Dependence of the distance between cut-wire-pair layers on resonance frequencies

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Abstract: We studied both experimentally and theoretically the influence of the distance between adjacent cut-wire-pair layers on the magnetic and the electric resonances in the microwave-frequency regime. Besides the dependence on the separation between cut-wire pairs, along the electric-field direction, the electric resonance strongly depends on the distance between cut-wire-pair layers, while the magnetic resonance is almost unchanged. This contrast can be understood by the difference in the distribution of induced-charge density and in the direction of the induced current between the electric and magnetic resonances. A simple model is proposed to simulate our experimental results and the simulation results are in good agreement with the experiment. This result provides important information in obtaining left-handed behavior when the cut-wire pairs are combined with the continuous wire.

OCIS codes: (160.1245) Artificially engineered materials; (160.3918) Metamaterials; (260.5740) Resonance.

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1. Introduction

Recently, left-handed materials (LHMs) or negative refractive-index materials have been investigated intensively because of their fascinating properties. Based on Pendry’s suggestion [1] and the original prediction by Veselago [2], Smith *et al.* [3] demonstrated the existence of LHM consisting of an array of split-ring resonators (SRRs) and continuous wires for the first time. Following this seminal paper, numerous reports, both theoretical and experimental, confirmed the existence of LHMs and their main properties. In regular LHMs, to apply the uniform-effective-medium theory in determining the effective electric permittivity ε and magnetic permeability μ, the size of the unit cell must be much smaller than the wavelength [4, 5]. Therefore, the development of geometry and fabrication technique is still an area of significant effort, especially for LHMs operating at optical frequencies. The design and construction of magnetic and electric components play a central role, mainly a magnetic one. At present, the search for magnetic systems with an effectively negative μ up to the optical range remains an actual task. Besides SRRs, several different structures have been employed to provide a negative μ [6, 7, 8]. Among them the cut-wire-pair (CWP) structure has received considerable interest since the electromagnetic (EM) response of the CWP structure has not yet been fully elucidated [9, 10, 11]. Furthermore, this topology provides significant technological advantages over the conventional SRR.

In essence, the CWP is an SRR with two gaps flattened in the wire-pair arrangement. This structure also exhibits both magnetic and electric resonances as in the SRR structures. By tuning the magnetic and/or the electric resonances, it might be possible to obtain the LHM using only an array of CWPs. Although Dolling *et al.*[12] have not shown evidence for negative refractive index n by using only CWPs, Shalaev *et al.*[13] have reported that the negative n at THz frequencies can be realized. However, the observed negative n was probably due to the significant imaginary parts of the complex ε and μ [14]. Recently, Zhou *et al*. [10] have theoretically studied the dependence of two resonances on the separation of neighboring pairs along the **E** direction on the cut-wire pair sheet (intra-layer distance d). It was found that the low-frequency edge of electric-resonance strongly depends on the separation of neighboring pairs [also see Fig. 1(b)]. However, it is very difficult to overlap two resonances to obtain the LH behavior using only an array of CWPs.

Recent experiments have revealed that an efficient approach to achieve the LH behavior by

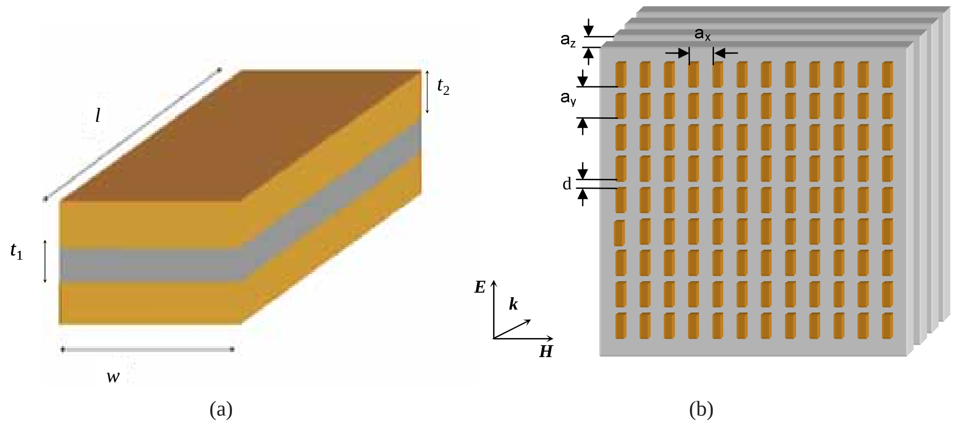

Fig. 1. (Color online) (a) Geometry of the cut-wire pair with the length of cut wire is l = 5.5 mm and the width w = 1.0 mm. t_1 is the thickness of the PCB board and t_2 that for the CU cut-wire. (b) Periods of cut-wire pairs.

employing the CWPs is to combine them with continuous wires [15, 16]. While the magnetic-resonance frequency of CWP is of primary interest, the electric-resonance frequency also plays an important role when combined with the continuous wire. As is well known, the plasma frequency of the combined structure is much lower than that of the continuous wire alone [16, 17]. If the plasma frequency of the combined structure is lower than the magnetic-resonance frequency, the transmission peak, conventionally interpreted as a LH peak, turns out to be right-handed [18]. Thus, it is necessary to investigate the electric-resonance frequency of CWP and its contribution to the effective permittivity of the combined structure.

In this work, we studied both experimentally and theoretically the influence of the distance between CWP layers (inter-layer distance a_z [also see Fig. 1(b)]) on the magnetic- and the electric-resonance frequencies. It was found that electric resonance strongly depends on the distance between layers, but the magnetic resonance is nearly unchanged. When the distance approaches a certain value, the magnetic resonance is degraded by the electric one or the two resonances kill each other.

2. Experiment

The geometrical structure of CWP is depicted in Fig. 1, similar to Refs. [11] and [16]. The CWP structures were fabricated on both sides of the printed-copper board (PCB) with a copper thickness of 36 μm. The thickness of the dielectric PCB is 0.4 mm with a dielectric constant of 4.8. The period of CWP in the x-y plane is kept constant to be $a_x = 3.5$ mm and $a_y = 7.0$ mm. The length l and the width w of CWP are 5.5 and 1.0 mm, respectively. The periodicity along the z direction was obtained by stacking a number of identically patterned boards with the distance between CWP layers varied from 1.0 to 5.0 mm. We performed the transmission measurements in free space, using a Hewlett-Packard E8362B network analyzer connected to microwave standard-gain horn antennas.

3. Results and discussion

It is well known that the CWP structure exhibits both magnetic and electric resonances similar to the case of SRR structures. Figure 2 shows the measured transmission spectra of the CWP structures with different numbers of layers, where the distance between layers is 1.0 mm.

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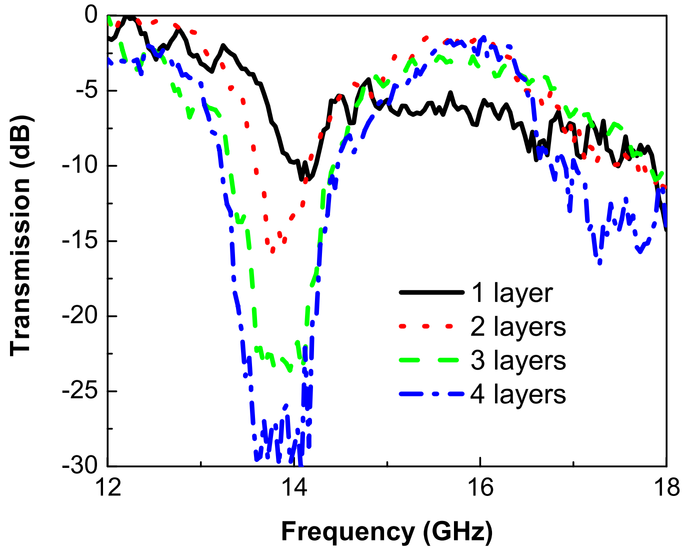

Fig. 2. (Color online) Measured transmission spectra of the cut-wire-pair structure with different numbers of layers in the propagation direction, where the distance between layers is kept 1.0 mm.

Clearly, there are two band gaps in the transmission spectra, similar to that of Ref. [16]. The two band gaps are separated by a certain amount of frequency. It is confirmed that the first band gap located at $13.4 - 14.4$ GHz is due to the magnetic resonance, providing a negative magnetic permeability, and the second band gap starting at $\sim$17 GHz is from the electric one, providing a negative electric permittivity. When the number of layers increases, the two resonances become more evident, but interestingly the separation between them is invariant. In this case, by combining the CWP with the continuous wire, it is better to obtain the LH behavior [15, 16].

Figure 3(a) shows the measured transmission spectra of various CWP structures with two layers, in which the period of CWP in the *x*-*y* plane is kept constant to be $a_x = 3.5$ mm and $a_y = 7.0$ mm, while in the *z* direction the distance between CWP layers varies from 1.0 to 5.0 mm. Clearly, the low-frequency edge of electric-resonance band is shifted to a lower frequency as the distance increases between CWP layers, but the magnetic-resonance frequency is practically unchanged. This indicates that the separation between the two resonances is reduced as the lattice constant a_z increases. One might argue that, if the distance between CWP layers increases, the coupling between layers is decreased and, consequently, the transmission in the region between two resonances is reduced in strength. Although this effect can explain the reduction of transmission between the two gaps, the observed red-shift of the low-frequency edge of electric resonance is not elucidated completely.

Very interesting results are obtained when a_z increases until it reaches 3.0 mm as the separation between two resonances is almost destroyed as shown in Fig. 3. This suggests two possible scenarios. Firstly, the electric and the magnetic resonances overlap in the frequency region between 14 and 15 GHz. If the two resonances overlap, it means that both ε and μ are negative; hence, there must be a transmission band in this frequency regime. However, if there is no transmission band found, this implies that the two resonances cannot overlap. The second scenario is where the two resonances might cancel each other. It is similar to that in Ref. [10], where Zhou *et al*. studied the dependence of the separation between the intra-layer CWPs

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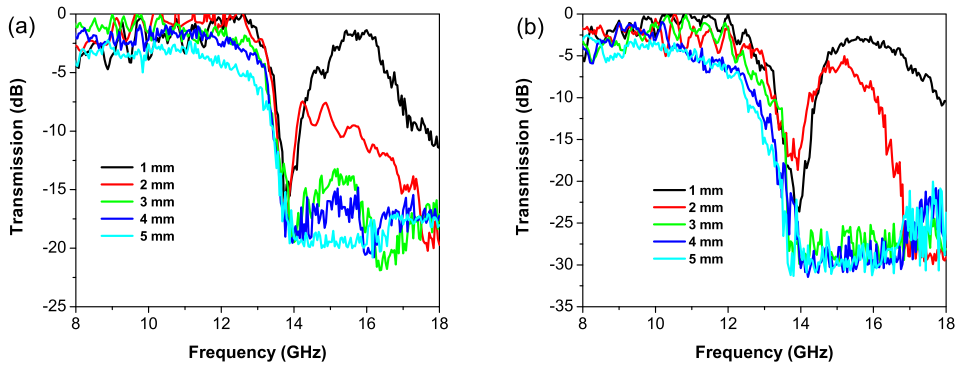

Fig. 3. (Color online) Measured transmission spectra of various cut-wire-pair structures; (a) two and (b) three layers. The period of cut-wire pair in the *x*-*y* plane is kept constant to be $a_x = 3.5$ mm and $a_y = 7.0$ mm; while the distance between layers is varied from 1.0 to 4.0 mm.

along the **E** direction. This effect will degrade or even destroy the LH behavior when the CWP is combined with the continuous wire. The dependence of the electric resonance on the distance between layers is also examined for a CWP structure with three layers as shown in Fig. 3(b). The observed result is consistent with that of the two-layer CWP structure.

One possible explanation on the observed result is that the shift of the low-frequency edge of electric-resonance band might be due to the influence of misalignment between the CWP boards along the propagation direction. This misalignment is achieved by shifting the CWP boards along the **H** direction with a deviation δ of 0.5 mm as shown in Fig. 4(a). To examine the potential effect of the misalignment between layers on the shift of the low-frequency edge of electric resonance, the transmission spectra of the misaligned CWP boards were measured and displayed in Fig. 4(b). For this study, the distance between layers is kept at 1.0 mm and $\delta = 0.5$ mm (half the width of cut-wire). Clearly, the band gaps between the magnetic and the electric resonances are nearly unchanged. This means that the electric resonance of CWP is not influenced by misalignment between CWP boards. Aydin *et al.* [19] also investigated the effect of misalignment in the SRR structures on the magnetic and the electric resonances. Thus, these results strongly suggest that the shift of the low-frequency edge of electric resonance observed in Fig. 3 is not caused by the misalignment of the CWP boards. Gay-Balmaz and Martin [20] also theoretically studied this fact for the SRR structure and pointed out that the inter-layer separation plays a more important role enhancing the coupling between SRRs than the misalignment of boards.

The influence of the distance between CWP layers on the resonances was also theoretically studied. In our theoretical study, we employed a similar model proposed by Zhou *et al.* [10] and used by us previously [11]. In the present paper, however, the model is realistically elaborated. We assume that the electric field of the incident radiation is along the length of a rectangular metallic stripe with length l, width w and thickness t_{f}, and the x-, the y- and the z-axis are in the direction of the width, the length, and the thickness, respectively, of the rectangular metallic stripe. Therefore, $\mathbf{E} = E_{\mathrm{o}}\hat{\mathbf{y}}$, where E_{o} is the amplitude of applied electric field by the incident electromagnetic wave. Since the charge density varies along the y-direction, the charge density

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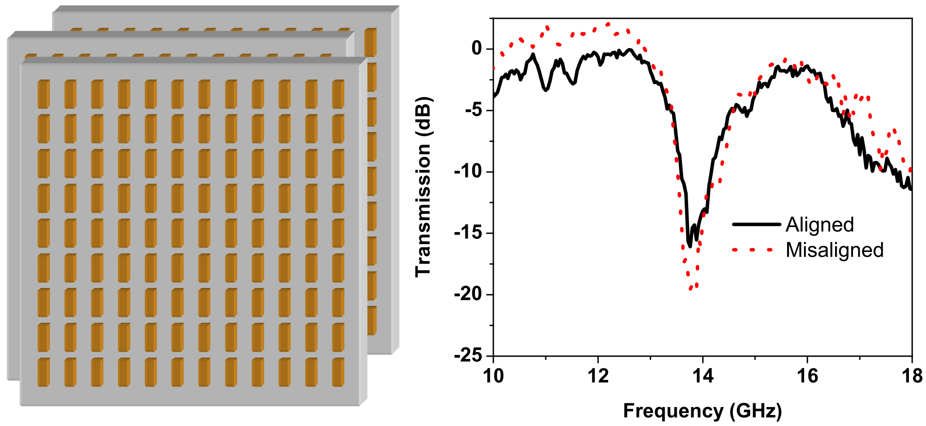

(a) (b)

Fig. 4. (Color online) (a) Schematic of misaligned cut-wire-pair boards. (b) Comparison of the measured transmission spectra between aligned and misaligned cut-wire-pair layers.

$\rho = \rho(y)$ and the current density are given by

$$\mathbf{J} = J(y)\hat{\mathbf{y}} = \sigma E_o \hat{\mathbf{y}} - \alpha \nabla \rho = \sigma E_o \hat{\mathbf{y}} - \alpha \frac{\partial \rho}{\partial y} \hat{\mathbf{y}}, \tag{1}$$

where σ is the electrical conductivity of the metallic stripe and α is a proportionality constant, which will be determined later. The first term in the rightmost side corresponds to the ordinary current density arising from the applied field and the second term is the current density driven by the gradient of induced charge density. By applying the continuity condition we can obtain

$$\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = \frac{\partial J}{\partial y} + \frac{\partial \rho}{\partial t} = -\alpha \frac{d^2 \rho}{dy^2} + \frac{\partial \rho}{\partial t} = 0. \tag{2}$$

Assuming that $\dfrac{\partial \rho}{\partial t} = -\omega \rho$, where ω is the angular frequency of the incident wave. The differential equation, which determines the charge density, is

$$\frac{d^2 \rho}{dy^2} + \frac{\omega}{\alpha} \rho = 0. \tag{3}$$

From now on, the time dependence of the field and current can be ignored without loss of the generality. Since the charge density is antisymmetric about the center of the metallic stripe along the y-axis, the charge density can be written as

$$\rho = \rho_o \sin\left(\sqrt{\frac{\omega}{\alpha}} y \right) \tag{4}$$

and the current density

$$\mathbf{J} = \sigma E_o \hat{\mathbf{y}} - \alpha \rho_o \sqrt{\frac{\omega}{\alpha}} \cos\left(\sqrt{\frac{\omega}{\alpha}} y \right) \hat{\mathbf{y}}, \tag{5}$$

where $\hat{\mathbf{y}}$ is the unit vector along y-axis. Since the charge distribution and the current density are entirely confined inside the metallic stripe, multiplying $\mathbf{r}$ by the continuity equation [Eq. (2)]

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and integrating over the volume of the metallic stripe, we can get the following expression:

$$\int \mathbf{r}\left[\nabla\cdot\mathbf{J}+\frac{\partial\rho}{\partial t}\right]d^3\mathbf{r}=\int \mathbf{J}d^3\mathbf{r}+\frac{\partial\mathbf{p}}{\partial t}=0, \tag{6}$$

where $\mathbf{p}\equiv\int\mathbf{r}\rho d^3\mathbf{r}$ is the dipole moment of the charge distribution. After some calculations, we obtain

$$\rho=\rho_{\mathrm{o}}\sin\left(\frac{\pi}{l}y\right), \tag{7}$$

and

$$\mathbf{J}=\frac{4\alpha\rho_{\mathrm{o}}}{l}\left[1-\frac{\pi}{4}\cos\left(\frac{\pi}{l}y\right)\right]\hat{\mathbf{y}}. \tag{8}$$

From these sources we can calculated the electric and the magnetic fields produced by 21×21 CWPs by directly applying Coulomb's law and Biot-Savart's law, respectively. Since these fields are periodic along the x and the y direction and the electric and the magnetic energies are symmetric about the origin, we calculate the fields only in the first quadrant confined by $a_x\times a_y$ and $0\leq z\leq 25$ cm. The effective capacitance C_{eff} and inductance L_{eff} are obtained from the following relations,

$$U_{\mathrm{E}}=\frac{1}{2}\int\varepsilon E^2dv=\frac{Q^2}{2C_{\mathrm{eff}}} \tag{9}$$

and

$$U_{\mathrm{B}}=\frac{1}{2}\int\frac{B^2}{\mu}dv=\frac{1}{2}L_{\mathrm{eff}}I^2. \tag{10}$$

The above electric and magnetic energies were obtained for the electric and the magnetic resonances separately by assuming that the currents of the cut-wire pair flow in the same and opposite directions, respectively. Finally, the electric and magnetic resonance frequencies are

$$\omega_{\mathrm{e}}=\frac{1}{\sqrt{L_{\mathrm{e,eff}}C_{\mathrm{e,eff}}}} \tag{11}$$

and

$$\omega_{\mathrm{m}}=\frac{1}{\sqrt{L_{\mathrm{m,eff}}C_{\mathrm{m,eff}}}}, \tag{12}$$

respectively.

The calculational results are displayed in Fig. 5. The magnetic resonance frequency does not change appreciably with the distance between cut-wire-pair layers (a_z), while the electric one changes significantly. As a_z approaches 4 mm, the electric resonance frequency keeps decreasing and reaches the magnetic resonance frequency. Then the electric-resonance frequency increases further with increasing a_z. These calculational results successfully reproduce the experimental results.

The reason for the variance of ω_{e} and the invariance of ω_{m} with a_z can be understood by the difference in the distribution of induced charge density and in the direction of the induced current between electric and magnetic resonances. For the magnetic resonance the induced-charge distribution on a metallic stripe is opposite of that on the counter metallic stripe in the same CWP. Hence, the electric field is mostly confined in the region between the two metallic stripes and thus the effective capacitance is determined by the CWP itself. The magnetic fields are also mostly concentrated in the region between two metallic stripes in CWP. Therefore, both the electric and magnetic energies do not vary with a_z, resulting in a constant magnetic resonance frequency independent of a_z. Our calculational results confirm this argument. For the electric resonance, on the other hand, the induced-charge distribution does not produce a

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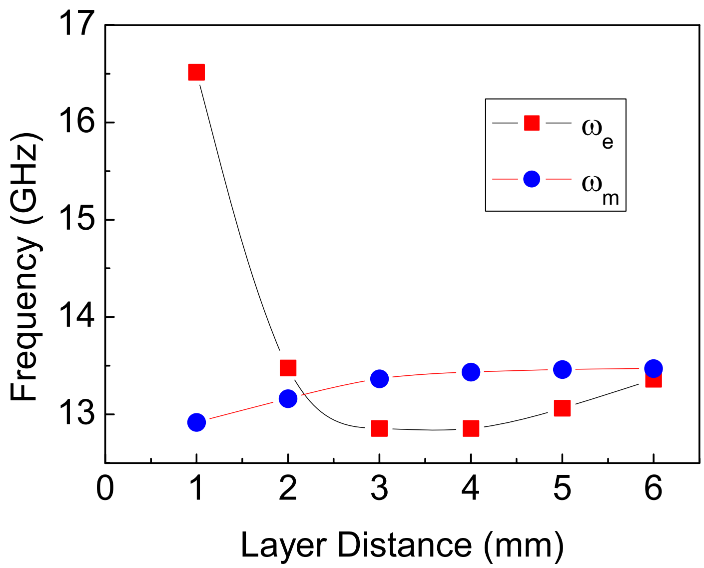

Fig. 5. Calculated electric and magnetic resonance frequencies as a function of the distance between cut-wire-pair layers for 3-layer structure. The solid lines are guide to the eyes only.

significant electric field in the region between the two metallic stripes and thus the effective capacitance is determined by the two adjacent metallic stripes in different CWPs. The magnetic fields are also mostly concentrated in the region between PCBs. Therefore, both electric and magnetic energies vary with a_z, resulting in a strong dependence of the low-frequency edge of electric resonance on a_z.

4. Conclusions

The influence of the distance between CWP layers on the electric and the magnetic resonances was investigated both experimentally and theoretically in the microwave-frequency regime. Good agreement between the theory and the experiment was obtained. It was found that the low-frequency edge of electric resonance strongly depends on the distance between layers, while the magnetic one is practically invariant. The difference can be understood by the difference in the distribution of the induced-charge density and the direction of the induced current between the electric and magnetic resonances. The magnetic resonance is degraded by the electric one or even two resonances might cancel each other with an increase in the distance between layers until it reaches 3.0 mm. This is of fundamental importance in understanding the electromagnetic response of the CWP at low frequencies and supports for the design on the LHMs working at high-frequency region. Furthermore, one has to avoid the crossing region where two resonances cancel each other or two resonances are too close. This effect will degrade or even destroy LH behavior when the cut-wire pairs are combined with the continuous wires.

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