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Research Article

The Stability of a Quadratic Functional Equation with the Fixed Point Alternative

Choonkil Park[1] and Ji-Hye Kim[2]

[1] *Department of Mathematics, Research Institute for Natural Sciences, Hanyang University, Seoul 133-791, South Korea*
[2] *Department of Mathematics, Hanyang University, Seoul 133-791, South Korea*

Correspondence should be addressed to Ji-Hye Kim, saharin@hanyang.ac.kr

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Lee, An and Park introduced the quadratic functional equation $f(2x+y)+f(2x-y) = 8f(x)+2f(y)$ and proved the stability of the quadratic functional equation in the spirit of Hyers, Ulam and Th. M. Rassias. Using the fixed point method, we prove the generalized Hyers-Ulam stability of the quadratic functional equation in Banach spaces.

1. Introduction

The stability problem of functional equations originated from a question of Ulam [1] concerning the stability of group homomorphisms. Hyers [2] gave a first affirmative partial answer to the question of Ulam for Banach spaces. Hyers' Theorem was generalized by Aoki [3] for additive mappings and by Th. M. Rassias [4] for linear mappings by considering an unbounded Cauchy difference.

Theorem 1.1 (Th. M. Rassias). *Let $f : E \to E'$ be a mapping from a normed vector space E into a Banach space E' subject to the inequality*

$$\left\| f(x+y) - f(x) - f(y) \right\| \leq \epsilon\left(\|x\|^p + \|y\|^p\right) \tag{1.1}$$

for all $x, y \in E$, where ϵ and p are constants with $\epsilon > 0$ and $p < 1$. Then the limit

$$L(x) = \lim_{n\to\infty} \frac{f(2^n x)}{2^n} \tag{1.2}$$

exists for all $x \in E$, and $L : E \to E'$ is the unique additive mapping which satisfies

$$\|f(x) - L(x)\| \leq \frac{2\epsilon}{2 - 2^p}\|x\|^p \tag{1.3}$$

for all $x \in E$. Also, if for each $x \in E$ the function $f(tx)$ is continuous in $t \in \mathbb{R}$, then L is $\mathbb{R}$-linear.

The above inequality (1.1) has provided a lot of influence in the development of what is now known as a *generalized Hyers-Ulam stability* of functional equations. Beginning around the year 1980, the topic of approximate homomorphisms, or the stability of the equation of homomorphism, was studied by a number of mathematicians. Găvruta [5] generalized the Rassias' result.

Theorem 1.2 (see [6–8]). *Let X be a real normed linear space and Y a real complete normed linear space. Assume that $f : X \to Y$ is an approximately additive mapping for which there exist constants $\theta \geq 0$ and $p \in \mathbb{R} - \{1\}$ such that f satisfies inequality*

$$\|f(x+y) - f(x) - f(y)\| \leq \theta \cdot \|x\|^{p/2} \cdot \|y\|^{p/2} \tag{1.4}$$

for all $x, y \in X$. Then there exists a unique additive mapping $L : X \to Y$ satisfying

$$\|f(x) - L(x)\| \leq \frac{\theta}{|2^p - 2|}\|x\|^p \tag{1.5}$$

for all $x \in X$. If, in addition, $f : X \to Y$ is a mapping such that the transformation $t \to f(tx)$ is continuous in $t \in \mathbb{R}$ for each fixed $x \in X$, then L is an $\mathbb{R}$-linear mapping.

The functional equation

$$f(x+y) + f(x-y) = 2f(x) + 2f(y) \tag{1.6}$$

is called a *quadratic functional equation*. In particular, every solution of the quadratic functional equation is said to be a *quadratic function*. A generalized Hyers-Ulam stability problem for the quadratic functional equation was proved by Skof [9] for mappings $f : X \to Y$, where X is a normed space and Y is a Banach space. Cholewa [10] noticed that the theorem of Skof is still true if the relevant domain X is replaced by an Abelian group. Czerwik [11] proved the generalized Hyers-Ulam stability of the quadratic functional equation. Several functional equations have been investigated in [12–25].

Let X be a set. A function $d : X \times X \to [0, \infty]$ is called a *generalized metric* on X if d satisfies

(1) $d(x, y) = 0$ if and only if $x = y$;

(2) $d(x, y) = d(y, x)$ for all $x, y \in X$;

(3) $d(x, z) \leq d(x, y) + d(y, z)$ for all $x, y, z \in X$.

We recall a fundamental result in fixed point theory.

Theorem 1.3 (see [26–28]). *Let (X, d) be a complete generalized metric space and let $J : X \to X$ be a strictly contractive mapping with Lipschitz constant $L < 1$. Then for each given element $x \in X$, either*

$$d\left(J^n x, J^{n+1} x\right) = \infty \tag{1.7}$$

for all nonnegative integers n or there exists a positive integer n_0 such that

(1) $d(J^n x, J^{n+1} x) < \infty$, *for all $n \geq n_0$;*

(2) *the sequence $\{J^n x\}$ converges to a fixed point y^* of J;*

(3) y^* *is the unique fixed point of J in the set $Y = \{y \in X \mid d(J^{n_0} x, y) < \infty\}$;*

(4) $d(y, y^*) \leq (1/(1-L)) d(y, Jy)$ *for all $y \in Y$.*

Lee et al. [29] proved that a mapping $f : X \to Y$ satisfies

$$f(2x + y) + f(2x - y) = 8f(x) + 2f(y) \tag{1.8}$$

for all $x, y \in X$ if and only if the mapping $f : X \to Y$ satisfies

$$f(x + y) + f(x - y) = 2f(x) + 2f(y) \tag{1.9}$$

for all $x, y \in X$.

Using the fixed point method, Park [14] proved the generalized Hyers-Ulam stability of the quadratic functional equation

$$f(2x + y) = 4f(x) + f(y) + f(x + y) - f(x - y) \tag{1.10}$$

in Banach spaces.

In this paper, using the fixed point method, we prove the generalized Hyers-Ulam stability of the quadratic functional equation (1.8) in Banach spaces.

Throughout this paper, assume that X is a normed vector space with norm $|| \cdot ||$ and that Y is a Banach space with norm $\| \cdot \|$.

2. Fixed Points and Generalized Hyers-Ulam Stability of a Quadratic Functional Equation

For a given mapping $f : X \to Y$, we define

$$Cf(x, y) := f(2x + y) + f(2x - y) - 8f(x) - 2f(y) \tag{2.1}$$

for all $x, y \in X$.

Using the fixed point method, we prove the generalized Hyers-Ulam stability of the quadratic functional equation $Cf(x, y) = 0$.

Theorem 2.1. *Let $f : X \to Y$ be a mapping for which there exists a function $\varphi : X^2 \to [0,\infty)$ with $f(0) = 0$ such that*

$$\|Df(x,y)\| \le \varphi(x,y) \tag{2.2}$$

for all $x, y \in X$. If there exists an $L < 1$ such that $\varphi(x,y) \le 4L\varphi(x/2, y/2)$ for all $x, y \in X$, then there exists a unique quadratic mapping $Q : X \to Y$ satisfying (1.8) *and*

$$\|f(x) - Q(x)\| \le \frac{1}{8-8L}\varphi(x,0) \tag{2.3}$$

for all $x \in X$.

Proof. Consider the set

$$S := \{g : X \longrightarrow Y\}, \tag{2.4}$$

and introduce the *generalized metric* on S:

$$d(g,h) = \inf\{K \in \mathbb{R}_+ : \|g(x) - h(x)\| \le K\varphi(x,0),\ \forall x \in X\}. \tag{2.5}$$

It is easy to show that (S,d) is complete.

Now we consider the linear mapping $J : S \to S$ such that

$$Jg(x) := \frac{1}{4}g(2x) \tag{2.6}$$

for all $x \in X$.

By [30, Theorem 3.1],

$$d(Jg, Jh) \le Ld(g,h) \tag{2.7}$$

for all $g, h \in S$.

Letting $y = 0$ in (2.2), we get

$$\|2f(2x) - 8f(x)\| \le \varphi(x,0) \tag{2.8}$$

for all $x \in X$. So

$$\left\|f(x) - \frac{1}{4}f(2x)\right\| \le \frac{1}{8}\varphi(x,0) \tag{2.9}$$

for all $x \in X$. Hence $d(f, Jf) \le 1/8$.

By Theorem 1.3, there exists a mapping $Q : X \to Y$ such that
(1) Q is a fixed point of J, that is,

$$Q(2x) = 4Q(x) \tag{2.10}$$

for all $x \in X$. The mapping Q is a unique fixed point of J in the set

$$M = \{g \in S : d(f, g) < \infty\}. \tag{2.11}$$

This implies that Q is a unique mapping satisfying (2.10) such that there exists $K \in (0, \infty)$ satisfying

$$\|f(x) - Q(x)\| \leq K\varphi(x, 0) \tag{2.12}$$

for all $x \in X$.
(2) $d(J^n f, Q) \to 0$ as $n \to \infty$. This implies the equality

$$\lim_{n\to\infty} \frac{f(2^n x)}{4^n} = Q(x) \tag{2.13}$$

for all $x \in X$.
(3) $d(f, Q) \leq (1/(1-L))d(f, Jf)$, which implies the inequality

$$d(f, Q) \leq \frac{1}{8 - 8L}. \tag{2.14}$$

This implies that the inequality (2.3) holds.
It follows from (2.2) and (2.13) that

$$\|CQ(x, y)\| = \lim_{n\to\infty} \frac{1}{4^n} \|Cf(2^n x, 2^n y)\| \leq \lim_{n\to\infty} \frac{1}{4^n} \varphi(2^n x, 2^n y) \leq \lim_{n\to\infty} L^n \varphi(x, y) = 0 \tag{2.15}$$

for all $x, y \in X$. So $CQ(x, y) = 0$ for all $x, y \in X$.
By [29, Proposition 2.1], the mapping $Q : X \to Y$ is quadratic, as desired. □

Corollary 2.2. *Let $0 < p < 2$ and θ be positive real numbers, and let $f : X \to Y$ be a mapping such that*

$$\|Cf(x, y)\| \leq \theta(\|x\|^p + \|y\|^p) \tag{2.16}$$

for all $x, y \in X$. Then there exists a unique quadratic mapping $Q : X \to Y$ satisfying (1.8) *and*

$$\|f(x) - Q(x)\| \leq \frac{\theta}{8 - 2^{p+1}} \|x\|^p \tag{2.17}$$

for all $x \in X$.

Proof. The proof follows from Theorem 2.1 by taking

$$\varphi(x,y) := \theta\left(\|x\|^p + \|y\|^p\right) \tag{2.18}$$

for all $x, y \in X$. Then $L = 2^{p-2}$, and we get the desired result. □

Theorem 2.3. *Let $f : X \to Y$ be a mapping for which there exists a function $\varphi : X^2 \to [0,\infty)$ satisfying (2.2) and $f(0) = 0$. If there exists an $L < 1$ such that $\varphi(x,y) \le (L/4)\varphi(2x,2y)$ for all $x, y \in X$, then there exists a unique quadratic mapping $Q : X \to Y$ satisfying (1.8) and*

$$\|f(x) - Q(x)\| \le \frac{L}{8-8L}\varphi(x,0) \tag{2.19}$$

for all $x \in X$.

Proof. We consider the linear mapping $J : S \to S$ such that

$$Jg(x) := 4g\left(\frac{x}{2}\right) \tag{2.20}$$

for all $x \in X$.

It follows from (2.8) that

$$\left\|f(x) - 4f\left(\frac{x}{2}\right)\right\| \le \frac{1}{2}\varphi\left(\frac{x}{2},0\right) \le \frac{L}{8}\varphi(x,0) \tag{2.21}$$

for all $x \in X$. Hence $d(f, Jf) \le L/8$.

By Theorem 1.3, there exists a mapping $Q : X \to Y$ such that

(1) Q is a fixed point of J, that is,

$$Q(2x) = 4Q(x) \tag{2.22}$$

for all $x \in X$. The mapping Q is a unique fixed point of J in the set

$$M = \{g \in S : d(f,g) < \infty\}. \tag{2.23}$$

This implies that Q is a unique mapping satisfying (2.22) such that there exists $K \in (0,\infty)$ satisfying

$$\|f(x) - Q(x)\| \le K\varphi(x,0) \tag{2.24}$$

for all $x \in X$.

(2) $d(J^n f, Q) \to 0$ as $n \to \infty$. This implies the equality

$$\lim_{n\to\infty} 4^n f\left(\frac{x}{2^n}\right) = Q(x) \tag{2.25}$$

for all $x \in X$.

(3) $d(f,Q) \leq (1/(1-L))d(f,Jf)$, which implies the inequality

$$d(f,Q) \leq \frac{L}{8-8L}, \tag{2.26}$$

which implies that the inequality (2.19) holds.

The rest of the proof is similar to the proof of Theorem 2.1. □

Corollary 2.4. *Let $p > 2$ and θ be positive real numbers, and let $f : X \to Y$ be a mapping satisfying* (2.16)*. Then there exists a unique quadratic mapping $Q : X \to Y$ satisfying* (1.8) *and*

$$\|f(x) - Q(x)\| \leq \frac{\theta}{2^{p+1}-8}\|x\|^p \tag{2.27}$$

for all $x \in X$.

Proof. The proof follows from Theorem 2.3 by taking

$$\varphi(x,y) := \theta\left(\|x\|^p + \|y\|^p\right) \tag{2.28}$$

for all $x, y \in X$. Then $L = 2^{2-p}$ and, we get the desired result. □

Theorem 2.5. *Let $f : X \to Y$ be a mapping for which there exists a function $\varphi : X^2 \to [0,\infty)$ satisfying* (2.2)*. If there exists an $L < 1$ such that $\varphi(x,y) \leq 9L\varphi(x/3, y/3)$ for all $x, y \in X$, then there exists a unique quadratic mapping $Q : X \to Y$ satisfying* (1.8) *and*

$$\|f(x) - Q(x)\| \leq \frac{1}{9-9L}\varphi(x,x) \tag{2.29}$$

for all $x \in X$.

Proof. Consider the set

$$S := \{g : X \longrightarrow Y\}, \tag{2.30}$$

and introduce the *generalized metric* on S:

$$d(g,h) = \inf\{K \in \mathbb{R}_+ : \|g(x) - h(x)\| \leq K\varphi(x,x),\ \forall x \in X\}. \tag{2.31}$$

It is easy to show that (S,d) is complete.

Now we consider the linear mapping $J : S \to S$ such that

$$Jg(x) := \frac{1}{9}g(3x) \tag{2.32}$$

for all $x \in X$.

By [30, Theorem 3.1],

$$d(Jg, Jh) \leq L d(g, h) \tag{2.33}$$

for all $g, h \in S$.

Letting $y = x$ in (2.2), we get

$$\|f(3x) - 9f(x)\| \leq \varphi(x, x) \tag{2.34}$$

for all $x \in X$. So

$$\left\| f(x) - \frac{1}{9} f(3x) \right\| \leq \frac{1}{9} \varphi(x, x) \tag{2.35}$$

for all $x \in X$. Hence $d(f, Jf) \leq 1/9$.

By Theorem 1.3, there exists a mapping $Q : X \to Y$ such that

(1) Q is a fixed point of J, that is,

$$Q(3x) = 9Q(x) \tag{2.36}$$

for all $x \in X$. The mapping Q is a unique fixed point of J in the set

$$M = \{g \in S : d(f, g) < \infty\}. \tag{2.37}$$

This implies that Q is a unique mapping satisfying (2.36) such that there exists $K \in (0, \infty)$ satisfying

$$\|f(x) - Q(x)\| \leq K \varphi(x, x) \tag{2.38}$$

for all $x \in X$.

(2) $d(J^n f, Q) \to 0$ as $n \to \infty$. This implies the equality

$$\lim_{n \to \infty} \frac{f(3^n x)}{9^n} = Q(x) \tag{2.39}$$

for all $x \in X$.

(3) $d(f, Q) \leq (1/(1 - L)) d(f, Jf)$, which implies the inequality

$$d(f, Q) \leq \frac{1}{9 - 9L}. \tag{2.40}$$

This implies that the inequality (2.29) holds.

It follows from (2.2) and (2.39) that

$$\|CQ(x, y)\| = \lim_{n \to \infty} \frac{1}{9^n} \|Cf(3^n x, 3^n y)\| \leq \lim_{n \to \infty} \frac{1}{9^n} \varphi(3^n x, 3^n y) \leq \lim_{n \to \infty} L^n \varphi(x, y) = 0 \tag{2.41}$$

for all $x, y \in X$. So $CQ(x, y) = 0$ for all $x, y \in X$.

By [29, Proposition 2.1], the mapping $Q : X \to Y$ is quadratic, as desired. □

Corollary 2.6. *Let $0 < p < 2$ and θ be positive real numbers, and let $f : X \to Y$ be a mapping satisfying* (2.16). *Then there exists a unique quadratic mapping $Q : X \to Y$ satisfying* (1.8) *and*

$$\|f(x) - Q(x)\| \le \frac{2\theta}{9 - 3^p}\|x\|^p \tag{2.42}$$

for all $x \in X$.

Proof. The proof follows from Theorem 2.5 by taking

$$\varphi(x, y) := \theta\left(\|x\|^p + \|y\|^p\right) \tag{2.43}$$

for all $x, y \in X$. Then $L = 3^{p-2}$ and, we get the desired result. □

Corollary 2.7. *Let $0 < p < 1$ and θ be positive real numbers, and let $f : X \to Y$ be a mapping such that*

$$\|Df(x, y)\| \le \theta \cdot \|x\|^p \cdot \|y\|^p \tag{2.44}$$

for all $x, y \in X$. Then there exists a unique quadratic mapping $Q : X \to Y$ satisfying (1.8) *and*

$$\|f(x) - Q(x)\| \le \frac{\theta}{9 - 9^p}\|x\|^{2p} \tag{2.45}$$

for all $x \in X$.

Proof. The proof follows from Theorem 2.5 by taking

$$\varphi(x, y) := \theta \cdot \|x\|^p \cdot \|y\|^p \tag{2.46}$$

for all $x, y \in X$. Then $L = 9^{p-1}$ and, we get the desired result. □

Theorem 2.8. *Let $f : X \to Y$ be a mapping for which there exists a function $\varphi : X^2 \to [0, \infty)$ satisfying* (2.2). *If there exists an $L < 1$ such that $\varphi(x, y) \le (L/9)\varphi(3x, 3y)$ for all $x, y \in X$, then there exists a unique quadratic mapping $Q : X \to Y$ satisfying* (1.8) *and*

$$\|f(x) - Q(x)\| \le \frac{L}{9 - 9L}\varphi(x, x) \tag{2.47}$$

for all $x \in X$.

Proof. We consider the linear mapping $J : S \to S$ such that

$$Jg(x) := 9g\left(\frac{x}{3}\right) \tag{2.48}$$

for all $x \in X$.

The rest of the proof is similar to the proof of Theorem 2.1. □

Corollary 2.9. *Let $p > 2$ and θ be positive real numbers, and let $f : X \to Y$ be a mapping satisfying* (2.16)*. Then there exists a unique quadratic mapping $Q : X \to Y$ satisfying* (1.8) *and*

$$\|f(x) - Q(x)\| \leq \frac{2\theta}{3^p - 9}\|x\|^p \tag{2.49}$$

for all $x \in X$.

Proof. The proof follows from Theorem 2.8 by taking

$$\varphi(x, y) := \theta\left(\|x\|^p + \|y\|^p\right) \tag{2.50}$$

for all $x, y \in X$. Then $L = 3^{2-p}$, and we get the desired result. □

Corollary 2.10. *Let $p > 1$ and θ be positive real numbers, and let $f : X \to Y$ be a mapping satisfying* (2.44)*. Then there exists a unique quadratic mapping $Q : X \to Y$ satisfying* (1.8) *and*

$$\|f(x) - Q(x)\| \leq \frac{\theta}{9^p - 9}\|x\|^{2p} \tag{2.51}$$

for all $x \in X$.

Proof. The proof follows from Theorem 2.8 by taking

$$\varphi(x, y) := \theta \cdot \|x\|^p \cdot \|y\|^p \tag{2.52}$$

for all $x, y \in X$. Then $L = 9^{1-p}$, and we get the desired result. □

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