. Therefore, the back-EMF vector can be used to indicate the motor design characteristics. The back-EMF

constant is affected by the shaft position. This consideration was verified by measuring the back-EMF in the three cases of the shaft position. Overall the back-EMF constant in a spherical PM motor is useful for designing and controlling the motor.

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