Dependence of the magnetic-resonance frequency on the cut-wire width of cut-wire pair medium

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Abstract: We investigated some magnetic metamaterials based on cut-wire pairs instead of split-ring resonators to quest for the possibility of a negative magnetic permeability. Several periodic structures of cut-wire pairs were designed and fabricated, and their transmission spectra were measured in the microwave-frequency regime. The width dependence of magnetic-resonance frequency was studied both theoretically and experimentally for the periodic structures of cut-wire pairs. It was found that, besides the length dependence, the magnetic-resonance frequency also depends significantly on the width of cut-wire pair. A simple equivalent-circuit model proposed by Zhou *et al.* [Opt. Lett. **31**, 3620-3622 (2006)] was employed to elucidate this interesting phenomenon and to simulate the width dependence. In the simulation the magnetic and the electric energies of the cut-wire pair were directly calculated to obtain the magnetic-resonance frequency. The theoretical results are in good agreement with the experimental ones. It reveals that there is a rather easy way to manipulate the magnetic-resonance frequency of magnetic-magnetic metamaterials.

OCIS codes: (160.1245) Artificially engineered materials; (160.3918) Metamaterials; (260.5740) Resonance.

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#87606 - $15.00 USD Received 17 Sep 2007; revised 26 Nov 2007; accepted 27 Nov 2007; published 30 Nov 2007

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1. Introduction

Left-handed material (LHM) has been studied extensively in recent years due to their unique physical properties [1–5] and novel applications [6, 7]. One of the most attractive features of LHM is the possibility of obtaining the negative refraction over a certain frequency range. If permeability and permittivity are simultaneously negative over a common frequency range, then a medium with left-handed behavior can be realized. It is well known that it is not difficult to obtain a medium with $\varepsilon < 0$, *e.g.*, by a periodic array of wires, for frequencies smaller than the plasma frequency. The medium with $\mu < 0$ is, however, still a major challenge for researchers, since it occurs only in a narrow frequency band. Therefore, the control of the magnetic-frequency band of the magnetic component for $\mu < 0$ plays an important role for the practical realization of LHM when it is combined with the electric component. To date, besides the early invented split-ring-resonator (SRR) structures, there are several different structures for achieving a negative magnetic permeability, such as S-shaped [8], Ω-shaped [9], and π-shaped [10], and cut-wire pair structures [11]. Together with metallic continuous wires, LH behavior was theoretically and experimentally investigated in the microwave and the optical regimes. Recently, some proposal and advances in magnetic metamaterials, which are different from the mainstream structures, were also studied by using the Mie theory [12–16]. These results provide important contributions to the community of metamaterials.

The cut-wire pair structure has been receiving a considerable interest, however, the electromagnetic (EM) response of the cut-wire pair structure has not been fully elucidated yet [17, 18]. Furthermore, the main advantages of the cut-wire pair structure, compared to the other structures, is its ability to produce strong magnetic resonance for normal-to-plane propagation with only one cut-wire pair layer. Thus, to obtain LHM working at optical frequency by employing the cut-wire pairs can be easily fabricated and experimentally characterized, considering the

#87606 - $15.00 USD   Received 17 Sep 2007; revised 26 Nov 2007; accepted 27 Nov 2007; published 30 Nov 2007

current nano-fabrication technology.

The cut-wire pair structure consists of a pair of finite-length wires separated by a dielectric layer. This structure also exhibits both a magnetic and electric resonances as in SRRs. Theoretically, it might be possible to obtain a LHM only using an array of cut-wire pairs. However, the recent experiments have revealed that an efficient approach to achieve the LH behavior by employing the cut-wire pairs is to combine them with continuous wires [11, 19].

Zhou *et al.* [11] have proposed a very simple *LC* model describing the cut-wire pair structure. The magnetic-resonance frequency of cut-wire pair is given by $f_m = \frac{1}{2\pi\sqrt{LC}} = \frac{c_o}{\pi l\sqrt{\varepsilon_r}}$, where c_o is the speed of light in vacuum, l is the length of cut-wire, and ε_r is the relative dielectric constant of the dielectric layer. The formula suggests that the magnetic-resonance frequency is inversely proportional to the length of cut-wire pair, but independent of the width of cut-wire. This argument, however, is based on a very simple assumption that all the electric and the magnetic fields are entirely confined into the space between the cut-wires. Since the edge effect might play a role and neighboring cut-wire pairs also influence the field, it is desirable to examine the width dependence of the magnetic-resonance frequency.

In this report, we report both theoretically and experimentally the width dependence of the magnetic-resonance frequency for periodic structures of cut-wire pair. It was found that besides the length dependence, the magnetic-resonance frequency also depends significantly on the width of cut-wire pair. We developed a simple model to directly calculate magnetic-resonance frequency, and the width dependence was elucidated. The results of simulation reveal that the width dependence of the magnetic-resonance frequency is significant and are in good agreement with the experimental ones.

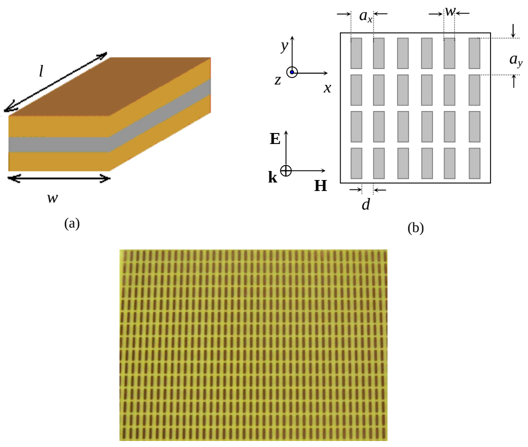
2. Experiment

For the experimental study, the cut-wire pair was fabricated using the conventional printed-circuit-board (PCB) process with copper patterns (36 μm thick) on both sides of a dielectric substrate (0.4 mm thick) with a dielectric constant of 4.8. The design of the cut-wire pair structure is similar to that of Refs. [11] and [19]. The geometrical parameters of the cut-wire pair (l, w, and others) are depicted in Fig. 1, along with the values for the cut-wire systems studied here. The periodicity along the x and the y direction was achieved by printing the 2-dimensional array of cut-wire pairs on a planar PCB. The period of cut-wire pair in the $x-y$ plane is kept constant to be a_x = 3.5 mm and a_y = 7.0 mm. The length of cut-wire pair is l = 5.5 mm, and the width varies from w = 0.7 to 1.8 mm. The periodicity along the z-direction was obtained by stacking a number of cut-wire pair boards with a lattice constant of a_z = 0.9 mm. We performed the transmission measurements in free space, using a Hewlett-Packard E8362B network analyzer connected to the microwave standard-gain horn antennas. In this measurement, the EM wave was incident normal to the sample surface [see Fig. 1(b)].

3. Results and discussion

Figure 2 displays the measured transmission spectra of the various cut-wire pair media with three layers. Clearly, there are two bandgaps in each transmission spectrum. The results observed here are similar to those in Ref. [19]. It is confirmed that the first bandgap is due to the magnetic resonance and the second bandgap, which begins to be formed at $\sim$ 17.5 GHz, is due to the electric resonance. Very interesting results are observed in the first bandgap. As can be seen that the stop band, where $\mu < 0$ in the transmission spectra, is shifted to lower-frequency region, from 13.9 - 15.1 GHz to 12.7 - 13.7 GHz as the width of cut-wire is increased from w = 0.7 to 1.8 mm. This result manifests that the magnetic-resonance band of the cut-wire pair medium strongly depends on the width of the cut-wire, which is contradictory to the model

#87606 - $15.00 USD   Received 17 Sep 2007; revised 26 Nov 2007; accepted 27 Nov 2007; published 30 Nov 2007



(a)

(b)

(c)

Fig. 1. (Color online) (a) Geometry of the cut-wire pair. (b) Top view of the cut-wire pair medium. The unit cell dimensions in the x and the *y* directions a_x = 3.5 mm and a_y = 7.0 mm. Length *l* = 5.5 mm, and width *w* = 0.7 mm, 1.0 mm and 1.8 mm. (c) Photo of one side of the fabricated cut-wire pair medium with *w* = 1.0 mm.

proposed by Zhou *et al*. [11, 18], where the magnetic-resonance frequency is independent of the width of cut-wire pair.

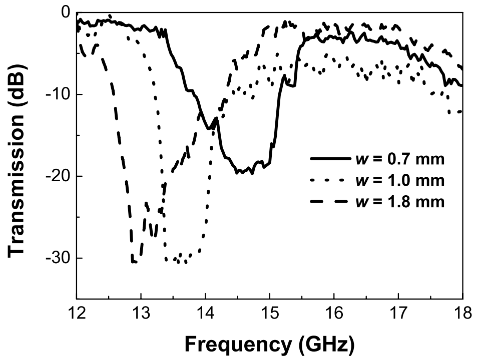


Fig. 2. Measured transmission spectra of the cut-wire pair medium with different widths of the cut-wire

One possible scenario of the observed result is that the shift of the magnetic-resonance band might be due to the magnetic-coupling effect between neighboring cut-wire pairs along the external magnetic field **H** [see Fig. 1(b)]. It is well known that the magnetic-coupling of two components occurs as the *ac* current flowing in a component induces a magnetic field, which, in turn, can influence neighboring components and excites a current in it. This effect induces a mutual inductance *M* between two magnetic components and depends on the separation distance between the magnetic components [20, 21]. However, as shown in Fig. 3 and the theoretical study (which we analyze in the rest of the paper), the magnetic coupling effect in these media is inconsiderable.

To examine the potential contribution of the magnetic coupling to the shift of the magnetic bandgap, the transmission spectra of various cut-wire media with three layers were measured, as shown in Fig. 3. For this study, the width of cut-wire is kept constant to be 1.0 mm but the distance between the cut-wire pair is changed from 1.9 to 2.7 mm. Figure 3 clearly exhibits that the magnetic-resonance frequency is almost unchanged (only from 13.3 to 14.0 GHz) with changing the distance between cut-wire pairs.

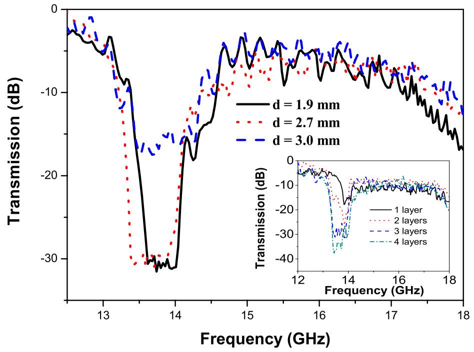


Fig. 3. Measured transmission spectra of the cut-wire pair medium with different distances between the cut-wire pairs. The inset shows the transmission spectra of the cut-wire pair media with different numbers of layers.

One considerable change here is that the bandgap becomes stronger when decreasing the distance between cut-wire pairs. It is quite reasonable since the density of cut-wire pair is increased.

The transmission spectra of the cut-wire pair media with different numbers of layers were also measured and displayed in the inset of Fig. 3. Similar to the case of changing the distance between cut-wire pairs, the magnetic-resonance frequency does not change significantly, as the number of layers increases, while the bandgap becomes stronger. We also calculated the magnetic-resonance frequency of the cut-wire pair medium with different length (l = 4.5 mm) but the same number of layers. Nearly no difference was found in the dependence of the magnetic-resonance frequency according to the length change of the cut-wire pair on the width of the cut-wire pair.

Thus, these results confirms that the shift of the magnetic-resonance band observed in Fig. 2 is not caused by the magnetic-coupling effect. It is due to the influence of the cut-wire width, implying that the magnetic-resonance band depends on the cut-wire's width.

These observations are in good agreement with our simulation. In the simulation, we use the same equivalent-circuit model proposed by Zou *et al.* (see Fig. 3(c) in Ref. [18]) and directly calculate the electric and the magnetic energies to derive the effective capacitance and inductance for the magnetic-resonance frequency. For the electric energy, the electric field at every point was calculated by assuming that the oppositely "induced" charges on the cut-wire pairs are uniform. Later, similar to Ref. [18], some geometrical factor should be taken into account to mimic the nonuniformity of the "induced" charge distribution. In the simulation, the number of layers was taken to be 3 and the geometrical factor of 0.222 was taken into account for the effective capacitance. The geometrical factor 0.222 is reasonable, as is estimated in Ref. [18]. In Ref. [18] the geometrical factor ranging from 0.2 to 0.3 was used *by inspection*. For the magnetic energy, the oppositely "induced" currents flowing through the cut-wire pairs are also assumed to be uniform. The results of simulation are displayed in Fig. 4 together with the experimental results.

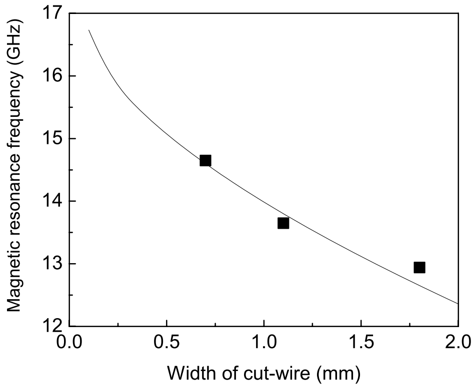


Fig. 4. Plot of simulated magnetic-resonance frequency as a function of the width of cut-wire pair. The experimental data are also plotted.

Although the simulation does not quantitatively reproduce the experimental results, we can clearly see that the magnetic-resonance frequency decreases as the width of cut-wires increases, which is qualitatively accordance with the experimental results. The discrepancy between the experiment and simulation can be ascribed to the ‘incorrect’ assumption of the oppositely “induced” charges on the metallic cut-wire pairs. In fact, the “induced” charge density is not uniform. Rather, the “induced” charge density varies along the length of cut-wire, negative at one half and positive at the other half and symmetric in magnitude about the bisector. This variation of charge density may cause the change in the geometric factor. The inclusion of the variation of charge density in the calculation of capacitance may improve the agreement between experiment and simulation.

4. Conclusions

We have studied experimentally and theoretically the width dependence of the magnetic-resonance frequency for the periodic structures of cut-wire pair. It was found that, besides the length dependence, the width of cut-wire pair affects remarkably the magnetic-resonance frequency. Direct derivation of resonance frequency by using a rather simple model correctly confirms the width dependence. For a given unit-cell size of combined structure, we can control the transmission band by changing the width of the cut-wire pair. This is of fundamental importance in understanding the EM response of the cut-wire pair in the low-frequency region and supports for the design of the LHMs working at high frequencies.

Acknowledgments

This work was supported by the Korea Science and Engineering Foundation(KOSEF) through Quantum Photonic Science Research Center at Hanyang University, Seoul, Korea, and MOST, Korea and by KOSEF grant funded by the Korea government (MOST) (No. R01-2007-000-20974-0).