Testing for risk spillover between stock market and foreign exchange market in Korea

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Abstract

We investigate how risk spills over between stock market and foreign exchange market in Korea where risk is defined by extreme negative values below 5% value at risk. For this purpose, we employ Granger causality tests in risk proposed by Hong, Liu, and Wang (2009). We compare the results from Granger causality test in risk with the results from traditional Granger causality test in mean. In the 1992-2009 sample periods, we find that causality in risk runs in both directions while Granger causality in mean runs only from stock returns to foreign exchange returns. This result suggests that joint dynamics of stock returns and foreign exchange returns in the left tail of the distribution are likely to be different from those in the rest of the distribution. The results, however, depend on whether to include the 1997 Asian crisis period and 2008 global financial crisis period.

Keywords : stock market; foreign exchange; risk spillover

JEL classification : G11, G15

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1 Introduction

During times of financial crisis, large swings of asset prices tend to spread across different markets. Such risk spillover is of great concern to investors for optimizing portfolio performance. Understanding risk spillover is also crucial to policy-makers for prudential supervision of financial markets and financial institutions.

In Korea, an important risk spillover may have taken place between stock market and foreign exchange market. Indeed, Korea has witnessed both a huge decline in stock prices and a rapid increase in exchange rate simultaneously during the period of financial crisis. Despite the clear contemporaneous relationship between stock prices and foreign exchange rates, however, the issue of joint dynamics in the periods of financial turmoil remains unanswered.

This paper attempts to examine the mechanism of risk spillover between stock market and foreign exchange market in Korea. For this purpose, we employ a new test procedure, Granger causality test in risk, proposed by Hong, Liu, and Wang (2009; HLW hereafter). The new concept of Granger causality in risk is designed to detect extreme downside risk spillover, where risk is measured by value at risk (VaR). The VaR has been popularly used as a standard measure of the risk. Formally, the risk is defined as a binary process, where it takes the value of one when the return goes below the VaR level. Then, the test makes use of sample cross correlations of the risks in two different markets. In this regard, the test is fundamentally different from tests for Granger causality in mean or in volatility.

It has long been argued that the mechanism governing the behavior of the tails may be different from that of the rest of the distribution in financial asset prices. Also, the risk is widely known to be closely related with the behavior of left tails in the return distributions. Thus, it is reasonable to use approaches based on VaR or conditional quantiles in analyzing possible risk spillover between two returns. In this respect, we compare the test results from Granger causality in risk with the test results from traditional Granger causality in mean to highlight the different joint behavior of asset prices in the left tails of distributions.

This paper is organized as follows. Section 2 describes testing procedure for Granger causality in risk. Section 3 provides empirical results on risk spillover between stock market and foreign exchange market in Korea. Section 4 concludes.

2 Methods to test for Granger causality in risk

In this section, we describe testing procedure for Granger-causality in risk. Denote the KOSPI index and Won/Dollar exchange rate (FX, hereafter) as X and Y, respectively. Define the stock return and FX return as

$$\begin{aligned} x_t &= \ln(X_t) - \ln(X_{t-1}) \\ y_t &= -(\ln(Y_t) - \ln(Y_{t-1})). \end{aligned} \tag{1}$$

We introduce conditional value at risk (VaR) in terms of conditional quantiles. The conditional VaR at the confidence level $\theta \in [0, 1]$ is defined as the value of x_t that is exceeded with the probability $1 - \theta$,

$$V_t(x;\theta) = \inf_v [v : P_t(x_t \leq v) \geq \theta], \tag{2}$$

thus, $V_t(x;\theta) = F_{x_t}^{-1}(\theta | x_t)$, that is to say, the inverse of conditional distribution function of x_t. The conditional VaR for y_t, denoted as $V_t(y;\theta)$ is similarly defined.

There have been some methods to compute the conditional VaR in terms of regression quantiles. In general, the $V_t(x;\theta)$ can be naturally modelled as a function of certain state (information) variables s_t,

$$V_t(x;\theta) = f(s_t, \theta), \tag{3}$$

where s_t is k-variate state variables $s_t = (s_{1t}, ..., s_{kt})'$. In particular, we narrow down our analysis to an autoregressive type linear model for conditional quantiles

$$V_t(x;\theta) = \beta_0(\theta) + \beta_1(\theta) x_{t-1}. \tag{4}$$

As is simple to use, linear models are popular in empirical studies (e.g., Chernouzukov and Umantsev (2001)). Other popular approximations include location-scale, non-location scale and polynomial models. See discussions in Engle and Manganellie (2004). The estimates of the parameter $\beta(\theta) = (\beta_0(\theta), \beta_1(\theta))'$ can be obtained through linear optimization by Koenker and Basset (1978),

$$\beta(\theta) = \arg\min_\beta [\sum_{t=1}^T \rho_\theta (x_t - \beta_0(\theta) - \beta_1(\theta) x_{t-1})], \tag{5}$$

where $\rho_\theta(u) = u[\theta - 1(u < 0)]$, for indicator function $1(.)$, and T is the sample size. The same method applies to computing $V_t(y;\theta)$ for FX returns. In our work, we fix the value $\theta = 0.05$, which is associated with the downside risk implied by the left-tails of the return distribution.

Next, define the risk indicator for KOSPI return and for FX return,

$$x_\theta(t) = 1[x_t \le -V_t(x;\theta)], \qquad y_\theta(t) = 1[y_t \le -V_t(y;\theta)]. \tag{6}$$

The risk variables take the value of one when the return goes below the VaR level. These binary processes are commonly called as hits. Testing for risk spillover is based on cross correlations of these risk variables.

Our main interests are whether there exists Granger-causality in risk between KOSPI return and FX return. The method to test for Granger-causality is basically employed from HLW. First, denote the information sets $I_x(t-1) = (x_{t-1}, x_{t-2}, ...)$, $I_y(t-1) = (y_{t-1}, y_{t-2}, ...)$, and $I_{xy}(t-1) = (I_x(t-1), I_y(t-1))$. Then, we formally write the null hypothesis that y does not Granger-cause x in risk at level θ,

$$H_0 : P[x_t \le -V_t(x;\theta)|I_x(t-1)] = P[x_t \le -V_t(x;\theta)|I_{xy}(t-1)]. \tag{7}$$

Simply put, the above null hypothesis implies that there exists no risk spillover from FX return to KOSPI return. The other directional Granger causlaity is also similarly formulated. Negation of this null hypothesis, due to comovement between two risk measures, can come not only from mean and variance but from higher-order moments in return processes (cf: HLW (2009) for more discussion).

Using the risk variable defined above, the null hypothesis is alternatively represented as

$$H_0 : E[x_\theta(t)|I_x(t-1)] = E[x_\theta(t)|I_{xy}(t-1)], \tag{8}$$

which makes it possible to test for the hypothesis in terms of cross correlations of risk variables. Under the null hypothesis, cross covariance between $x_\theta(t)$ and $y_\theta(t-j)$ for $j \ge 1$ should be equal to zero. By similar reasoning, we can test Granger-causality in risk from KOSPI return to FX return by looking at cross covariance between $y_\theta(t)$ and $x_\theta(t-j)$ for $j \ge 1$. In this regard, a natural testing procedure is based on suitably standardized sample cross covariances.

Specifically, steps for testing are as follows. Given conditional quantile estimates, we obtain risk processes, $\widehat{x}_\theta(t) = 1[x_t \le -V_t(x;\widehat{\theta})]$ and $\widehat{y}_\theta(t) = 1[y_t \le -V_t(y;\widehat{\theta})]$. Then, compute the sample cross-covariances

$$\widehat{R}(j) = T^{-1}\sum_{t=1+j}^{T}(\widehat{x}_\theta(t) - \overline{\widehat{x}_\theta})(\widehat{y}_\theta(t-j) - \overline{\widehat{y}_\theta}), \text{ for } 0 \le j \le T-1, \tag{9}$$

where $\overline{\widehat{x}_\theta} = T^{-1}\sum_{t=1}^{T}\widehat{x}_\theta(t)$ and $\overline{\widehat{y}_\theta} = T^{-1}\sum_{t=1}^{T}\widehat{y}_\theta(t)$.

Next, we construct a valid test statistic based on sample cross correlations. We adopt a kernel-based nonparametric test proposed by HLW,

$$Q(M) = [T\sum_{j=1}^{T}k^2(j/M)\widehat{\rho}^2(j) - C(M)]/D(M)^{1/2}, \tag{10}$$

where k is a kernel function with bandwidths M, and $\widehat{\rho}(j) = \widehat{R}(j)/(\widehat{S}_1\widehat{S}_2)$, $\widehat{S}_1 = \overline{\widehat{x}_\theta}(1 - \overline{\widehat{x}_\theta})$, and $\widehat{S}_2 = \overline{\widehat{y}_\theta}(1 - \overline{\widehat{y}_\theta})$. The quantities $C(M)$ and $D(M)$ are kernel-specific values related with corrections of mean and variance of the test statistic. See HLW(2009) for expressions of such values. The test can be viewed as the normal approximation of Chi-squared random variables. Note that HLW simply use truncated kernel mainly because they use high-frequency intra daily data. On the other hand, our data is daily return data, then it seems reasonable to allow weights to cross correlations at different lags. Thus, in our work, we use Bartlett kernel given as

$$k(x) = \begin{cases} 1 - |x|, \text{ for } |x| < 1 \\ 0, \text{ otherwise.} \end{cases} \tag{11}$$

Other popular kernels including quadratic kernels do not affect the asymptotic distribution for the test statistics. As for the bandwidths, the values of M often affect the finite sample performance of the test statistics. It is seen from simulation studies in HLW that larger values of M tend to lower the power of the tests, whereas the size of the tests relatively remains stable. Thus, we choose several small values of bandwidths, say $M = 3, 5$, and 10, in our analysis.

Under the null hypothesis as well as certain conditions including growth rate of the bandwidths, we have asymptotic normality result for the test,

$$Q(M) \to N(0, 1). \tag{12}$$

As the test achieves a standard normal distribution in limit, it is convenient to use in practical works. For example, one can reject the null hypothesis at the 5% level if the absolute value of $Q(M)$ exceeds 1.96.

3 Empirical Results

3.1 Data and Summary statistics

We use the daily time-series of KOSPI and Won/Dollar exchange rate obtained from the Bank of Korea[3]. Stock return is defined by log difference of KOSPI while FX return is defined by log difference of Won/Dollar exchange rate multiplied by -1. Thus, a negative value of FX return implies a depreciation of Korean Won or an increase in exchange rate. The sample of daily returns starts on January 3 1992 when Korean stock market was opened to foreign investors and ends on June 25 2009 with a total of 4641 observations.

[3]The low frequency data can mask the joint dynamics of the different financial markets where daily capital flows play an important role.

Thus, the sample period includes both the 1997 Asian currency crisis period and the recent global financial crisis period.

Table 1 provides sample statistics, including correlation coefficient between stock returns and FX returns. During the sample period, we observe positive average stock returns and negative FX returns. The volatility of daily stock returns is much greater than the volatility of daily FX returns in the full sample, but the relatively lower volatility of FX returns is mainly from the earlier period of the sample. Notably, the large kurtosis indicates the distributions of both stock return and FX return have fat tails. The contemporaneous correlation between two returns implies that higher stock returns is associated with higher FX returns (decrease in exchange rate).

Table 1. Summary Statistics for KOSPI return (x) and Won/Dollar exchange rate return (y)

	Mean	Standard Deviation	Skewness	Kurtosis
x	0.000173	0.0186	-0.1593	7.1203
y	-0.000112	0.0087	1.5252	141.2610
$\mathrm{corr}(x, y) = 0.1001$				

Before a statistical inference on the risk spillover between stock market and foreign exchange market, we examine whether an extremely negative stock return today is associated with an extremely negative FX returns in the near future and vice versa. To do this, following the methodology explained in the previous section, we first identify the periods when stock returns and FX returns exhibit extreme negative values below 5% value at risk. Then, we compute the conditional probability that FX returns have at least one extreme negative value in the periods from $t+1$ to $t+k$ given that stock returns have extreme negative value at time t. We label this conditional probability as $P_1^k(y|x)$. Similarly, we also compute the conditional probability that stock returns have at least one extreme negative value in the periods from $t+1$ to $t+k$ given that FX returns have extreme negative value at time t. Such probability is denoted as $P_1^k(x|y)$.

Table 2 reports the results. They tell us that, given that stock returns show the worst 5% performance at time t, the probabilities that FX return also show the worst 5% performance within the next 3, 5, and 10 days are 41.20%, 45.49%. and 52.36%, respectively. Alternatively, the probabilities of extreme negative FX returns within the next 3, 5, and 10 days conditional on the extreme negative stock returns today are 32.19%, 47.64%, and 61.80%.

We note that these conditional probabilities are much higher than the unconditional probability of 5%, clearly implying that extreme negative stock returns (FX returns) are more likely to be followed by extreme negative FX returns (stock returns).

Table 2. Conditional Probabilities

	$k = 3$	$k = 5$	$k = 10$
$P_1^k(y\|x)$	0.4120	0.4549	0.5236
$P_1^k(x\|y)$	0.3219	0.4764	0.6180

Note: 1. $P_1^k(y|x) = P(y$ hits at least once from $t+1$ to $t+k|$ x hits at $t)$, and $P_1^k(x|y) = P(x$ hits at least once from $t+1$ to $t+k|$ y hits at $t)$.

3.2 Empirical Results: Full Sample

Table 3 reports the test statistics for the traditional Granger causality in mean and the new Granger causality in risk between stock returns and FX returns[4]. The value of m indicates the number of lags in linear regression model of Granger causality test in mean or the number of bandwidths in Granger causality test in risk. The arrow in Table 3 denotes one-way Granger causality from the former to the latter.

The results from Granger causality test in mean reported in Panel A of Table 3 show that stock returns predict FX returns but FX returns do not predict stock returns. The F-test statistics for the null hypothesis that stock returns do not Granger cause FX returns is strongly rejected at the 1% significance level regardless of the number of lags in the linear regression model. Yet, the null hypothesis that FX returns do not Granger cause stock returns cannot be rejected at usual level of significance for m =3 and 5.

In contrast to Granger causality in mean, the test statistics of Granger causality in risk, reported in Panel B, show a very different result. We find that causality in risk runs both ways. The one-way test of risk causality from stock returns to FX returns yields test statistics of 180.52, 185.82, and 175.79 for m =3, 5, and 10, respectively. The highly significant test statistics suggest that risk spills over from stock market to FX market. At the same time, the test of risk causality from FX returns to stock returns yields the test statistics of 22.48, 34.17, and 61.62 for m =3, 5, and 10, which are also statistically significant. This result suggests that risk also spills over from FX market to stock market.

[4]Ajayi, Friedman, and Mehddian (1998), Granger, Huang, and Yang (1998), and Lee (2002) also examine Granger causality in mean between stock return and foreign exchange rate return in Korea.

Table 3. Testing Granger causality in mean and in risk between KOSPI return (x) and Won/Dollar exchange rate return (y): Full Sample

m	3	5	10
Panel A: Test results of Granger causality in mean			
$x \Longrightarrow y$	111.3734**	64.9524**	32.9447**
$x \Longleftarrow y$	0.8186	2.0111	1.8853*
Panel B: Test results of Granger-causality in risk			
$x_{0.05} \Longrightarrow y_{0.05}$	180.5198**	185.8214**	175.7912**
$x_{0.05} \Longleftarrow y_{0.05}$	22.4821**	34.1747**	61.6160**

Notes for Panel A: 1. The symbol " $\Longrightarrow$ " (" $\Longleftarrow$ ") denotes one-way Granger-causality from the former (the latter) to the latter (the former). 2. The value of m is the number of lags in linear regression models in Panel A and the number of bandwidths for the test statistic in Panel B. 3. * (**) denotes rejection of the null of no Granger-causality at the at the 5% (1%) significance level. In Panel A: The F critical values at the 5%(1%) level are 2.60(3.78), 2.21(3.02) and 1.83(2.32) for $m = 3, 5$ and 10. In Panel B: Normal critical values are $\pm 1.96(\pm 2.57)$ at the 5%(1%) level for $m = 3, 5$ and 10.

In Table 3, we find a causality in Granger sense from stock returns to FX return both in mean and in risk, which may be explained by large scale capital outflows. If foreign investors sell large amount of domestic stocks and then sell the proceed of domestic currency in foreign exchange market on the subsequent days, we may observe that stock returns leads FX returns if these large transactions affect the market prices. In addition, a decrease in stock price reduces the wealth of domestic investors which in turn lowers the demand for money and interest rate. It then can lead to capital outflow and currency depreciation.

In contrast, the causality from FX returns to stock returns is only found between the left tails of the distributions. This result suggests that the joint dynamics of the tails may be different from those of the rest of the distribution. In normal times, changes in exchange rate are largely viewed temporary and thus do not necessarily affect future stock prices[3]. In times of financial crisis, however, a large depreciation of exchange rate may influence investor's expectation on the future course of exchange rate. In particular, if foreign investors foresee a further depreciation of local currency, they would cut the demand for domestic assets particularly for stocks and convert the proceed to foreign currency. This process validates their own expectation resulting in a self-fulfilling phenomenon.

[3]In principle, a depreciation of local currency is likely to increase competitiveness of exporting goods and thus increase stock prices of exporting firms. However, the test result of Granger causality in mean suggest that this mechanism does not work in our high frequency data.

3.3 Empirical Results: Sub-samples

This sub-section investigates risk spillover in sub-samples. As a measure of risk, we again employ a 5% value at risk in stock returns and FX returns for each sub-sample. Thus, the risk measure depends on the sample period. Since the full sample period includes two financial crisis periods, 1997 Asian crisis and 2008 global financial crisis, we may expect that extreme negative values below 5% value at risk are mostly observed during such two crisis periods. Then, we may ask whether the patterns of risk spillover would be the same in the sample periods excluding the two crisis periods.

For this purpose, we divide the full sample period into two sub-sample periods: a sample period from January 3 1992 to September 30 1997 and a sample period from October 1 1997 to June 25 2009. The former sub-sample corresponds to the period before 1997 Asian crisis and the latter sub-sample includes two crisis periods. We also consider a sub-sample from January 4 2000 to August 29 2008 which corresponds to the period after 1997 Asian currency crisis but before 2008 global financial crisis.[5]

Another important reason that we attempt to examine sub-sample results is related to the change in exchange rate system which occurred in 1997. On December 17, 1997, the daily band on the won/dollar exchange rate was completely abolished and the exchange rate system in Korea was changed to a completely free-floating system. In the full sample, thanks to the daily band on exchange rate, extreme negative values of FX returns below 5% value at risk may rarely observed before 1997.

Table 4 displays the sub-sample results of Granger causality in mean (Panel A) and Granger causality in risk (Panel B). Panel A shows that, in 1992-1997 sample period, we cannot reject the null of no Granger causality in mean from stock returns to FX returns when the number of lags are three and ten days. Only a weak causality is found when lags are set five days. In both 1997-2009 and 2000-2008 sample periods, however, stock returns strongly predict the subsequent FX returns. In contrast, the null of no Granger causality from FX returns to stock returns cannot be rejected at any level of significance in all of the three sub-sample periods. Except for one-way causality from stock returns to FX returns in 1992-1997 sample, the sub-sample results are qualitatively the same as the results in the full sample.

The sub-sample results of Granger causality in risk reported in Panel B, however, are sometimes very different from the results of Granger causality in risk in full sample and/or the results of Granger causality in mean in Panel A. In 1992-1997 sample, the results documents that risk spills over from stock returns to FX returns but not vice versa. Unlike Granger causality test in mean, extreme downturn of stock returns turns out to cause the subsequent extreme depreciation of Korean Won.[6] This result suggests that the behavior of joint dynamics in the left tail are different from the rest of the distribution even with the binding constraint imposed on the FX return

[5]We do not consider subsamples during 1997.10.1-2000.1.3 and during 2008.8.30-2009.6.25 for Asian and global financial crises, as the sample size of each period is too small to draw meaningful inferences.

[6]Note that 5% value at risk of FX returns is calculated within the daily band in 1992-1997 sample periods.

fluctuations.

The results on Granger causality in risk for 1997-2009 sample period are qualitatively the same as the results for full sample period. The results indicate that risk causality runs in both ways as the test statistics for risk causality both from stock returns to FX returns and from FX returns to stock returns are highly significant at the 1% significance level.

The results for 2000-2008 sample period, in contrast, show that risk spills over only from stock returns to FX returns as in the 1992-1997 sample period. This result is also comparable to the result of Granger causality in mean, suggesting that joint dynamics of tail distribution may not much different from the rest of the distribution in this sample period.

Table 4. Testing Granger causality in mean and in risk between KOSPI return (x) and Won/Dollar exchange rate return (y): Sub-samples

m	3	5	10
Panel A: Test results of Granger causality in mean			
(a) 1992.1.3.-1997.9.30.			
$x \Longrightarrow y$	2.1101	2.2691*	1.5612
$x \Longleftarrow y$	1.2735	1.0636	0.9429
(b) 1997.10.1.-2009.6.25.			
$x \Longrightarrow y$	84.0467**	48.8690**	28.6615**
$x \Longleftarrow y$	0.4929	1.3620	1.3538
(c) 2000.1.4.-2008.8.29			
$x \Longrightarrow y$	44.6406**	28.3634**	14.6630**
$x \Longleftarrow y$	0.7309	0.9214	1.9012
Panel B: Test results of Granger-causality in risk			
(a) 1992.1.3.-1997.9.30.			
$x_{0.05} \Longrightarrow y_{0.05}$	2.7249**	2.1362*	1.2873
$x_{0.05} \Longleftarrow y_{0.05}$	1.2813	0.9298	0.5800
(b) 1997.10.1.-2009.6.25.			
$x_{0.05} \Longrightarrow y_{0.05}$	111.9134**	108.0144**	96.5680**
$x_{0.05} \Longleftarrow y_{0.05}$	15.9928**	27.3776**	41.0931**
(c) 2000.1.4.-2008.8.29			
$x_{0.05} \Longrightarrow y_{0.05}$	13.4746**	13.7837**	11.4573**
$x_{0.05} \Longleftarrow y_{0.05}$	0.0328	-0.2549	-0.4760

See the Notes in Table 3.

3.4 Concluding Remarks.

With the growing cross-border capital flows in the past decade, we can expect strong dynamic association between stock market and foreign exchange market. However, the joint dynamics of asset prices in the periods of financial turmoil are likely to be different from those in normal times. In this paper, we employ a statistical testing procedure for Granger causality test in risk to examine the joint dynamics of extreme negative values in the left tail of distribution. This test enables us to investigate how risk spills over between stock market and foreign exchange market in Korea. In addition, we compare the results from Granger causality test in risk with the results from traditional Granger causality test in mean. We find that causality in risk runs in both directions while causality in mean runs only from stock returns to foreign exchange returns for the sample between 1992 and 2008. The results, however, depend on whether to include the 1997 Asian crisis period and 2008 global financial crisis period.

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