

# Bulk filling branes and the baryon density in AdS/QCD with gravity back-reaction

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# Bulk filling branes and the baryon density in AdS/QCD with gravity back-reaction

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**ABSTRACT:** We consider the gravity back reaction on the metric due to the baryon density in effective ads/qcd model by reconsidering the role of the charged AdS black hole. Previously it has been known that the U(1) charge is dual to the R-charge. Here we point out that if we consider the case where  $AdS_5$  is completely filled with  $N_f$  flavor branes, the gravity back reaction produces charged AdS black hole where the effect of charge on the metric is proportional to  $N_f/N_c$ . As a consequence, phase diagram changes qualitatively if we allow  $N_f/N_c$  finite: it closes at the finite density unlike the probe brane embedding approach. Another issue we discuss here is the question whether there is any chemical potential dependence in the confining phase. We consider this problem in the hard wall model with baryon charge. We conclude that there is a non-trivial dependence on the chemical potential in this case also.

**KEYWORDS:** Gauge-gravity correspondence, Black Holes in String Theory, AdS-CFT Correspondence.

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## 1. Introduction

It is of great interest to see how far one can use AdS/CFT [1, 2] to understand the strongly interacting nuclear force. So far it has been suggested to apply in the strongly interacting quark gluon plasma [3–8] and phenomenological aspect of chiral dynamics [9–15]. Especially interesting is to investigate the baryon density effect [16–22] on the phases of the Yang-Mill theory.

The purpose of the paper is to study the gravity back reaction to the baryon density in the effective ads/qcd model by reconsidering the role of the charged AdS black hole in AdS/CFT. Previously this background has been studied [23] and its local U(1) charge is identified as dual to the R-charge. See also [24]. The gravity back reaction for the general flavor branes has been studied in various contexts [25–29] and usually it is very non-trivial problem. Here we point out that in the case of effective AdS/QCD model [14], which can be interpreted to have bulk-filling-flavor-branes, the gravity back reaction is simple and the result is nothing but the charged AdS black hole, where the effect of baryon charge on the metric is proportional to  $(N_f/N_c)N_B$ . This  $N_f/N_c$  suppression can be attributed to the fact that R-charge is carried by adjoint scalar while baryon charge is carried by the bi-fundamentals. If we consider strictly large  $N_c$  limit, the metric correction is negligible. However, if we are interested in  $N_f/N_c$  correction, this is important. Such a  $N_f/N_c$

correction to the metric results in corresponding corrections in all physical observables. Especially, in phenomenological applications where one calculate using the finite value of  $N_c$  as well as finite  $N_f$ , this  $N_f/N_c$  correction will be essential. One of the consequence of back reaction of the metric is the qualitative change in phase diagram: it closes at the finite density, while in the probe brane embedding approach [17] or in the hard wall model without gravity back reaction [22] the phase diagrams does not close. The main reason for this change can be traced back to the charge dependence of the temperature.

We also consider an issue which is confusing in the present literature. We first describe the problem by summarizing the relevant history of baryon density problem in AdS/QCD: In [16], it is suggested that the baryon/quark chemical potential should be treated as the boundary value of the brane electric potential  $A_0$  and electric potential was determined by the vector domination. In [17], it was suggested that one can use equation of motion of DBI action to determine the electric potential. Non-trivial Electric potential can exist only with some sources. However, the authors of [17] imposed a smoothness condition on the electric potential  $A_0$  for the in the chiraly asymmetric phase. As a result, constant potential was obtained. This is the same as assuming that there is no source on the brane. In [20], the same boundary condition is chosen and it is related to the earlier work on Hawing-Page transition of charged AdS black hole [23], where for the fixed chemical potential, low temperature phase is associated with the thermal AdS space. As a consequence, electric potential is constant and there is no chemical potential dependence of thermodynamics and average baryon density is zero in that phase phase. We believe this is not physically acceptable.<sup>1</sup>

However, the Hawking-Page transition in [23] is for the case with  $S^3$  boundary, and for the flat boundary, there is no phase transition. Therefore there is no necessity to consider thermal AdS for the low temperature, and the work in [23] is not relevant to this case. However, when we consider the round boundary or introduce the confining phase with hard wall, this is still an issue. One of the purpose of this paper is to consider above issue in the context of hard wall model.

The rest of the paper is in following order. In section 2, we set up the AdS/QCD from brane embedding point of view and derive the gauge coupling on the in terms of basic variables  $N_c, N_f$  and describe charged ads black hole as a back reaction of gravity to the flavor branes. We also and determine the metric dependence of the baryon charge density. In section 3, we describe the thermodynamics of the theory without confinement. In section 4, we discuss the issue of chemical potential dependence of grand potential and we explicitly workout the phase diagram in temperature and chemical potential  $(T, \mu)$  for the theory with confinement. In section, we summarize and describe some future projects.

## 2. Baryon density and the gravity back-reaction

In this section we set up the AdS/QCD from brane embedding point of view and derive the gauge coupling on the in terms of basic variables  $N_c, N_f$  and consider the metric back reaction to the presence of baryon charge density.

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<sup>1</sup>There is another issue regarding baryon density in AdS/CFT, which we will not be treated in this paper and will be mentioned in the discussion section.

## 2.1 Bulk filling branes and AdS/QCD.

We start from the DBI action of the D7-brane. For a given background and embedding, we can write the DBI action in terms of the induced metric  $g_{ab}$  and gauge field on the brane.<sup>2</sup>

$$I_{DBI} = -N_f \mu_7 \int d\sigma^8 \sqrt{-\det(g_{ab} + 2\pi\alpha' F_{ab})}, \quad (2.1)$$

where  $\mu_7$  is the D7-brane tension. The D7 embedding of our concern is those wrapping  $S^3$  of  $S^5$  and cover our own spacetime. If we neglect the  $S^3$  dependence of the field variable, (2.1) defines a 5 dimensional model. The DBI action can be expanded in the power of  $\alpha' F_{ab}$  and the quadratic part is

$$I_2 = -N_f \mu_7 (2\pi\alpha')^2 \Omega_3 \cdot \frac{1}{4} \int d^5\sigma \sqrt{-\det g^{(8)}} F_{ab} F^{ab}, \quad (2.2)$$

where  $\Omega_3$  is the volume of  $S^3$  and we have used  $\sqrt{\det(1+A)} = 1 - \frac{1}{4}\text{Tr}A^2 - \frac{1}{8}\text{Tr}A^4 + \frac{1}{32}(\text{Tr}A^2)^2 + \dots$ , for traceless matrix  $A = g^{-1}F$ . Eq. (2.2) can be used to define phenomenological models as an approximation to the original DBI action.

We first take the D3 background in the near horizon limit:

$$\begin{aligned} ds^2 &= \frac{r^2}{R^2} (-dt^2 + d\vec{x}^2) + R^2 \left( \frac{dr^2}{r^2} + d\Omega_5^2 \right), \\ &= \frac{r^2}{R^2} (-dt^2 + d\vec{x}^2) + \frac{R^2}{r^2} (d\rho^2 + \rho^2 d\Omega_3^2 + dy^2 + y^2 d\varphi^2), \end{aligned} \quad (2.3)$$

In this background, the flat embedding  $y = y_0 = 2\pi\alpha' m_q$  is allowed, for which  $r^2 = y_0^2 + \rho^2$  and the induced metric on the brane is given by

$$ds_{D7}^2 = \frac{r^2}{R^2} (-dt^2 + d\vec{x}^2) + \frac{R^2}{r^2} (d\rho^2 + \rho^2 d\Omega_3^2). \quad (2.4)$$

For this case the volume factor is simplified  $\sqrt{-\det g^{(8)}} = \rho^3$ . One should notice that if we further assume that  $y_0 = 0$ , then the D7 fills all the  $AdS_5$  and the background radial coordinate  $r$  is identical to the world volume radial coordinate, the induced metric is identical to the AdS bulk metric. The action now becomes precisely the gauge action part of AdS/QCD model of [14]

$$I_{\text{gauge}} = -\frac{1}{4g^2} \int d^4x dr \sqrt{-\det g^{(5)}} F_{\mu\nu} F^{\mu\nu} \quad (2.5)$$

with identification

$$\frac{1}{g^2} = \mu_7 \Omega_3 (2\pi\alpha')^2 R^3 = \frac{N_c N_f}{(2\pi)^2 R}, \quad (2.6)$$

and  $\sqrt{-\det g^{(5)}} = \sqrt{-\det g^{(8)}}/R^3$ . Notice that the action contains the metric through the volume factor as well as through two  $g^{\mu\nu}$  factors in the  $F^{\mu\nu}$ . For a general background and a general embedding, eq.(2.5) will contain the correction coming from the difference

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<sup>2</sup>The Chern-Simons term vanishes in the present case.

of induced metric from the background metric. One can build models considering such corrections, which is a deviation of original hard wall model.

Here, instead of considering such correction we ask following question: When the probe brane is filling the 5 dimensional bulk which is asymptotically AdS space, can the induced metric be the same with the bulk metric? The answer positive. For AdS Schwarzschild case, we can work out the answer explicitly. Take the black brane solution in the form where the radial coordinate of bulk and brane is the same and also D7's winding structure is manifest:

$$ds^2 = \frac{r^2}{R^2} (-f(r)dt^2 + d\vec{x}^2) + R^2 \left( \frac{dr^2}{f(r)r^2} + d\theta^2 + \sin^2 \theta d\psi^2 + \cos^2 \theta d\Omega_3^2 \right), \quad (2.7)$$

Then the induced metric is specified if we determine  $\theta$  in terms of  $r$ :

$$ds_{D7}^2 = \frac{r^2}{R^2} (-f(r)dt^2 + d\vec{x}^2) + R^2 \left( \left( \frac{1}{f(r)r^2} + \theta'(r)^2 \right) dr^2 + \cos^2 \theta d\Omega_3^2 \right), \quad (2.8)$$

For the space filling embedding we are interested,

$$r \sin \theta = 2\pi\alpha' m_q \rightarrow 0, \quad (2.9)$$

therefore  $\theta' = 0$ . In this case, the world volume measure factor is reduced to that of 5 dimensional background metric:

$$\sqrt{-\det g^{(8)}/R^3} \rightarrow (r/R)^3 \quad (2.10)$$

which is precisely the volume factor of  $AdS_5$  Schwarzschild solution.

For more practical purpose, we can simply assume that (2.5) is the definition of the model and add necessary terms like scalars and without further consideration of brane embedding [14]. The rest of the paper is essentially following this philosophy by taking the space filling branes as an assumption.

## 2.2 The baryon chemical potential and charged AdS black hole.

The chemical potential can be treated as the tail of the electric potential living on the probe brane [16, 17]. In the brane embedding approach, considering back reaction of the metric is very hard to study analytically. However, when the brane is completely filling the background space-time, it can be easily done. This is because there is a unique way for the U(1) gauge potential to couple to the gravity: the solution is nothing but the Reissner-Nordstrom metric considered in ref. [23], where the charge was identified as the R-charge, whose carrier is adjoint representation of  $N = 4$  gauge theory. Therefore its gravitational coupling's order of magnitude goes like  $1/8\pi G \sim N_c^2$ . In our case the charge is weighted by  $N_f/N_c$ , and so is the back reaction.

Our starting point is to add gravity plus U(1) gauge interaction term to the phenomenological model of AdS/QCD [14] action:

$$I = \frac{1}{2\kappa^2} \int d^5x \sqrt{-g} \left( \mathcal{R} + \frac{12}{R^2} \right) - \frac{1}{4g^2} \int d^5x \sqrt{-g} F_{\mu\nu} F^{\mu\nu} + I_{\text{flavor}}[A_L, A_R; \Phi]. \quad (2.11)$$

with  $\kappa^2 = 8\pi G_5$ , and then neglect the original scalar and non-abelian flavour gauge interaction part  $I_{\text{flavor}}[A_L, A_R; \Phi]$ . Since we want to utilize the known charged AdS black hole solution [23], the ansatz for the metric and the potential is

$$ds^2 = \frac{r^2}{R^2} (-f(r)dt^2 + d\vec{x}^2) + \frac{R^2}{r^2} \frac{dr^2}{f(r)}, \quad \text{with} \quad f(r) = \frac{r^2}{R^2} - \frac{m}{r^2} + \frac{q^2}{r^4} \quad (2.12)$$

$$A_0 = \mu - \frac{Q}{r^2}. \quad (2.13)$$

Notice that  $q$  is a parameter describing the metric deformation while  $Q$  is the actual charge. We ask what is the precise relation of the parameter  $q$  and  $Q$  to satisfy the equation of motion:

$$\mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R} - \frac{6}{R^2} = \kappa^2 T_{\mu\nu}, \quad (2.14)$$

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\partial I}{\partial g^{\mu\nu}} = \frac{1}{g^2} \left( F_{\mu a} F^{\nu a} - \frac{1}{4} g_{\mu\nu} F_{ab} F^{ab} \right) + T_{\mu\nu}^{L,R}. \quad (2.15)$$

Since we consider back reaction to baryon charge, which is the diagonal U(1) charge of the flavor brane, we set non-abelian fields zero:  $T_{\mu\nu}^{L,R} = 0$ . The answer is

$$q^2 = \frac{2\kappa^2}{3g^2} Q^2 = \frac{2}{3} \frac{N_f}{N_c} R^2 Q^2 := a^2 R^2 Q^2, \quad (2.16)$$

confirming  $N_c/N_f$  dependence as we stated above. Here we have used  $\frac{R^3}{\kappa^2} = \frac{N_c^2}{4\pi^2}$ , and  $\frac{R}{g^2} = \frac{N_c N_f}{4\pi^2}$ . This suppression in gravity coupling of the baryons relative to R-charges is, of course, due to the difference in degrees of freedom between the fundamentals and adjoints. If we consider strictly large  $N_c$  limit, the metric correction is negligible. However, if we are interested in  $N_f/N_c$  correction, this is important. Such a  $N_f/N_c$  correction to the metric results in corresponding corrections in all physical observables. Especially, in phenomenological applications where one calculate using the finite value of  $N_c$  as well as finite  $N_f$ , this  $N_f/N_c$  correction will be essential and this will most important implication of this work. In this way, the baryon density problem in AdS/QCD can have a simple description including the back reaction of the metric. Without metric correction, it is difficult for the AdS/QCD defined as a quadratic action of gauge fields to encode the density effect unless one add higher order terms ( $\mathcal{O}(F^4)$ ) [16] or Wess-Zumino term [21].

### 3. Thermodynamics without hard-wall.

For the spherical boundary, there is a Hawking-Page transition (HPT) associated with deconfinement phase transition [2] and the HPT for the charged black hole in AdS space (AdSRN) was discussed in [23]. For the flat boundary we are interested here, there is no other scale than the temperature and therefore there is no such transition: the system is always in a de-confined phase, if we do not install hard wall. There will be some differences in fixed charge case compared with ref. [23] apart from the topology of the boundary.

### 3.1 Fixed chemical potential

The total action is evaluated to be<sup>3</sup>

$$I = \frac{V_3}{2\kappa^2} \int_{r_+}^{\infty} dr \int_0^{\beta} \sqrt{g^{(5)}} \left[ \frac{2q^2}{r^6} + \frac{8}{R^2} - \frac{4\kappa^2}{g^2} \frac{Q^2}{r^6} \right]. \quad (3.1)$$

where  $L$  is a large cutoff at large radius. The last term can be written  $-6q^2/r^6$  using  $\frac{Q^2}{g^2} = \frac{3q^2}{2\kappa^2}$ . If we define the horizon radius  $r_+$  as the larger zero of  $f(r) = 0$ , we can write the mass in terms of  $r_+$

$$m = \frac{r_+^4}{R^2} + \frac{q^2}{r_+^2}. \quad (3.2)$$

The temperature is defined by the singularity free condition:

$$T = \frac{f'(r_+)}{4\pi} = \frac{r_+}{\pi R^2} - \frac{q^2}{2\pi r_+^5} = \frac{r_+}{\pi R^2} - \frac{a^2 R^2 \mu^2}{2\pi r_+}. \quad (3.3)$$

where we used the relation between the chemical potential and the charge  $\mu = Q/r_+^2$ , determined by the condition that electric potential is vanishing at the horizon [30, 23, 17, 18]:

$$A_0(r_+) = \mu - Q/r_+^2 = 0, \quad (3.4)$$

and

$$q = aRQ = aRr_+^2\mu, \quad \text{with} \quad a = \sqrt{\frac{2N_f}{3N_c}}. \quad (3.5)$$

The action is evaluated to be,

$$I_{RN} = \frac{\beta V_3}{2\kappa^2 R^3} \left[ -\frac{q^2}{r^2} + \frac{2r^4}{R^2} \right]_{r_+}^L, \quad (3.6)$$

which diverge as  $L \rightarrow \infty$ . We regularize by the value of the thermal AdS action:

$$I_{AdS} = \frac{V_3 \beta'}{2\kappa^2 R^3} \frac{2L^4}{R^2}. \quad (3.7)$$

We determine  $\beta' (\simeq \beta)$  such that the volume of the two space outside cutoff  $r = L$  is equal:

$$\beta' = \beta \left( 1 - mR^2/2L^4 \right). \quad (3.8)$$

Then, the regularized action is given by,

$$I_{RN}^{(reg)} = -\frac{\beta V_3}{2\kappa^2 R^3} \left( \frac{r_+^4}{R^2} + \frac{q^2}{r_+^2} \right), \quad (3.9)$$

where we used (3.2).

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<sup>3</sup>Here we should work with Euclidean action which is obtained by taking a overall - sign in the Minkowski action. To make the electric potential real, we also have to change  $F^2 \rightarrow -F^2$ .



Now, we can calculate thermodynamic functions like pressure  $P$ , energy  $E$ , total Mass  $M$ , entropy  $S$  and total charge  $Q_{\text{tot}}$  can be calculated to be

$$E = \left( \frac{\partial I}{\partial \beta} \right)_{\mu} - \frac{\mu}{\beta} \left( \frac{\partial I}{\partial \beta} \right)_{\beta} = 3mb = M = 3P, \quad (3.10)$$

$$S = \beta \left( \frac{\partial I}{\partial \beta} \right)_{\mu} - I = 4\pi r_+^3 b = \frac{A}{4G}, \quad (3.11)$$

$$\langle Q \rangle = -\beta^{-1} \left( \frac{\partial I}{\partial \mu} \right)_{\beta} = 6qb, \quad (3.12)$$

where  $A$  is volume of horizon and constant  $b$  is given by

$$b = \frac{V_3}{2\kappa^2 R^3} = \frac{N_c^2 V_3}{8\pi^2 R^6}. \quad (3.13)$$

It is interesting to find an expression of entropy and energy in terms of temperature and chemical potential in small charge and high temperature region.

$$s = \frac{S}{V_3} = \frac{\pi^2}{2} N_c^2 T^3 + \frac{1}{2} N_c N_f \mu^2 T + \frac{1}{54\pi^4} \frac{N_f^3 \mu^6}{N_c T^3} - \frac{1}{54\pi^6} \frac{N_f^4 \mu^8}{N_c^2 T^5} \dots, \quad (3.14)$$

$$\epsilon = \frac{E}{V_3} = \frac{3\pi^2}{8} N_c^2 T^4 + \frac{3}{4} N_c N_f \mu^2 T^2 + \frac{N_f^2 \mu^4}{4\pi^2} + \frac{1}{36\pi^4} \frac{N_f^3 \mu^6}{N_c T^2} + \frac{1}{72\pi^6} \frac{N_f^4 \mu^8}{N_c^2 T^4} \dots \quad (3.15)$$

Notice that coupling dependence is hidden in  $N_f$  in this expression through  $R/g^2 = N_c N_f / 4\pi^2$ . It would be interesting to compare this with the gauge theory calculation that can be done along the line of recent paper by Yaffe et.al [33].

### 3.2 Fixed charge

For the canonical ensemble, The quark number is conserved under the deconfinement-phase- transition. Let's assume that  $\mu$  as the quark number and treat it as a conserved quantum number under the deconfinement phase transition. We need to add a boundary term [30] to guarantee the equation of motion with the fixed charge.

$$I_A = -\frac{1}{4g^2} \int d^5x \sqrt{-g} F^2 \rightarrow I_A + \frac{1}{g^2} \int_{\Sigma} F_{\mu\nu} n^{\mu} A^{\nu}. \quad (3.16)$$

The added surface term has the effect that is precisely changing the sign of the electromagnetic action:<sup>4</sup>  $I_A + I_{\text{surface}} = -I_A$  at on-shell. Therefore the value of action  $I_{RN}$  with cut off at large radius  $L$  is

$$\tilde{I}_{RN}^{(reg)} = \frac{\beta V_3}{2\kappa^2 R^3} \left( -\frac{r_+^4}{R^2} + \frac{5q^2}{r_+^2} \right). \quad (3.17)$$

One should notice that the result is precisely equal to that of Legendre transformation:

$$\tilde{I} = I - \mu \left( \frac{\partial I}{\partial \mu} \right)_{\beta}. \quad (3.18)$$

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<sup>4</sup>This addition of surface charge does not have essential effect in discussing the issues above.

This relation is not accidental:  $\tilde{I}$  is describing the fixed charge sector where Gauss law constrains the field configurations and equations of motion should be derived not from the action but its Legendre transformation. Identifying grand potential  $\Omega$  by  $I = \beta\Omega$ , the above relation corresponds to the thermodynamic relation between the free energy  $F = \tilde{I}/\beta$  and the grand potential  $\Omega$ .

$$F = \Omega + \mu\bar{Q}. \quad (3.19)$$

Once we realize the thermodynamic potential in terms of action and its Legendre transformation, we can use them to calculate physical quantities.

$$S = -\left(\frac{\partial F}{\partial T}\right)_q = b \cdot 4\pi r_+^3 = \frac{A}{4G}, \quad (3.20)$$

$$E = F + TS = 3bm = M, \quad (3.21)$$

$$\mu = \left(\frac{\partial F}{\partial Q_{\text{tot}}}\right)_q = \frac{Q}{r_+^2}. \quad (3.22)$$

It turns out that canonical (fixed charge) and grand canonical (fixed chemical potential) give completely equivalent description in this case. In this section, there is no phase transition and charge free AdS space is used just for the regularization. Since the relation between the free energy and grand canonical potential is consequence of charge conservation and gauss law, it should hold regardless of presence of phase transition.

Our result in this subsection should be compared with the result of ref. [23], where the authors used extremal charged black hole as a background to subtract, making some difference with the result here. Since extremal black hole can not have arbitrary temperature, it can not be used as a regularizing background without introducing conical singularity.

## 4. Charged background with Hard wall: Hawking-Page transition

To discuss the confinement in the context of D3 background, one has to introduce the IR cutoff, called hard wall [10, 14]. In gravity background and brane embedding picture, this corresponds to the situation where D7's (as well as the baryon vertices) are expelled by the repulsive core of the confining background [32]. The repulsive hard core is singular [32], and physically relevant region must be somewhat away from the singularity because the singularity invalidate the solution's reliability. One has to put a boundary by hand to cut off the unreliable region around the singularity. The IR brane in phenomenological model with can be understood as this boundary with background simplified to  $AdS_5$ . We assume that probe flavor brane is occupying all the relevant region of bulk of  $AdS_5$ .

In the interesting paper [31], it was pointed out that in the presence of IR brane, HPT exists even for the flat boundary case and worked out explicitly for the hard wall model without charge [10, 14]. Here we study the analogue of it in the presence of baryon charges.

### 4.1 Fixed charge

For high temperature, the system is described by a charged AdS black hole and for low temperature, confining configuration takes over by set-up. The key question is what is the

low temperature pair of charged black hole for the purpose of the HPT. There are a few possibilities.

The first possibility is to take the extremal AdS RN black hole to take care of the charge conservation. In ref. [23], phase diagram was worked out with this choice. Then  $f'(r_+) = 0$  and  $f(r_+) = 0$  so that for the given value of charge the horizon and the mass has fixed value  $m_* = \frac{3}{2} \frac{q^2}{r_*^2}$ , and  $r_* = 2^{-1/6} q^{1/3} R^{1/3}$ . Therefore

$$I_{\text{extremal RN}} = \frac{\beta_* V_3}{2\kappa^2 R^3} \left( \frac{2(L^4 - r_*^4)}{R^2} + \frac{4q^2}{r_*^2} \right). \quad (4.1)$$

Under this choice, we need to take the difference between (3.17) and (4.1) with a “common” inverse temperature  $\beta$ , instead of  $\beta_*$  in (4.1). However, while thermal AdS can be in any temperature, the extremal black hole has a fixed temperature, therefore it can not be used as a background at an arbitrary temperature competing with non-extremal black hole.<sup>5</sup>

The only background we know that allows arbitrary temperature at low temperature is the thermal AdS background. But, this background does *not* have any charge. How can we interpret this situation? Where the charges can go as temperature cools down? There are two options to interpret this:

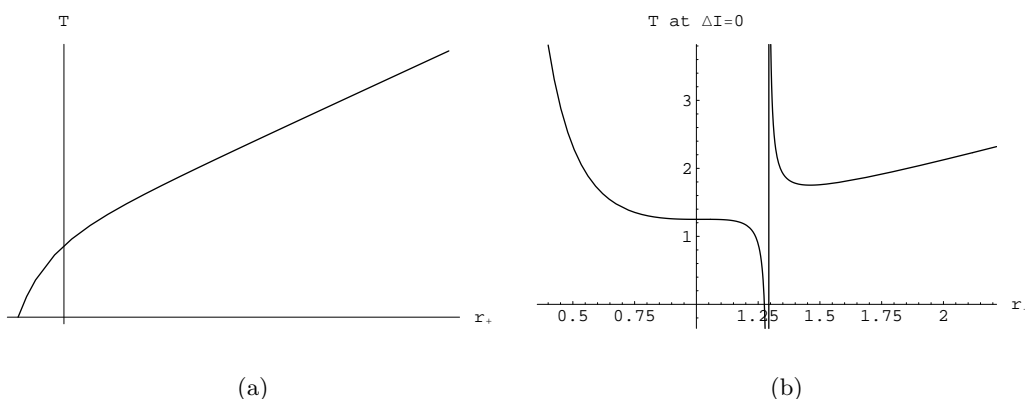
1. If we keep the charge fixed, there is no confinement phase transition as temperature goes down.
2. We take thermal AdS as a reference background for the low temperature and take care of the charge conservation by IR-brane.

Let’s consider the former possibility: When we do not introduce the hard wall this is what happens as we discussed in section 3. However, in the presence of hard wall or confinement, it is physically implausible for a few reasons. First, in the absence of charge we know there is a confinement phase transition [31]. Now, suppose phase transition disappear by adding baryons as it says. Then, this phenomena is independent of how much charge is there, so even in the extreme low density disappearance of HPT is implied in this case. How just a few charges can change the phase diagram of a system discontinuously? Secondly, the confinement is determined by the dynamics of gluon degree of freedom which are  $\mathcal{O}(N_c^2)$ , and we do not expect it can be abolished by introducing quark degree of freedom which is just  $\mathcal{O}(N_c)$ . Therefore in the presence of confining set-up in low temperature, we do not take the first possibility.

The second possibility is the only one left and it corresponds to storing the charges outside the repulsive gravitational core. In this paper, for the convenience of treatment, we assume that all charges are at the IR brane located at  $r_m$ . With this set up, we can now calculate the phase diagram.

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<sup>5</sup>In the round boundary case [23] with high chemical potential, there is no phase transition in high high chemical potential. In the flat boundary limit only high chemical potential case survive and the result is reduced to the previous section. So, although the boundary condition is different to ours, their result itself is not in conflict with ours.



**Figure 1:** Temperature as function of  $r_+$ . (a) fixed  $q$ , (b)  $T$  v.s  $r_+$  at the phase boundary.

The value of AdS action with charge at IR brane is given by

$$I_{AdS+Q} = \frac{\beta V_3}{2\kappa^2 R^3} \left( -m + \frac{2(L^4 - r_m^4)}{R^2} + \frac{2\kappa^2 Q^2}{g^2 r_m^2} \right), \quad (4.2)$$

where  $m$  comes from the condition the temperature of AdS was tuned by (3.8). Now the difference of the actions of two phases is

$$\Delta I = \frac{\beta V_3}{2\kappa^2 R^3} \left[ m - \frac{2(r_+^4 - r_m^4)}{R^2} + \frac{q^2}{r_+^2} + \frac{2\kappa^2 Q^2}{g^2} \left( \frac{1}{r_+^2} - \frac{1}{r_m^2} \right) \right], \quad \text{for } r_+ > r_m \quad (4.3)$$

$$= \frac{\beta V_3}{2\kappa^2 R^3} \left[ m + \frac{q^2}{r_m^2} \right] > 0, \quad \text{for } r_+ < r_m. \quad (4.4)$$

The phase boundary is given by  $\Delta S = 0$ . It is helpful to figure the temperature as a function of the horizon radius  $r_+$ . See figure 1. Notice that unlike the round boundary case, temperature is monotonically increasing function of  $r_+$  for all fixed value of  $q$  starting from  $r_+ = q^{1/3}/2^{1/6}$  where  $T = 0$  (See figure 1(a)). However it is monotonically (rapidly) decreasing function of  $r_+$  for  $2^{1/4} < r_+/r_m < 1.27517$  which is the relevant region for the phase boundary. ( See figure 1(b).) The phase boundary is parametrically given by

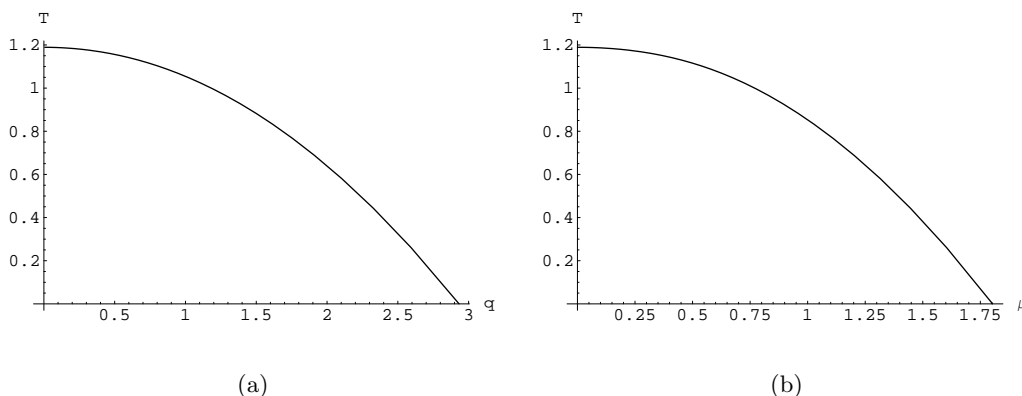
$$\tilde{q} = \tilde{r}_+ \sqrt{\frac{\tilde{r}_+^4 - 2}{5 - 3\tilde{r}_+^2}}, \quad \tilde{T} = \tilde{r}_+ \left( 1 - \frac{1}{2} \frac{\tilde{r}_+^4 - 2}{\tilde{r}_+^4 (5 - 3\tilde{r}_+^2)} \right), \quad (4.5)$$

with  $\tilde{r}_+ = r_+/r_m$ ,  $\tilde{q} = qR/r_m^3$  and  $\tilde{T} = \pi T R^2/r_m$ . See figure 2(a).

Notice that the phase diagram closes at finite density unlike the case without gravity back reaction [17, 22], where phase diagram is open. The main reason of this change is attributable to the change of the definition of the temperature by the second term of eq. (3.3).

## 4.2 Fixed chemical potential

When we fix chemical potential, charge itself is not fixed. Only average charge is fixed. So one might choose, as the low temperature phase, the thermal AdS without charge. The



**Figure 2:** Phase diagrams for the fixed charge (a) and for the fixed chemical potential (b). In  $q$ - $T$  space, two ends points of phase boundary are given by  $(q, T) = (0, 2^{1/4}r_m/\pi R^2)$  and  $(2.9314r_m^3/R, 0)$ . Below (over) the boundary is the (de-)confinement region.

phase diagram of resulting system is worked out in the appendix. The issue here is that as a consequence of the choice, there would be no chemical potential dependence of the system in the low temperature phase. One might argue that  $\mu$  independence is analogue of the temperature independence of the low temperature phase of large  $N_c$  limit. For the round boundary, this choice was actually taken in [23]. In [20] this consequence is used to support the chemical potential independence in chirally asymmetric case.

However, comparing confinement phase relative deconfinement phases, thermodynamics functions of baryons are not so suppressed compared with that of glueballs. The gluons are adjoint so degree of freedom in confined phase relative to that in deconfined phase is  $\mathcal{O}(1/N_c^2)$ , while for baryon case, it is suppressed by  $\mathcal{O}(1/N_c)$ . So it is not clear whether baryon density independence can be implied by the temperature independence, especially when we consider the gravity back reaction. Also, if there is no charge in the system at all, electric potential is globally constant and if we calculate the average charge, it is zero.

$$\langle Q \rangle = - \left( \frac{\partial \Omega}{\partial \mu} \right)_\beta = 0. \quad (4.6)$$

It vanishes because there is no chemical potential dependence of the system. It is not sensible to have zero charge density for any chemical potential, since the latter case will have charge dependence as we will show shortly. Furthermore, this means the relation with fixed charge case by the Legendre transformation would be lost. Therefore we should not take the pure AdS space describe the low temperature phase. In this paper, we take the view that we should take the Legendre transform of the fixed charge system, which discussed in the previous subsection. Then, the only change is replacing  $q \rightarrow aRr_+^2\mu$ , whose jacobian does not involve any singularity over the  $r_+$  region of the phase boundary. The phase diagram has the same topology as one can see in figure 2(b). So we do not repeat the analysis here.

The point of interest is that the thermodynamic potential defined by the Legendre transformation of the previous sections have non-trivial chemical potential dependence,

simply because the fixed charge case has non-trivial charge dependence. This is the answer to the issue we raised in the beginning of this section.

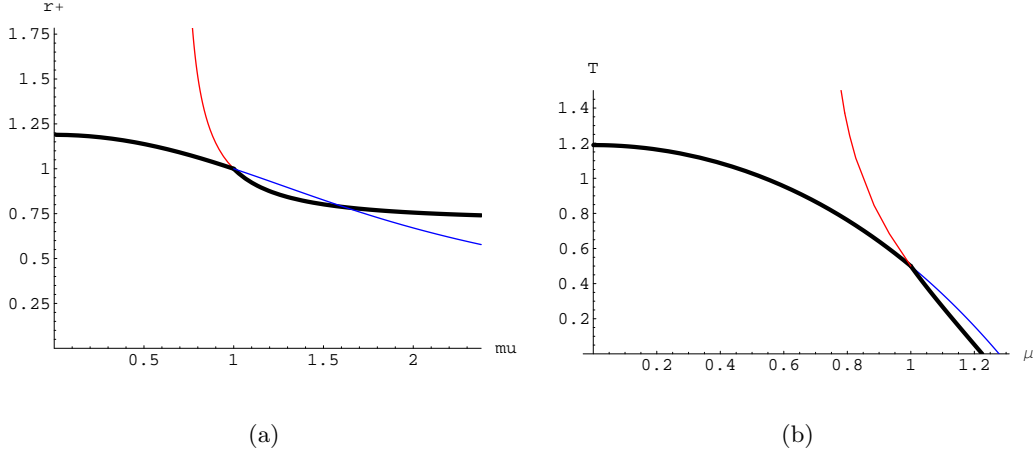
Finally one may want to compare the phase diagram in this section with those in the appendix, where we analyzed the choice taking the AdS space with no charge as the low temperature background.

## 5. Discussion

In this paper, we considered the back reaction of the metric to the baryon charge for the case where  $AdS_5$  bulk is completely filled with flavor brane. For the high temperature phase, the unique gravity solution coupled with  $U(1)$  electric potential is the charged AdS black hole with flat boundary. We also addressed the issue whether we have a chemical potential dependence in low temperature confining phase and concluded there is non-trivial chemical potential dependence. Phase diagram for hard wall model in the presence of baryon charge is worked out and it turns out that phase diagram closes at finite baryon density. This is very different from the previous treatment without gravity back reaction. We emphasize that the general consideration of gravity back reaction in top down models are non-trivial [25, 26] and the simplicity of present model is coming from the bulk-filling nature of the flavor branes in the effective ads/qcd models in the bottom-up approach.

We also want to point out that with the charge dependence of the metric, density dependence of physical quantities can be encoded in quadratic action of AdS/QCD without adding higher order  $\alpha'$  correction, due to the metric change, while in brane embedding approach, one has to include the higher order  $\mathcal{O}(F^4)$  effect to see it [34]. It is interesting to workout the density dependence of physical quantities (mass and couplings) in this model.

There is another issue in the baryon density in AdS/CFT which is not treated in this paper. IN reference, [18], electric charges were added with boundary condition such that adding charges does not change smooth surface structure. The phases allowed in this assumption is rich. Shortly after the paper [18], the authors of ref. [19], pointed out that electric charges on flavor branes are fundamental strings' end points. The force balancing condition is imposed and it was concluded that due to the spiky structures of the surface, branes should always touch the horizon, even at very low temperature. As a consequence, even a tiny amount of baryons should change the whole phase diagram which is reminiscent to the situation treated here as a puzzle. We believe that this issue need to be discussed more carefully, since baryon vertex staying outside the horizon can change the conclusion very easily. Here our set up is such that brane is filling the bulk so that we can not address this issue. It will be treated in a separate paper [35].



**Figure 3:** Phase diagrams for the fixed chemical potential with AdS without charge as the low temperature phase. (a)  $r_+$  v.s  $\mu$ . In actual plot  $r$  means  $r_+/r_m$  and  $\mu$  means  $\mu aR/r_m$ ; (b) Temperature v.s chemical potential; Actual plot is  $TaR/r_m$  v.s  $\mu aR/r_m$ . In each case, the thick line is the phase boundary and the connection point is  $(T, \mu, r_+) = (r_m/(2aR), r_m/(aR), r_m)$  respectively. Notice that phase diagram closes in  $T$ - $\mu$  diagram.

### A. Phase diagram for grand canonical system with thermal AdS as the low temperature phase

The difference of actions is

$$I_{RN} - I_{AdS} = \frac{\beta V_3}{2\kappa^2 R^3} \left( - \left( \frac{r_+^4}{R^2} + \frac{q^2}{r_+^2} \right) + \frac{2r_m^4}{R^2} \right), \quad \text{for } r_+ > r_m \quad (\text{A.1})$$

$$= \frac{\beta V_3}{2\kappa^2 R^3} \left( \left( \frac{r_+^4}{R^2} + \frac{q^2}{r_+^2} \right) - \frac{2q^2}{r_m^2} \right), \quad \text{for } r_+ < r_m. \quad (\text{A.2})$$

One can study the phase diagram by looking at the locus of the  $\Delta S = 0$ :

$$(r_+/r_m)^4 + (r_+/r_m)^2 (aR\mu/r_m)^2 = 2, \quad \text{for } r_+ > r_m, \quad (\text{High T branch}), \quad (\text{A.3})$$

$$r_m^2/r_+^2 + r_m^2/(aR\mu)^2 = 2, \quad \text{for } r_+ < r_m, \quad (\text{Low T branch}). \quad (\text{A.4})$$

The phase diagram in  $(\mu, r_+)$  is shown in figure 3(a). Notice that there is a discontinuity in derivative at the connection point  $(\mu, r_+) = (r_m/(aR), r_m)$ .

For the large chemical potential the phase boundary seems to saturate asymptotically:

$$r_+ \rightarrow \frac{r_m}{\sqrt{2}} \left( 1 + \frac{r_m^2}{4a^2} \frac{1}{\mu^2} \right), \quad (\text{A.5})$$

which is similar to the results in [17, 22], where back reaction of the probe brane is not considered with identification  $T = \frac{r_+}{\pi R^2}$ . However, this similarity is just apparent one. In our case, the temperature is given by  $T = \frac{r_+}{\pi R^2} - \frac{1}{2\pi} \frac{(a\mu)^2}{r_+}$ , and we need to consider the implication of the correction given by the second term. For the positivity of temperature

we need  $r_+ \geq \frac{Ra\mu}{\sqrt{2}}$ , which requires  $r \geq \frac{\sqrt{3}}{2}r_m$  for the low temperature branch and  $r \geq (2/3)^{1/4}r_m$  for high temperature branch. One can easily work out the phase boundary in the parametric form:

$$\tilde{T}_H = \tilde{r}_+ \left( 3/2 - 1/\tilde{r}_+^4 \right), \quad (\text{A.6})$$

$$\tilde{\mu}_H = \sqrt{2 - \tilde{r}_+^4/\tilde{r}_+}, \quad (\text{A.7})$$

for the high temperature branch and

$$\tilde{T}_L = \tilde{r}_+ \frac{\tilde{r}_+^2 - 3/4}{\tilde{r}_+^2 - 1/2}, \quad (\text{A.8})$$

$$\tilde{\mu}_L = \frac{\tilde{r}_+}{\sqrt{2\tilde{r}_+^2 - 1}}, \quad (\text{A.9})$$

for the low temperature branch. Here we used the scaled variables,  $\tilde{r}_+ = r_+/r_m$ ,  $\tilde{T} = TaR/r_m$  and  $\tilde{\mu} = \mu aR/r_m$ . By Plotting temperature v.s chemical potential, we see that the phase diagram closes. See figure 3(b). This difference of results can be easily understood: The chemical potential is related to the charge by  $\mu = Q/r_+^2$  and charge is of order  $N_f/N_c$ , therefore in AdS/CFT with probe brane approach the range of the chemical potential is rather limited. However, our result indicates that, with large  $N_f$  and large gravity back reaction, the phase diagram closes.

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