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Design of a computationally efficient dc-notch FIR filter

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Abstract: A new design of a computationally efficient sharp dc-notch FIR filter is proposed, whereby a modified $\alpha-$ scaled sampling kernel and a complementary filter concept are utilized to implement a programmable dc-notch FIR filter. Design examples demonstrate that the proposed approach provides a practical and efficient design procedure for the realization of a sharp dc-notch FIR filter of lower order, satisfying the given filter specification.

Keywords: modified $\alpha-$ scaled sampling kernel, analytical design, dc-notch filter, finite-impulse response (FIR)

Classification: Science and engineering for electronics

References

[1] P. Zahradnik and M. Vlček, "Note on the design of an equiripple DC-notch FIR filter," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 54, no. 2, pp. 196–199, Feb. 2007.

[2] S. K. Mitra, *Digital Signal Processing: A Computer-Based Approach*, 3rd ed., New York, McGraw-Hill, 2006.

[3] K. J. Kim, J. B. Seo, S. W. Nam, and Y. Lian, "Design of a sharp linear-phase FIR filter using the $\alpha-$ scaled sampling kernel," *Proc. of SIP'06 (The 6th WSEAS Int. Conf. on Signal Processing)*, Dallas, Texas, U.S.A., pp. 129–133, 22–24 March 2007.

[4] S. C. Pei and M. P. Cao, "A two-channel nonuniform perfect reconstruction filter bank with irrational down-sampling factors," *IEEE Signal Process. Lett.*, vol. 12, pp. 116–119, Feb. 2005.

[5] Y. Neuvo, D. C. Yu, and S. K. Mitra, "Interpolated finite impulse response filters," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. ASSP-32, no. 3, pp. 563–570, June 1984.

[6] M. Bellanger, *Digital Processing of Signals*, New York, John Wiley & Sons, 1984.

[7] S. Winder, *Analog and Digital Filter Design*, 2nd ed., Elsevier Science (USA), 2002.

1 Introduction

A dc-notch FIR filter suppresses dc-noise in the digital communication signals. In particular, the dc-offset is such a case in the zero mark of the waveform, and it may be undesirable and rather troublesome because undesirable

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pops can occur when editing a file with a large dc-offset. Thus, the dc-offset should be removed after recording.

Recently, a fast and robust dc-notch FIR filter design employing a simple algebraic recursive procedure was reported [1]. In particular, the analytical design procedure leads to an effective evaluation of the impulse response of a highly selective optimal dc-notch FIR filter, but the number of filter coefficients required can increase rapidly with sharpness of the corresponding transition band. Also, several computationally efficient FIR filter designs have been proposed to reduce the total number of multipliers needed to implement a sharp FIR filter [1, 2, 3], including (i) the prefiler-equalizer and the interpolated FIR (IFIR) approaches for the narrow-band sharp filter design [2, 3] and (ii) the frequency-response masking (FRM) technique for the sharp filter design with arbitrary bandwidth [2]. They are effective for the design of sharp FIR filters with fixed coefficients, but, since those conventional methods do not provide a closed-form solution to FIR filter coefficients, it may be difficult to apply them further for the design of programmable FIR filters. Recently, the $\alpha-$scaled sampling kernel [4] was utilized for the design of linear-phase FIR filters with narrow transition [3], where a closed-form expression is introduced for FIR filter coefficients. In this paper, we propose a new design procedure for a more computationally efficient dc-notch FIR filter when compared with the analytical design procedure [1]. For that purpose, a modified $\alpha -$ scaled sampling kernel and a complementary filter concept are utilized to derive a closed-form expression for filter coefficients. This paper is concerned with a practical and efficient realization of a dc-notch FIR filter of lower order.

2 Sharp linear-phase FIR filter design using a modified α-scaled sampling kernel

Let's denote $x[n]$ and $x_\alpha[n]$ by two discrete-time signals sampled at different sampling rates (here, T and $T' = T/\alpha$, respectively).

$$x[n] = x(nT), \quad x_\alpha[n] = x(nT'), \quad T' = T/\alpha \tag{1}$$

where $\alpha > 1$ is for interpolation and $0 < \alpha < 1$ for decimation. Recently, the $\alpha -$ scaled sampling kernels [4] were suggested to obtain in a single step $x_\alpha[n]$ from $x[n]$. Also, an efficient design of linear-phase FIR filters with sharp transition was developed [3], which utilizes the $\alpha-$scaled interpolation kernel $\mathrm{sinc}((n/\alpha) - k)$. Furthermore, when the prototype model filter $h[n]$ is a linear-phase (Type 1) FIR filter of length N, consider the following FIR filter $h_{(\alpha)}[n]$:

$$h_{(\alpha)}[n] = \frac{1}{\alpha} h_\alpha[n] = \sum_{k=0}^{N-1} h[k] \bullet \frac{1}{\alpha}\mathrm{sinc}\left(\frac{n}{\alpha} - k\right) \tag{2}$$

By realizing the fact that (i) $H_{(\alpha)}[z] = H[z^\alpha]$ and (ii) (2) can be expressed in a convolution form when α is an integer, we can design a programmable linear-phase FIR filter with computational complexity similar to or a bit more

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than conventional computational efficient approaches (e.g., IFIR or FRM). We prove that (2) can be derived in a convolution form when α is an integer, which can be obtained as follows:

$$h_{(\alpha)}[n] = \frac{1}{\alpha}\sum_{k=0}^{N-1} h[k]\mathrm{sinc}\left(\frac{n}{\alpha} - k\right) \tag{3}$$

$$= \frac{1}{\alpha}\sum_{k=0}^{N-1} h[k]\mathrm{sinc}\left(\frac{1}{\alpha}(n - \alpha k)\right) \tag{4}$$

$$= \frac{1}{\alpha} g[n] * \mathrm{sinc}\left(\frac{n}{\alpha}\right) \tag{5}$$

In (5), $g[n]$ can be obtained by inserting $(\alpha - 1)$ zeros between the adjacent samples of $h[n]$ and $*$ denotes the convolution operation. Note that the z transform of $g[n]$ corresponds to $H[z^\alpha]$. Thus, the sampling frequency of $H[z^\alpha]$ need not to be α times larger than $H[z]$ and a sharp FIR filter can be obtained without increase of sampling frequency and without increase of computational complexity. From (3)-(5), we can see that (3) can be expressed in a convolution form when α is integer as shown in (5).

Conventional IFIR and FRM methods [2] are effective for the design of sharp FIR filters with fixed coefficients, but, since they do not provide a closed-form solution for FIR filter coefficients, it may be difficult to extend them further for the design of programmable FIR filters. Since a programmable FIR filter allows us a great flexibility for the filter design, a closed-form expression for filter coefficients will be derived next by introducing a modified $\alpha -$ scaled interpolation kernel and it will be utilized to design a programmable dc-notch FIR filter.

We can see from (2) that $\mathrm{sinc}(n/\alpha)$ is an ideal low-pass filter of doubly infinite length. Accordingly, that kernel should be modified to design sharp linear phase FIR filter of finite order. Since a raised-cosine filter with the desired roll-off and its impulse response of finite length is a low-pass filter widely used for pulse shaping in the communication field, it is employed in this paper as a modified sampling kernel and its time-domain impulse response is given by

$$p[n] = \frac{\sin \pi n}{\pi n}\frac{\cos \pi R n}{1 - 4R^2 n^2} \tag{6}$$

where R is defined as the roll-off factor $(0 \le R \le 1)$. The sharpness of the filter is controlled by R. Thus, we can obtain a modified $\alpha -$scaled sampling kernel $p_1(n) = p(n/\alpha)/\alpha$ by replacing $\mathrm{sinc}((n/\alpha) - k)$ in (2) by $p((n/\alpha) - k)$. Then, the new filter $h_{(\alpha)}[n]$ can be given by

$$h_{(\alpha)}[n] = \sum_{k=0}^{N-1} h[k] \bullet \frac{1}{\alpha} p\left[\frac{n}{\alpha} - k\right] \tag{7}$$

Also, the filter order of $h_{(\alpha)}[n]$ can be determined by using some equations including the *Bellanger* equation as in [6, 7].

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3 Design of a computationally efficient dc-notch FIR filter using a prototype model filter designed by an analytical procedure

Recently, an optimal dc-notch FIR filter design using the simple algebraic recursive procedure was proposed [1], which is robust and fast. However, the length of filter coefficients should increase rapidly with an increase of the passband maximal attenuation a and a decrease of the passband edge frequency $\omega_p T$. By utilizing (7) when α is an integer, we develop a new design procedure for a more computationally efficient dc-notch FIR filter when compared with the analytical design. For that purpose, (7) and a complementary filter concept are employed to derive a closed-form expression for filter coefficients. To make the stopband of a dc-notch filter sharper with only a little increase in computational complexity, a (Type 1 FIR) prototype model filter $H[z]$ as in Fig. 1 (a) should be designed first. It can be designed by many conventional FIR filter design methods. $H[z]$ with less-sharp transition (e.g., compare Fig. 1 (a) with Fig. 1 (d)) was designed in this paper by the analytical design [1], since it provides a closed-form solution to filter coefficients. Also, the complementary filter $H^c[z]$ of $H[z]$ is obtained as follows (see Fig. 1 (b)):

$$H^c[z] = z^{-\frac{N-1}{2}} - H[z] \tag{8}$$

where N is the filter length of $H[z]$. In Fig. 1 (c), $H^c_{(\alpha)}[z]$ is a sharp low-pass FIR filter, obtained by substituting $H^c[z]$ into $H[z]$ in (7) (see Fig. 1 (c)).

$$h^c_{(\alpha)}[n] = \sum_{k=0}^{N-1} h^c[k] \bullet \frac{1}{\alpha} p\left[\frac{n}{\alpha} - k\right] \tag{9}$$

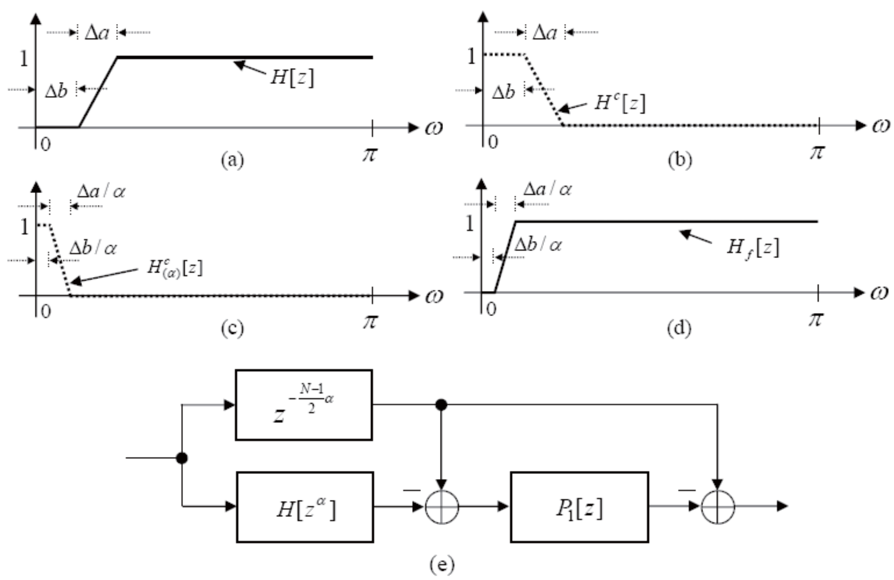

Fig. 1. The frequency responses (a)-(d) and (e) the structure of the proposed dc-notch filter

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Since (9) can be expressed in a convolution form when α is an integer, it can be represented in the z domain by

$$H^c_{(\alpha)}[z] = H^c[z^\alpha]P_1[z] \tag{10}$$

In (10), $H^c[z^\alpha]$ can be obtained from $H^c[z]$ by inserting $(\alpha-1)$ zeros between the adjacent samples of $H^c[z]$. The overall structure of the following filter $H_f[z]$ is given in Fig. 1 (e).

$$H_f[z] = z^{-\frac{N-1}{2}\alpha} - H^c_{(\alpha)}[z]. \tag{11}$$

Given the desired specifications, we should determine the scaling factor α in (7), the maximal passband attenuation a and the passband edge frequency $\omega_p T$ of $H[z]$. Since a is given from the filter specification and $\omega_p T$ depends on the values of α and the passband edge frequency of $H_f[z]$ (both are closely related), we need to control only α. Its determination is demonstrated in the next section.

4 Design examples

To demonstrate the filter performance, we consider two dc-notch filter examples designed by the analytical design and the proposed approach. Same filter specifications as in [1] are used: The passband edge frequency $\omega_p T = 0.01\pi$ of $H_f[z]$ and the passband maximal attenuation $a = 0.01$ dB. The filter length of the final filter is 227 when the analytical procedure [1] is applied. When the proposed approach is applied, the passband edge frequency of $H[z]$ becomes 0.05π when $\alpha = 5$, the filter length of $H[z]$ is 47, and that of $P_1[z]$ is

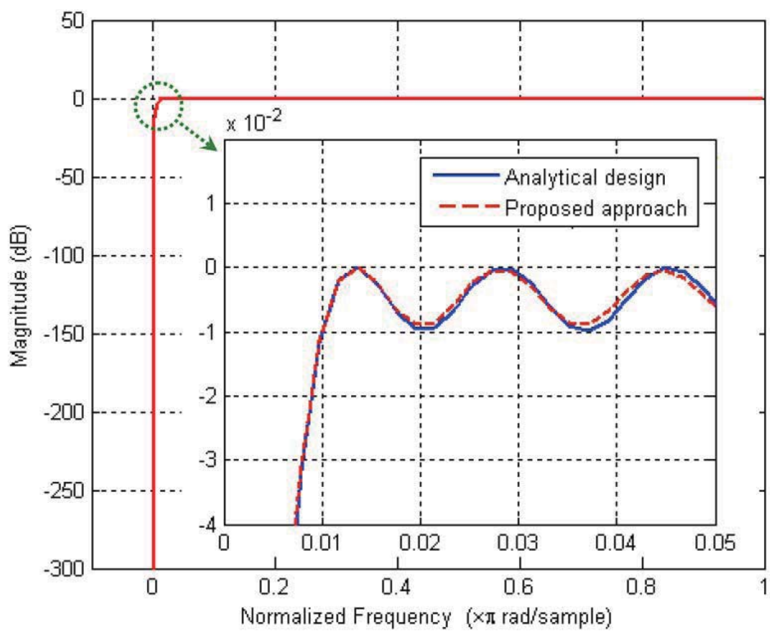

Fig. 2. Gain response of a design example (Small figure: passband details)

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Table I. Filter coefficients of $H[z]$ and $P_1[z]$ ($\alpha = 5$)

n	h[n] (model)	n	p_1[n] (raised-cosine)
0, 46	−0.029400	0, 40	0.000000
1, 45	−0.008300	1, 39	0.003500
2, 44	−0.009400	2, 38	0.005700
3, 43	−0.010600	3, 37	0.005000
4, 42	−0.011700	4, 36	0.002000
5, 41	−0.012900	5, 35	0.000000
6, 40	−0.014100	6, 34	0.003000
7, 39	−0.015300	7, 33	0.011900
8, 38	−0.016600	8, 32	0.021400
9, 37	−0.017700	9, 31	0.021100
10, 36	−0.018900	10, 30	0.000000
11, 35	−0.020000	11, 29	−0.044100
12, 34	−0.021100	12, 28	−0.098100
13, 33	−0.022200	13, 27	−0.132400
14, 32	−0.023100	14, 26	−0.109500
15, 31	−0.024000	15, 25	0.000000
16, 30	−0.024800	16, 24	0.200800
17, 29	−0.025500	17, 23	0.463400
18, 28	−0.026200	18, 22	0.728900
19, 27	−0.026700	19, 21	0.926800
20, 26	−0.027100	20	1.000000
21, 25	−0.027400		
22, 24	−0.027500		
23	0.921400		

Table II. The computational complexity comparison

Implementation	No. of additions	No. of multiplications
Analytical design [1]	226	114
Proposed method	88	45

41 for R = 0.5 [6]. We can verify from Fig. 2 that the dc-notch FIR filters by both the conventional and proposed approaches satisfy the filter specifications. The filter coefficients by the proposed approach are summarized in Table I. As mentioned in the section 2, no increase of sampling frequency is required in designing the proposed sharp FIR filter. As described in [5], the number of multipliers and adders in the structure can be approximately the same as in the model filter if additional computational complexity for the hardware needed to implement the interpolator is small. In IFIR filter, the passband and stopband gains are the same as in the model filter, but the passband and stopband widths are only $(1/\alpha)$-th of those of the model filter when the interpolation factor is α. Thus, the effect of the interpolation of

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the impulse response (i.e., $H[z^\alpha]$) is to make narrow the passband and transition bands without any increase in the number of arithmetic operations. We employed the same method as in the IFIR filter design approach to calculate computational complexity. Thus, we can see from Table II that the dc-notch FIR filter design by the proposed approach saves about 61% adders and about 60% multipliers, compared with that by the analytical procedure only [1].

5 Conclusion

We proposed a more computationally efficient dc-notch FIR filter design when compared with the conventional analytical design. For that purpose, a modified $\alpha -$ scaled sampling kernel is introduced and the complementary filter concept is utilized. Finally, the proposed filter design yields a practical and efficient realization of a dc-notch FIR filter of lower order.

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