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String cosmology of the *D*-brane universe

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We analyze homogeneous anisotropic cosmology driven by the dilaton and the self-interacting ''massive'' antisymmetric tensor field which are indispensable bosonic degrees with the graviton in the NS-NS sector of string theories with *D*-branes. We found the attractor solutions for this system, which show the overall features of general solutions, and confirmed it through numerical analysis. The dilaton possesses the potential due to the presence of the *D*-brane and the curvature of extra dimensions. In the presence of the nonvanishing antisymmetric tensor field, the homogeneous universe expands anisotropically while the *D*-brane term dominates. The isotropy is recovered as the dilaton rolls down and the curvature term dominates. With the stabilizing potential for the dilaton, the isotropy can also be recovered.

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I. INTRODUCTION

Low energy effective theories derived from the NS-NS sector of string theories contain the gravity, $g_{\mu\nu}$, the dilation, Φ, and the antisymmetric tensor field, $B_{\mu\nu}$. The existence of the last degree of freedom leads to intriguing implications in string cosmology [1]. In four spacetime dimensions, the massless antisymmetric tensor field is dual to the a pseudoscalar (axion) field [2], and the axion-dilaton system is known to develop an unobserved anisotropy in our Universe, which can be diluted away at late times only in a contracting universe [3].

Such a disastrous cosmological situation can be resolved also in string theory which contains another dynamical object called *D*-brane [4]. The gauge invariance on the *D*-brane is maintained through the coupling of the gauge field strength to the antisymmetric tensor field [4–6]. Then, the effective action derived on the *D*-brane describes the antisymmetric tensor field as a massive and self-interacting 2-form field. Cosmological evolution of such a tensor field has been investigated in Ref. [7] assuming that the dilaton is fixed to a reasonable value. Although the time-dependent magnetic *B* field existing in the early universe develops an anisotropy in the universe, it was realized that the matterlike behavior of the *B* field (*B*-matter) ensures a dilution of the anisotropy at late times and thus the isotropy is recovered in reasonable cosmological scenarios [7]. In such sense the effect of antisymmetric tensor field on the *D*-brane is distinguished from that of field strength of the U(1) gauge field [8].

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In this paper, we investigate the cosmological evolution of the *B*-matter-dilaton system in our Universe which is assumed to be imbedded in the *D*-brane. The usual string cosmology with the dilaton suffers from the notorious runaway problem, which is also troublesome in the *D*-brane universe. In our study, the dilaton obtains two exponential potential terms due to the curvature of extra dimensions Λ, and the *D*-brane tension (the mass term of the *B*-matter) m_B. It is interesting to observe that the dilaton can be stabilized for negative Λ [9] which, however, leads to a contracting universe due to the effective negative cosmological constant in our Universe. When Λ is positive, the *B*-matter dominance will be overturned by the Λ dominance as the dilaton runs away to the negative infinity. As a consequence of this, the initial anisotropy driven by the *B*-matter can also be diluted away at late times.

For a realistic low energy effective theory, string theory must be endowed with a certain mechanism generating an appropriate vacuum expectation value for the dilaton. In such a situation, the dilaton is expected to be stabilized at some stage of the cosmological evolution affecting the dynamics of the antisymmetric tensor field. Taking an example of the dilaton stabilization, we will also examine the cosmological evolution of the *B*-matter and the dilaton in which the essential features of Ref. [7] are reproduced.

This paper is organized as follows. In Sec. II, we describe the low energy effective action of the *D*-brane universe and the corresponding field equations. Before our main discussion, Sec. III is devoted to presenting homogeneous solutions in flat spacetime illustrating some interesting features of the *B*-matter-dilaton system in a simple way. In Sec. IV, we find semianalytic and numerical cosmological solutions to observe an intriguing interplay of the curvature Λ and the D-brane tension m_B. In Sec. V, we consider the evolution of the anisotropic universe with the dilaton stabilization which leads to a satisfactory cosmology of the D-brane universe. We conclude in Sec. VI.

II. STRING EFFECTIVE THEORY ON THE *D*-BRANE

The main idea of the D-brane world is that we reside on a Dp-brane imbedded in 10 (or 11) dimensional spacetime with extra-dimensions compactified. The bosonic NS-NS sector of the D-brane world consists of the U(1) gauge field A_μ living on the Dp-brane and the bulk degrees including the graviton $g_{\mu\nu}$, the dilaton Φ, and the antisymmetric tensor field of rank-two $B_{\mu\nu}$. In the presence of the brane, the gauge invariance of $B_{\mu\nu}$ is restored through its coupling to a U(1) gauge field A_μ and the gauge invariant field strength is [4]

$$\mathcal{B}_{\mu\nu} \equiv B_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}, \tag{1}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

Even though we are assumed to live on the D-brane, we adopt the conventional compactification in the sense that the extra dimensions are compact and stabilized, and thus static. In the presence of the D-brane, this would in general require an additional setup like additional branes and fluxes along the extra dimensions [10], and the working of nonperturbative effects [11]. The extra dimensions are warped due to branes and fluxes. The warping of extra dimensions gives a warp factor in the definition of the four-dimensional Planck scale [12], and also induces the potential for the dilaton [10] as described below. If the fluxes in the extra dimensions significantly affect the compactification, the existence of the fluxes on the D-brane seems also natural and it is intriguing to tackle their effect in the early universe. However, simultaneous consideration of the fluxes along both the extra dimensions and the D-brane lets the computation almost intractable. Therefore, the conventional compactification is an appropriate setup at the present stage, and then we take the following four-dimensional effective action of the bosonic sector in the string frame [7]

$$\begin{aligned} S_{\mathrm{S}} = \frac{1}{2\kappa_4^2}\int d^4\tilde{x}\sqrt{-\tilde{g}}\Big[& e^{-2\Phi}\Big(\tilde{R} - 2\Lambda + 4\tilde{\nabla}_\mu\Phi\tilde{\nabla}^\mu\Phi \\ & - \frac{1}{12}\tilde{H}_{\mu\nu\rho}\tilde{H}^{\mu\nu\rho}\Big) \\ & - m_B^2 e^{-\Phi}\sqrt{1 + \frac{1}{2}\tilde{\mathcal{B}}_{\mu\nu}\tilde{\mathcal{B}}^{\mu\nu} - \frac{1}{16}(\tilde{\mathcal{B}}^*_{\mu\nu}\tilde{\mathcal{B}}^{\mu\nu})^2}\Big], \end{aligned} \tag{2}$$

where the tilde denotes the string-frame quantity, $H_{\mu\nu\rho} = \partial_{[\mu}\mathcal{B}_{\nu\rho]}$ and $\mathcal{B}^*_{\mu\nu} = \frac{1}{2}\sqrt{-g}\epsilon_{\mu\nu\alpha\beta}\mathcal{B}^{\alpha\beta}$ with $\epsilon_{0123} = 1$. The parameter m_B is defined by $m_B^2 = 2\kappa_4^2\mathcal{T}_p$ where $\mathcal{T}_p$ is the effective brane tension. Note that we omitted the Chern-Simons like term assuming the trivial R-R background. If we assume that the six extra-dimensions are compactified with a common radius R_c, one finds $m_B = \pi^{1/4}(g_s^2/4\pi)^{(p-3)/16}(R_c M_P)^{(15-p)/8}M_P$ where $g_s = e^\Phi$ and $M_P = 2.4\times 10^{18}$ GeV is the four-dimensional Planck mass. The qualitative features of our results do not depend on specific values of p. Thus, we take $p = 3$ for simplicity. The Λ term comes from the scalar curvature or the condensate $\langle H^2\rangle$ of extra dimensions integrated over the whole extra dimensions.

The action, and thus also field equations, can be written in a more familiar form in the Einstein metric, which is defined by

$$g_{\mu\nu} = e^{-2\Phi}\tilde{g}_{\mu\nu}. \tag{3}$$

We will work in this metric from now on. In terms of Einstein metric, the action becomes

$$\begin{aligned} S_{\mathrm{E}} = \frac{1}{2\kappa_4^2}\int d^4x\sqrt{-g}\Big[& R - 2\Lambda e^{2\Phi} - 2(\nabla\Phi)^2 \\ & - \frac{1}{12}e^{-4\Phi}H^2 \\ & - m_B^2 e^{3\Phi}\sqrt{1 + \frac{1}{2}e^{-4\Phi}\mathcal{B}^2 - \frac{1}{16}e^{-8\Phi}(\mathcal{B}^*\mathcal{B})^2}\Big]. \end{aligned} \tag{4}$$

The field equations derived from the action (4) are

$$\nabla^\lambda H_{\lambda\mu\nu} - 4H_{\lambda\mu\nu}\nabla^\lambda\Phi - m_B^2 e^{3\Phi}\frac{\mathcal{B}_{\mu\nu} - \frac{1}{4}e^{-4\Phi}\mathcal{B}^*_{\mu\nu}(\mathcal{B}\mathcal{B}^*)}{\sqrt{1 + \frac{1}{2}e^{-4\Phi}\mathcal{B}^2 - \frac{1}{16}e^{-8\Phi}(\mathcal{B}\mathcal{B}^*)^2}} = 0, \tag{5}$$

$$-\nabla^2\Phi + \frac{\partial V(\Phi)}{\partial\Phi} = 0, \tag{6}$$

$$G_{\mu\nu} = \kappa_4^2 T_{\mu\nu}, \tag{7}$$

where the dilaton potential is

$$V(\Phi) = \frac{1}{4}\Big[2\Lambda e^{2\Phi} + \frac{1}{12}e^{-4\Phi}H^2 + m_B^2 e^{3\Phi}\sqrt{1 + \frac{1}{2}e^{-4\Phi}\mathcal{B}^2 - \frac{1}{16}e^{-8\Phi}(\mathcal{B}^*\mathcal{B})^2}\Big], \tag{8}$$

and the energy-momentum tensor is given by

$$\begin{aligned} \kappa_4^2 T_{\mu\nu} = & -g_{\mu\nu}\Lambda e^{2\Phi} + 2\nabla_\mu\Phi\nabla_\nu\Phi - g_{\mu\nu}(\nabla\Phi)^2 \\ & + \frac{1}{12}e^{-4\Phi}\Big(3H_{\mu\lambda\rho}H_\nu^{\lambda\rho} - \frac{1}{2}g_{\mu\nu}H^2\Big) \\ & + \frac{1}{2}m_B^2 e^{3\Phi}\frac{-g_{\mu\nu} - \frac{1}{2}g_{\mu\nu}e^{-4\Phi}\mathcal{B}^2 + e^{-8\Phi}\mathcal{B}_{\mu\lambda}\mathcal{B}^\lambda_\nu}{\sqrt{1 + \frac{1}{2}e^{-4\Phi}\mathcal{B}^2 - \frac{1}{16}e^{-8\Phi}(\mathcal{B}\mathcal{B}^*)^2}}. \end{aligned} \tag{9}$$

In the subsequent sections, we examine the equations of motion (5)–(7) and find homogeneous solutions. Cosmological implication of the D-brane universe is of our main interest, including stabilization of the dilaton and condensation of the antisymmetric tensor field.

III. HOMOGENEOUS SOLUTIONS IN FLAT SPACETIME

For the sake of the simplicity, let us first look into the dynamics of our system in flat spacetime. With $g_{\mu\nu} = \eta_{\mu\nu}$, the Einstein Eqs. (7) become simple and gauge invariance of the field variables $\mathbf{E}^i \equiv \mathcal{B}_{i0}$ and $\mathbf{B}^i \equiv \epsilon_{0ijk}\mathcal{B}_{jk}/2$ which directly appear in the expression of energy-momentum (9) allows a homogeneous configuration

$$\Phi = \Phi(t), \qquad \mathbf{E} = \mathbf{E}(t), \qquad \mathbf{B} = \mathbf{B}(t). \tag{10}$$

For homogeneous configuration (10), $0i$-components (*electric* components) of the equation of the antisymmetric tensor field (5) reduce to an algebraic equation

$$m_B^2 e^{3\Phi} \frac{\mathbf{E} + e^{-4\Phi}\mathbf{B}(\mathbf{E}\cdot\mathbf{B})}{\sqrt{1 - e^{-4\Phi}(\mathbf{E}^2 - \mathbf{B}^2) - e^{-8\Phi}(\mathbf{E}\cdot\mathbf{B})^2}} = 0. \tag{11}$$

ij-components (*magnetic* components) give

$$\ddot{\mathbf{B}} - 4\dot{\mathbf{B}}\,\dot{\Phi} = -e^{3\Phi}\frac{\mathbf{B} - e^{-4\Phi}\mathbf{E}(\mathbf{E}\cdot\mathbf{B})}{\sqrt{1 - e^{-4\Phi}(\mathbf{E}^2 - \mathbf{B}^2) - e^{-8\Phi}(\mathbf{E}\cdot\mathbf{B})^2}}, \tag{12}$$

where the overdot denotes differentiation with respect to the rescaled dimensionless time

$$\tilde{t} = m_B t. \tag{13}$$

These equations (11) and (12) tell us that the *magnetic* components $\mathbf{B}$ are *dynamical* but the *electric* components are determined by the *constraint* Eq. (11). Dynamics of the dilaton is governed by (6) which becomes

$$\ddot{\Phi} + \frac{1}{4}e^{-4\Phi}\dot{\mathbf{B}}^2 = -\lambda e^{2\Phi} - \frac{1}{4}e^{3\Phi}\frac{3 - e^{-4\Phi}(\mathbf{E}^2 - \mathbf{B}^2) + e^{-8\Phi}(\mathbf{E}\cdot\mathbf{B})^2}{\sqrt{1 - e^{-4\Phi}(\mathbf{E}^2 - \mathbf{B}^2) - e^{-8\Phi}(\mathbf{E}\cdot\mathbf{B})^2}}, \tag{14}$$

where the dimensionless parameter λ is defined by

$$\lambda = \frac{\Lambda}{m_B^2}. \tag{15}$$

The constraint Eq. (11) forces its numerator to vanish except a trivial solution $\Phi = -\infty$, however vanishing numerator allows only a *vanishing* electric field solution $\mathbf{E} = 0$. Then the equations for the magnetic components (12) and the dilaton (14) reduce to

$$\ddot{\mathbf{B}} - 4\dot{\mathbf{B}}\,\dot{\Phi} = -e^{3\Phi}\frac{\mathbf{B}}{\sqrt{1 + e^{-4\Phi}\mathbf{B}^2}}, \tag{16}$$

$$\ddot{\Phi} + \frac{1}{2}e^{-4\Phi}\dot{\mathbf{B}}^2 = -\lambda e^{2\Phi} - \frac{1}{4}e^{3\Phi}\frac{3 + e^{-4\Phi}\mathbf{B}^2}{\sqrt{1 + e^{-4\Phi}\mathbf{B}^2}}. \tag{17}$$

In the following, we will find solutions of (16) and (17) to see how the dilaton and the magnetic component behave.

A. Dilaton potential with B = 0

When the magnetic components of the antisymmetric tensor field $\mathbf{B}$ vanish, we have only the dilaton equation from (17)

$$\ddot{\Phi} = -\frac{dU}{d\Phi}, \tag{18}$$

where the dilaton potential $U(\Phi)$ is given by the sum of the curvature term and the brane tension term

$$U(\Phi) = \frac{\lambda}{2}e^{2\Phi} + \frac{1}{4}e^{3\Phi} \tag{19}$$

as shown in Fig. 1.

For $\lambda \geq 0$, the dilaton potential $U(\Phi)$ monotonically increases, so the dilaton finally rolls to the minimum $\Phi(t) \to -\infty$ as $t \to \infty$. For $\lambda < 0$, the dilaton potential $U(\Phi)$ has the minimum value $U_{\rm min} = 8\lambda^3/27$ at $\Phi_{\rm min} = \ln(-4\lambda/3)$. If $E = U_{\rm min}$, the dilaton is stuck at $\Phi = \Phi_{\rm min}$. When $U_{\rm min} < E < 0$, the dilaton oscillates around the minimum. Therefore, the dilaton is stabilized with mass $m_\Phi = \frac{4}{3}\lambda^{3/2}m_B$, and due to exponential potential terms, its mass can be much different from the vacuum expectation

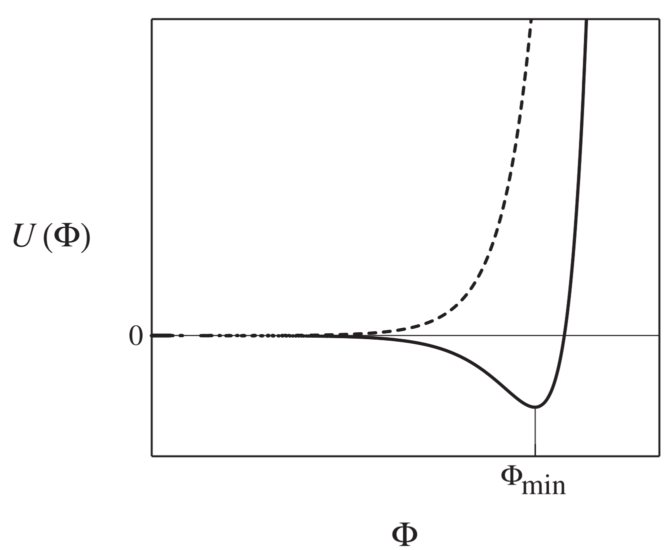

FIG. 1. The graph of $U(\Phi)$. For $\lambda \geq 0$, $U(\Phi)$ monotonically increases (dashed line). For $\lambda < 0$, $U(\Phi)$ has a minimum at $\Phi = \ln(-4\lambda/3)$ (solid line).

value of the dilaton $m_B\Phi_{\rm min}$. As λ increases, the dilaton mass rapidly grows. When $E \geq 0$, the dilaton again rolls to negative infinity. It means that the dilaton is easily destabilized by its fluctuations, if the dilaton mass becomes too small. Though the negative vacuum energy $U_{\rm min} < 0$ does not affect the dynamics of the dilaton and the antisymmetric tensor field in flat spacetime, its coupling to gravity leads to singular cosmological evolution as we shall see in Sec. IV. Therefore, for the proper description of cosmology, some additional term is needed to adjust the value of $U_{\rm min}$ to vanish as we shall discuss in Sec. V.

The runaway or stabilized behavior of the dilaton remains essentially unchanged even with $B \neq 0$ although the dependence on the initial conditions alters as we will see in the next subsection.

B. Magnetic solution

Without loss of generality, let us assume that the direction of the magnetic component of antisymmetric tensor field is fixed as $\mathbf{B}(t) = B(t)\mathbf{k}$. Although the equations of motion (16) and (17) are still complicated even under this assumption, the presence of the magnetic components do not alter the story of the dilaton much.

First, let us consider a simple case of zero brane tension ($m_B = 0$), which is analytically tractable. Then in (16) and (17), only the λ-dependent term survives on the right-hand side. (For this case, we will use the original Λ and time t since $m_B = 0$.) The solutions to these equations are

$$\Phi(T) = T - \frac{1}{2}\ln\left[\left(\frac{1}{4}e^{2T} + \Lambda\right)^2 + b_1^2c_1^2\right] + \ln c_1, \quad (20)$$

and

$$\pm B(T) = 16\frac{c_1}{b_1}\frac{\Lambda e^{2T} + 4(\Lambda^2 + b_1^2c_1^2)}{e^{4T} + 8\Lambda e^{2T} + 16(\Lambda^2 + b_1^2c_1^2)} + 4\frac{\Lambda}{b_1^2}\tan^{-1}\left(\frac{e^{2T} + 4\Lambda}{4b_1c_1}\right) + b_2, \quad (21)$$

where b_1, b_2, and c_1 are integration constants, and the time has been rescaled to $T = c_1(t + c_2)$ by absorbing another constant c_2. The solutions are still remaining valid under $T \rightarrow -T$, i.e., under the change of the sign of c_1. Therefore, we can fix the time T to flow to the positive direction and $c_1 > 0$. The solutions approach their asymptotic configurations at large T,

$$\Phi(T) = -T + \ln(4c_1), \quad (22)$$

$$\pm B(T) = {\rm sign}(b_1)\frac{2\pi\Lambda}{b_1^2} + b_2 \equiv B_{\rm st}. \quad (23)$$

We observe that the dilaton rolls linearly in time to negative infinity, and the antisymmetric tensor field condensates to a constant $\pm B_{\rm st}$ which depends on Λ. There are two topologically distinct condensation processes depending on the signature of b_1 which is to be set by the initial conditions; the change in Λ contributes to $B_{\rm st}$ oppositely.

With the brane tension term turned on, we numerically study due to the complexity of field equations. The solutions are classified into two classes as in the pure dilaton case in Sec. III A, depending on the signature of λ.

For $\lambda \geq 0$, the dilaton approaches negative infinity irrespective of initial conditions, Φ_0 and B_0. The condensed value of the magnetic component deviates from the initial value in the beginning, and then stabilizes to a constant as shown in Fig. 2. We observed from the numerical results that there exist two branches of the magnetic condensation depending on the initial conditions as discussed above, but we show only one branch in the figure. Like in the $m_B = 0$ case, once the magnetic component is condensed, the condensation survives as a constant value. It is different from the case without the dilaton, in which the magnetic component permanently oscillates in flat spacetime [7].

When λ is negative, there are two classes of solutions, depending on the initial value of the dilaton Φ_0 (or equivalently the energy density) for a fixed initial magnetic condensation B_0. For large Φ_0 or B_0, the configurations are almost the same as those for nonnegative λ (see Fig. 2). For small enough Φ_0 and B_0, the dilaton shows the oscil-

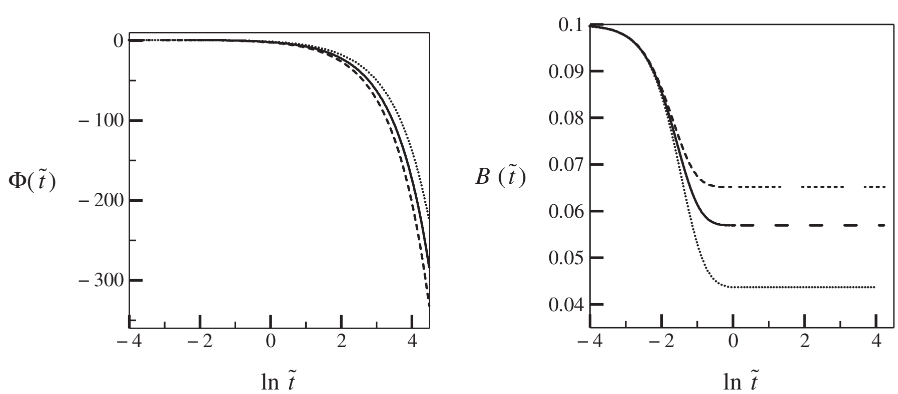

FIG. 2. Φ and B in the Einstein frame for $\lambda = 0$ (solid lines), $\lambda = 1/2$ (dashed lines), and $\lambda = -1/2$ (dotted lines). The initial conditions are $\Phi_0 = 1$ and $B_0 = 0.1$. The configurations are very similar regardless of λ.

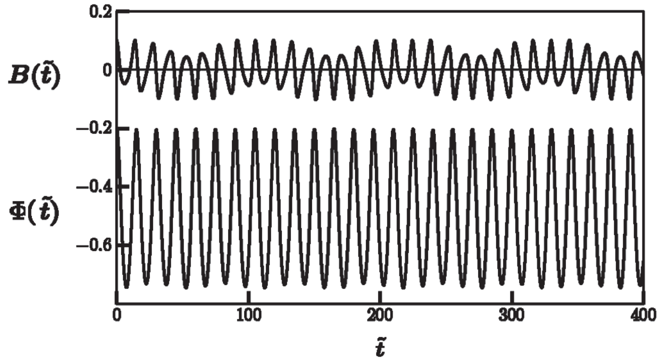

FIG. 3. Φ and B in the Einstein frame for $\lambda = -1/2$, $B_0 = 0.1$, and $\Phi_0 = -0.2$. The dilaton stabilization is observed as well as the periodically waving oscillation of B.

lating behavior as expected from the dilaton potential in Fig. 1. According to the dilaton oscillation, the magnetic component is also oscillating around a condensed value (see Fig. 3). If we involve the radiation of perturbative modes of the dilaton and the antisymmetric tensor field, which is missing in this classical configuration, the classical solution will damp to a stabilized value of the dilaton and the final condensed value of the magnetic component.

IV. COSMOLOGICAL HOMOGENEOUS SOLUTIONS

In this section we study cosmological solutions of the *D*-brane universe in the presence of both the dilaton and the antisymmetric tensor field. As we have done in the previous paper [7], we assume spatially homogeneous configurations for the antisymmetric tensor field and the dilaton, and look for the time evolution of these fields and the expansion of the universe.

The nonvanishing homogeneous antisymmetric tensor field, in general, implies the anisotropic universe. To consider the simplest form of anisotropic cosmology, we take only a single magnetic component of $\mathcal{B}_{\mu\nu}$ to be nonzero, namely $\mathcal{B}_{12}(t) \equiv B(t)$ and $\mathcal{B}_{0i}(t) = \mathcal{B}_{23}(t) = \mathcal{B}_{31}(t) = 0$. Then the metric consistent with this choice of field configuration is of Bianchi type-I

$$ds^2 = -dt^2 + a_1(t)^2(dx^1)^2 + a_2(t)^2(dx^2)^2 + a_3(t)^2(dx^3)^2. \tag{24}$$

Then, in the Einstein frame, the field equation for B (5) reads

$$\frac{d^2B}{dt^2} + \left(-\frac{1}{a_1}\frac{da_1}{dt} - \frac{1}{a_2}\frac{da_2}{dt} + \frac{1}{a_3}\frac{da_3}{dt} - 4\frac{d\Phi}{dt}\right)\frac{dB}{dt} + \frac{m_B^2 e^{3\Phi}B}{\sqrt{1 + B^2/e^{4\Phi}a_1^2a_2^2}} = 0, \tag{25}$$

the dilaton-field Eq. (6) is

$$\frac{d^2\Phi}{dt^2} + \left(\frac{1}{a_1}\frac{da_1}{dt} + \frac{1}{a_2}\frac{da_2}{dt} + \frac{1}{a_3}\frac{da_3}{dt}\right)\frac{d\Phi}{dt} = -2\varrho_B - \frac{1}{2}\varrho_b - \tilde{\varrho}_b - \varrho_\Lambda, \tag{26}$$

and Einstein Eqs. (7) are

$$\frac{1}{a_1}\frac{da_1}{dt}\frac{1}{a_2}\frac{da_2}{dt} + \frac{1}{a_2}\frac{da_2}{dt}\frac{1}{a_3}\frac{da_3}{dt} + \frac{1}{a_3}\frac{da_3}{dt}\frac{1}{a_1}\frac{da_1}{dt} = \varrho_\Phi + \varrho_B + \varrho_b + \varrho_\Lambda, \tag{27}$$

$$\frac{1}{a_2}\frac{d^2a_2}{dt^2} + \frac{1}{a_3}\frac{d^2a_3}{dt^2} + \frac{1}{a_2}\frac{da_2}{dt}\frac{1}{a_3}\frac{da_3}{dt} = -\varrho_\Phi + \varrho_B + \tilde{\varrho}_b + \varrho_\Lambda, \tag{28}$$

$$\frac{1}{a_3}\frac{d^2a_3}{dt^2} + \frac{1}{a_1}\frac{d^2a_1}{dt^2} + \frac{1}{a_3}\frac{da_3}{dt}\frac{1}{a_1}\frac{da_1}{dt} = -\varrho_\Phi + \varrho_B + \tilde{\varrho}_b + \varrho_\Lambda, \tag{29}$$

$$\frac{1}{a_1}\frac{d^2a_1}{dt^2} + \frac{1}{a_2}\frac{d^2a_2}{dt^2} + \frac{1}{a_1}\frac{da_1}{dt}\frac{1}{a_2}\frac{da_2}{dt} = -\varrho_\Phi - \varrho_B + \varrho_b + \varrho_\Lambda, \tag{30}$$

where

$$\varrho_\Phi = \left(\frac{d\Phi}{dt}\right)^2, \qquad \varrho_B = \frac{e^{-4\Phi}}{4a_1^2a_2^2}\left(\frac{dB}{dt}\right)^2, \qquad \varrho_\Lambda = \Lambda e^{2\Phi}, \tag{31}$$

$$\varrho_b = \frac{1}{2}m_B^2 e^{3\Phi}\left(1 + \frac{e^{-4\Phi}B^2}{a_1^2a_2^2}\right)^{1/2}, \quad \tilde{\varrho}_b = \frac{1}{2}m_B^2 e^{3\Phi}\left(1 + \frac{e^{-4\Phi}B^2}{a_1^2a_2^2}\right)^{-1/2}. \tag{32}$$

It is more convenient to employ the dimensionless time variable $\tilde{t}$ of (13) and to introduce the variables α_i and b defined by

$$\alpha_i = \ln a_i, \qquad b = \frac{e^{-2\Phi}B}{a_1a_2}. \tag{33}$$

Then the above Eqs. (25)–(30) are rewritten as

$$\ddot{b} + (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3)\dot{b} + \left[2\ddot{\Phi} + 2(-\dot{\alpha}_1 - \dot{\alpha}_2 + \dot{\alpha}_3 - 2\dot{\Phi})\dot{\Phi} + \ddot{\alpha}_1 + \ddot{\alpha}_2 + (\dot{\alpha}_1 + \dot{\alpha}_2)\dot{\alpha}_3 + \frac{e^{3\Phi}}{\sqrt{1+b^2}}\right]b = 0, \tag{34}$$

$$\ddot{\Phi} + (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3)\dot{\Phi} = -2\rho_B - \rho_\Lambda - \frac{1}{2}\rho_b - \tilde{\rho}_b, \tag{35}$$

$$\dot{\alpha}_1\dot{\alpha}_2 + \dot{\alpha}_2\dot{\alpha}_3 + \dot{\alpha}_3\dot{\alpha}_1 = \rho_\Phi + \rho_B + \rho_\Lambda + \rho_b, \tag{36}$$

$$\ddot{\alpha}_1 + \dot{\alpha}_1(\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3) = \rho_\Lambda + \rho_b, \quad (37)$$

$$\ddot{\alpha}_2 + \dot{\alpha}_2(\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3) = \rho_\Lambda + \rho_b, \quad (38)$$

$$\ddot{\alpha}_3 + \dot{\alpha}_3(\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3) = 2\rho_B + \rho_\Lambda + \tilde{\rho}_b, \quad (39)$$

where, by dividing m_B^2 from each ϱ in Eqs. (31) and (32), we have

$$\rho_\Phi = \dot{\Phi}^2, \qquad \rho_B = \frac{1}{4}[\dot{b} + (\dot{\alpha}_1 + \dot{\alpha}_2 + 2\dot{\Phi})b]^2,$$
$$\rho_\Lambda = \lambda e^{2\Phi}, \qquad \rho_b = \frac{1}{2}e^{3\Phi}(1+b^2)^{1/2}, \quad (40)$$
$$\tilde{\rho}_b = \frac{1}{2}e^{3\Phi}(1+b^2)^{-1/2},$$

where λ is defined in (15). In the subsequent subsections, we look for the solutions to the above field equations for various cases beginning with some simple solutions.

A. Attractor solutions

The dilaton in the system of Eqs. (34)–(39) has the exponential potential up to the correction due to the antisymmetric tensor field. It is well-known that the scalar field with the exponential potential possesses the scaling solution in which the energy density of the scalar field mimics the background fluid energy density [13,14]. This scaling solution is also an attractor, so that the late time behavior of the solutions are universal irrespective of initial conditions. This is an attractive property of the exponential potential. For the potential of the form $V(\Phi) = V_0 e^{\beta\Phi}$, there is an attractor solution

$$\Phi = -\frac{2}{\beta}\ln\left[\frac{\beta V_0^{1/2} t}{\sqrt{2(12-\beta^2)}}\right], \qquad \alpha = \left(\frac{2}{\beta}\right)^2 \ln t, \quad (41)$$

for $0 \le \beta < \sqrt{12}$. The scale factor obeys the power-law time-dependence, implying that the rolling of Φ constitutes the matter having an equation of state $p = w\rho$ where $w = \beta^2/6 - 1$ varies from -1 to $+1$ for the aforementioned range of β.

We found this type of particular solutions of the Eqs. (34)–(39), which can be found when we have a single exponential term in the potential, that is, for the case of $\Lambda = 0$ and for the case of $m_B = 0$. For both cases we start from an ansatz of the form

$$\Phi(\tilde{t}) = \gamma_\Phi \ln\tilde{t} + \Phi_0, \qquad \alpha_1 = \gamma_1 \ln\tilde{t},$$
$$\alpha_3 = \gamma_3 \ln\tilde{t}, \qquad b = \text{constant}, \quad (42)$$

where we suppressed constant terms in α_1 and α_3 which correspond to simple rescaling of coordinates. For the case of $\Lambda = 0$, we obtain two distinguished solutions

$$\Phi(\tilde{t}) = -\frac{2}{3}\ln\tilde{t} + \ln\frac{2}{3}, \qquad \alpha_1(\tilde{t}) = \alpha_3(\tilde{t}) = \frac{4}{9}\ln\tilde{t}, \qquad b = 0, \quad (43)$$

and

$$\Phi(\tilde{t}) = -\frac{2}{3}\ln\tilde{t} + \frac{2}{3}\ln\left(\frac{8\cdot 5^{1/4}}{21}\right), \qquad \alpha_1(\tilde{t}) = \frac{10}{21}\ln\tilde{t},$$
$$\alpha_3(\tilde{t}) = \frac{3}{7}\ln\tilde{t}, \qquad b = \pm\frac{1}{2}. \quad (44)$$

The first solution is nothing but the solution (41) with $\beta = 3$. The second solution has the nonvanishing antisymmetric tensor field. For the case of $m_B = 0$, we introduce a new rescaled dimensionless time variable $\bar{t} = \Lambda^{1/2} t$ instead of $\tilde{t}$ of (13) and then get the continuous set of solutions from Eqs. (25)–(30)

$$\Phi(\bar{t}) = -\ln\bar{t} + \frac{1}{2}\ln 2, \qquad \alpha_1(\bar{t}) = \alpha_3(\bar{t}) = \ln\bar{t},$$
$$b = \text{arbitrary constant}. \quad (45)$$

$\Phi(\bar{t})$ and $\alpha(\bar{t})$ are same as those in Eq. (41) with $\beta = 2$, while we have the nonvanishing B field condensate.

These solutions are the solutions to the Eqs. (34)–(39) for the specific initial conditions. However, the importance of these solutions, as noted in the first paragraph, arises from the fact that they are attractors, which means that after enough time the solutions with different initial conditions approach these solutions. We will confirm this through numerical analysis in the next subsection.

The solution (43) applies for the brane tension dominated case where the dilaton potential is approximated by $m_B^2 U(\Phi) = \frac{1}{2}m_B^2 e^{3\Phi}$ from Eq. (19). The evolution of the dilaton under this potential produces matter with the equation of state $p = \frac{1}{2}\rho$.

Once the antisymmetric tensor field is turned on, the anisotropy appears as in the solution (44). The measure of anisotropy is

$$\frac{\dot{\alpha}_3}{\dot{\alpha}_1} = \frac{9}{10}. \quad (46)$$

This result is contrasted with that in Ref. [7] where the dilaton is assumed to be stabilized. The rolling of the dilaton makes the difference. It affects the dynamics of B field in such a way that b remains constant at $b = \pm 1/2$ instead of oscillating about the potential minimum $b = 0$ and the anisotropy is maintained.

Let us turn to the third solution (45). It is relevant when the dilaton potential arising from the curvature of extra dimensions, $V(\Phi) = \Lambda e^{2\Phi}$, dominates over other contributions. In our scheme this happens as the dilaton rolls down the potential. When b vanishes, the transition point at which $\Lambda e^{2\Phi}$ starts to dominate over $\frac{1}{2}m_B^2 e^{3\Phi}$ is at $\Phi_t = \ln(2\lambda)$. Thus, this solution describes the late time behavior of all the solutions with various initial conditions when Λ is positive. It is very intriguing since we achieve the isotropic universe in the end. The rolling of dilaton makes the brane tension term which causes the anisotropy when we have nonvanishing B field less important than the extra dimension curvature term which recovers the isotropy. The rolling of the dilaton under the potential $\Lambda e^{2\Phi}$ now forms the matter having the equation of state $p = -\frac{1}{3}\rho$, thus giving marginal inflation.

B. Numerical analysis

1. Initial conditions

We have the second order differential equations for four variables $\Phi(t)$, $B(t)$, $\alpha_1(t) = \alpha_2(t)$, $\alpha_3(t)$. Thus we need eight initial values Φ_0, $\dot{\Phi}_0$, B_0, $\dot{B}_0$, α_{10}, $\dot{\alpha}_{10}$, α_{30}, $\dot{\alpha}_{30}$ to specify the solution. Among these, α_{i0} can always be set to zero by coordinate rescaling. $\dot{\alpha}_{i0}$ must obey the constraint Eq. (36), but this does not fix the ratio $\dot{\alpha}_{10}/\dot{\alpha}_{30}$. We choose the isotropic universe as a natural initial condition which leads to $\dot{\alpha}_{10} = \dot{\alpha}_{30}$.

Since the dilaton potential is composed of exponential terms, the shift of the dilaton field by a constant can be traded for the redefinition of mass scale. We use this property to take the initial value of the dilaton to be zero without loss of generality. In our numerical analysis, we take the dimensionless time variable as $\tilde{t} \equiv \tilde{m}t$ where $\tilde{m} = m_B e^{3/2\Phi_0}$ and use the variable $\tilde{\Phi} \equiv \Phi - \Phi_0$ with its initial value $\tilde{\Phi}_0 = 0$. This means that the proper time scale for the cosmological evolution is not m_B^{-1}, but $\tilde{m}^{-1}$. For convenience's sake, we take the initial time as $\tilde{t}_0 = 1$. The other mass scale Λ is also affected by this shift and we can treat it by replacing the parameter λ with $\tilde{\lambda} \equiv \lambda e^{-\Phi_0}$.

Now we need three initial values $\dot{\Phi}_0$, B_0, and $\dot{B}_0$, to fix the functional form of the solution. The initial values b_0 and $\dot{b}_0$ are related to B_0 and $\dot{B}_0$ by $b_0 = B_0$ and $\dot{b}_0 = \dot{B}_0 - 2(\dot{\alpha}_{10} + \dot{\Phi}_0)B_0$.

2. Numerical solutions for $B = 0$

If we assume $\mathcal{B}_{\mu\nu} = 0$, we can study the role of the dilaton interacting with gravity more clearly as was also true in the subsection III A. We keep the brane term with the tension m_B. The spacetime is now isotropic. The parameter Λ in the string-frame action, which can be interpreted as the curvature of the compact internal manifold, plays a very important role, and the solutions are topologically different depending on the signature of Λ.

When $\Lambda = 0$, we have an attractor solution (43). Since we consider the dilaton only, the only relevant initial condition is $\dot{\Phi}_0$. The initial condition for the attractor solution is $\dot{\Phi}_0 = -\sqrt{3/2}$ in the setup described in the previous subsection. For a few other values of $\dot{\Phi}_0$, we plot the numerical solutions in Fig. 4. Regardless of the initial value $\dot{\Phi}_0$, all the solutions approach universally the attractor solution (up to rescaling for the scale factor) after some time.

When $\Lambda > 0$, the evolution is divided into two stages. When Φ is large so that $\rho_b = \frac{1}{2}e^{3\Phi}$ is much larger than $\rho_\Lambda = \lambda e^{2\Phi}$, the solution approaches the attractor (43). As Φ rolls down to around $\Phi_t = \ln(2\lambda)$, ρ_Λ becomes larger than ρ_b. After that point the solution approaches the attractor (45) with $b = 0$. In Fig. 5, numerical solutions show this two stage evolution explicitly.

Since Λ measures the curvature of the extra dimensions, its value can be negative. When $\Lambda < 0$, the dilaton potential has the global minimum at $\Phi_{\min} = \ln(-4\lambda/3)$ with negative cosmological constant $U_{\min}$ as discussed in Subsec. III A. The disastrous consequence of this negative energy minimum is that the scale factor collapses in the end. This cannot be avoided because the dilaton rolls down toward the minimum and stays where the potential is negative while the kinetic energy is diluted by the expansion, thus the total energy becomes negative and it derives the universe to collapse. It is different from the case of flat spacetime in Subsec. III A, where the dilaton experiences either the permanent oscillation around $\Phi_{\min}$ for negative initial energy density or the attractor solution for nonnegative initial energy density.

3. Numerical solutions for $B \neq 0$

Now, we turn on the antisymmetric tensor field along the x^3 direction, $\mathcal{B}_{12} = B \neq 0$. The spacetime becomes aniso-

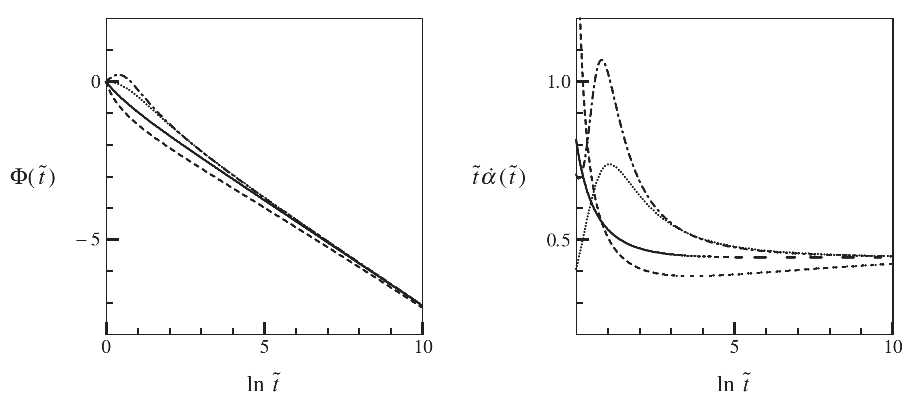

FIG. 4. Numerical solutions for $B = 0$ and $\Lambda = 0$ with initial value $\dot{\Phi}_0 = -\sqrt{3/2}$ (solid curve, the attractor), 0 (dotted curve), -3 (dashed curve) and 1 (dash-dotted curve), respectively. For the evolution of the scale factor, we plot $\tilde{t}\,\dot{\alpha}(\tilde{t})$ for it becomes constant γ when the scale factor behaves as $a \propto t^\gamma$.

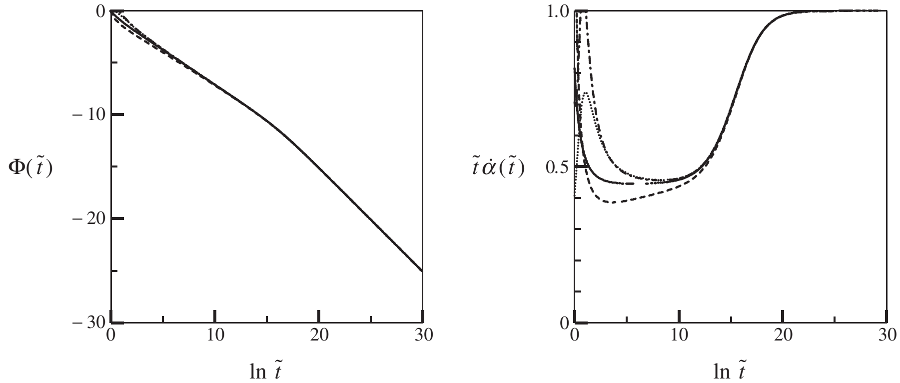

FIG. 5. Numerical solutions for $B = 0$ and $\Lambda > 0$. When we set the parameter $\lambda = 10^{-4}$, the curves stand for the solutions with the initial value $\dot{\Phi}_0 = -\sqrt{3/2}$ (solid curve) and 0 (dotted curve), -3 (dashed curve) and 1 (dash-dotted curve), respectively.

tropic, $a_1(t) = a_2(t) \neq a_3(t)$, in general. The solutions are classified again by the signature of Λ.

For $\Lambda = 0$, we have an attractor (44) under the initial conditions of $\dot{\Phi}_0 = -7/4 \cdot 5^{1/4}$, $b_0 = \pm\frac{1}{2}$, and $\dot{b}_0 = 0$. In Fig. 6, we plotted numerical solutions for a few different initial conditions in addition to the attractor. It is confirmed again that all the solutions approach the attractor as the time goes on. The final value of b is either $+\frac{1}{2}$ or $-\frac{1}{2}$ depending on the initial conditions. In the lower-right panel in Fig. 6 we provide the evolution of each component of energy density. The kinetic energy of the dilaton ρ_Φ catches up the potential energy ρ_b and the ratio of them becomes constant. This is a characteristic feature of the scaling solution [14]. The kinetic energy of B field is kept much smaller than both of them, but the anisotropy is still maintained due to the difference between ρ_b and $\tilde{\rho}_b$.

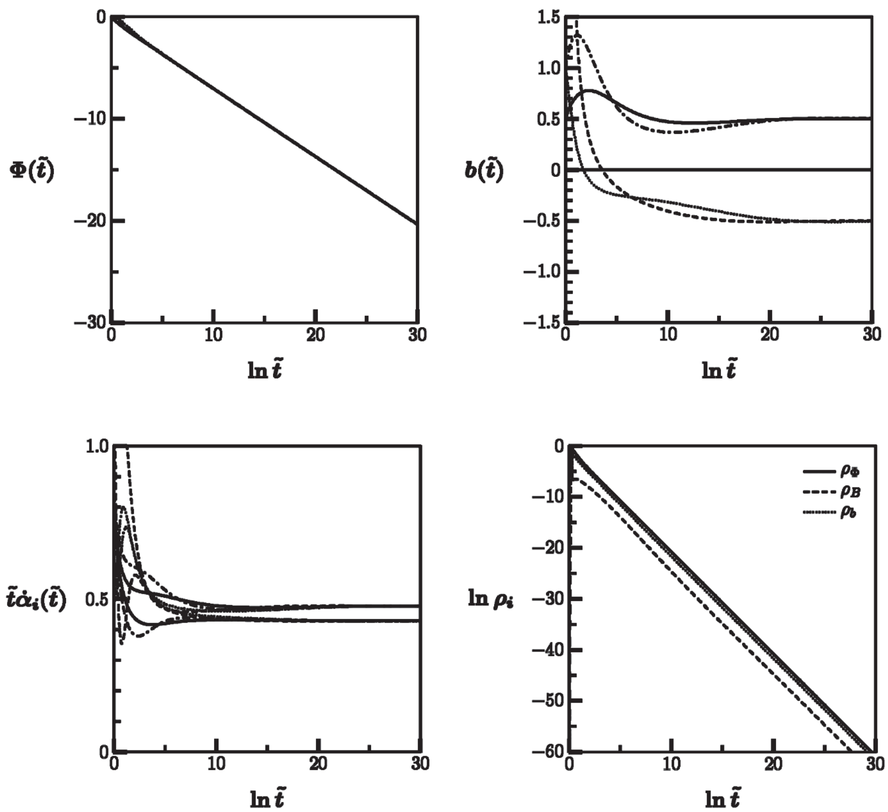

FIG. 6. Numerical solutions for $B \neq 0$ and $\Lambda = 0$. The initial conditions are as follow. Solid line: $\dot{\Phi}_0 = -7/4 \cdot 5^{1/4}$, $b_0 = 1/2$, $\dot{B}_0 = 0$; Dotted line: $\dot{\Phi}_0 = 0$, $b_0 = 1$, $\dot{B}_0 = 0$; Dashed line: $\dot{\Phi}_0 = 0$, $b_0 = 10$, $\dot{B}_0 = 0$; Dash-dotted line: $\dot{\Phi}_0 = -\sqrt{3/2}$, $b_0 = 1$, $\dot{B}_0 = 0$. The lower-right panel shows the evolution of each component of energy density for initial conditions $\dot{\Phi}_0 = -7/4 \cdot 5^{1/4}$, $b_0 = 1/2$, $\dot{B}_0 = 0$.

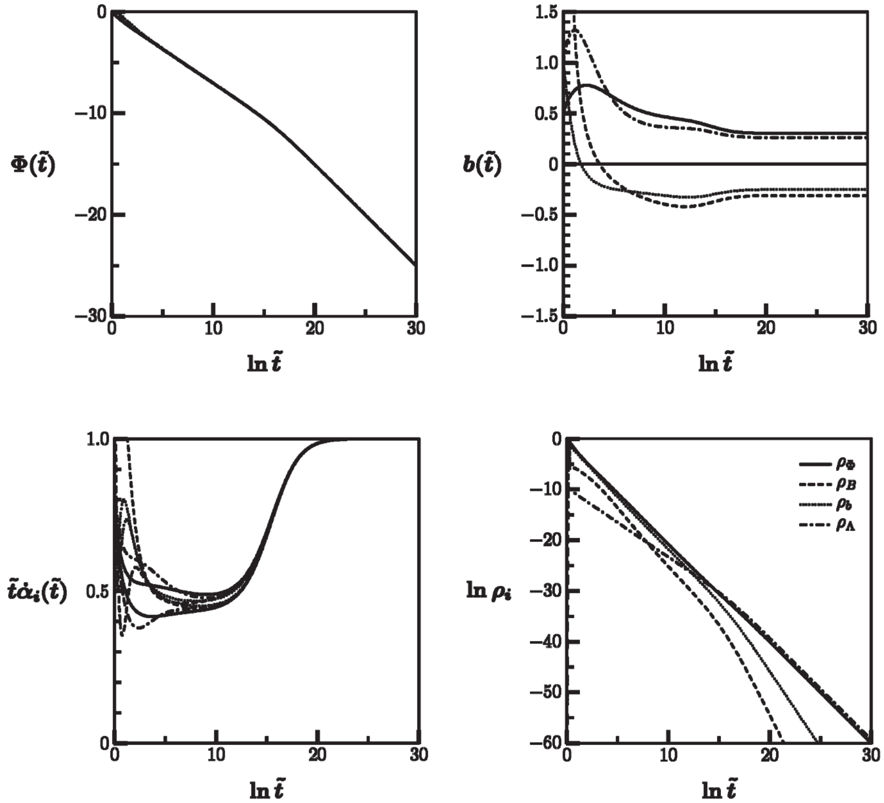

FIG. 7. Numerical solutions for $B \neq 0$ and $\Lambda > 0$. When $\lambda = 10^{-4}$, the solutions of four initial conditions are given: $\dot{\Phi}_0 = -7/4 \cdot 5^{1/4}$, $b_0 = 1/2$, $\dot{B}_0 = 0$ for the solid curves, $\dot{\Phi}_0 = 0$, $b_0 = 1$, $\dot{B}_0 = 0$ for the dotted curves, $\dot{\Phi}_0 = 0$, $b_0 = 10$, $\dot{B}_0 = 0$ for the dashed curves, and $\dot{\Phi}_0 = -\sqrt{3/2}$, $b_0 = 1$, $\dot{B}_0 = 0$ for the dash-dotted curves. The lower-right panel shows the evolution of each component of energy density for initial conditions $\dot{\Phi}_0 = -\sqrt{3/2}$, $b_0 = 1$, $\dot{B}_0 = 0$.

For $\Lambda > 0$, the evolution is again divided into two stages as in the $B = 0$ case. In the first stage where ρ_b is dominant, the solution approaches an attractor (44). In the second stage where ρ_Λ is dominant, it approaches another attractor (45). Thus the universe recovers the isotropy. The final value of b is a certain constant which is determined by initial conditions and can differ from $\pm\frac{1}{2}$. Numerical solutions for a few initial conditions are shown in Fig. 7. The kinetic energy of the dilaton ρ_Φ now catches up the potential energy ρ_b in the first stage and ρ_Λ in the second stage. The ratios, ρ_Φ/ρ_b and ρ_Φ/ρ_Λ, approach constants in each stage. The kinetic energy of B field is kept much smaller as in $\Lambda = 0$ case.

For $\Lambda < 0$, the solution becomes singular as in the $B = 0$ case. Here we skip the description of such singular solutions which are not suitable for the evolution of our Universe.

V. STABILIZED DILATON

The vacuum expectation value of the dilaton determines both the gauge and gravitational coupling constants of the low energy effective theory. Therefore, the dilaton must be stabilized at some stage of the evolution for the action (4) to have something to do with the reality. In this section, we study the cosmological evolution when the dilaton is stabilized. As for the correct mechanism of dilaton stabilization, the consensus has not been made yet. Our goal here is to illustrate an example of the dilaton stabilization and look into the effect of it on the dynamics of B field and the cosmological evolution, since the overall features of which are insensitive to the detailed mechanism of stabilization.

Our starting point is the dilaton potential (8). This potential possesses the minimum for $\Lambda < 0$, but the value of the potential at the minimum is negative and need to be set to zero by fine-tuning of the constant shift. However, the constant shift of the potential has no motivation in the context of string theory. Instead, we introduce a term $\frac{1}{4} m_{\mathrm{F}}^2 e^{-3\Phi}$ in the potential, which can arise from the effect of various form field fluxes in extra dimensions [15]. Thus, our potential for the dilaton for $B = 0$ looks like

$$V_{\mathrm{F}}(\Phi) = \frac{1}{4}(m_B^2 e^{3\Phi} + 2\Lambda e^{2\Phi} + m_{\mathrm{F}}^2 e^{-3\Phi}). \tag{47}$$

This potential has a global minimum for any value of Λ and m_{F}. To have sensible cosmology, the potential at the minimum must be zero. For $\Lambda < 0$, this can be done through a fine-tuning of the parameters in the potential

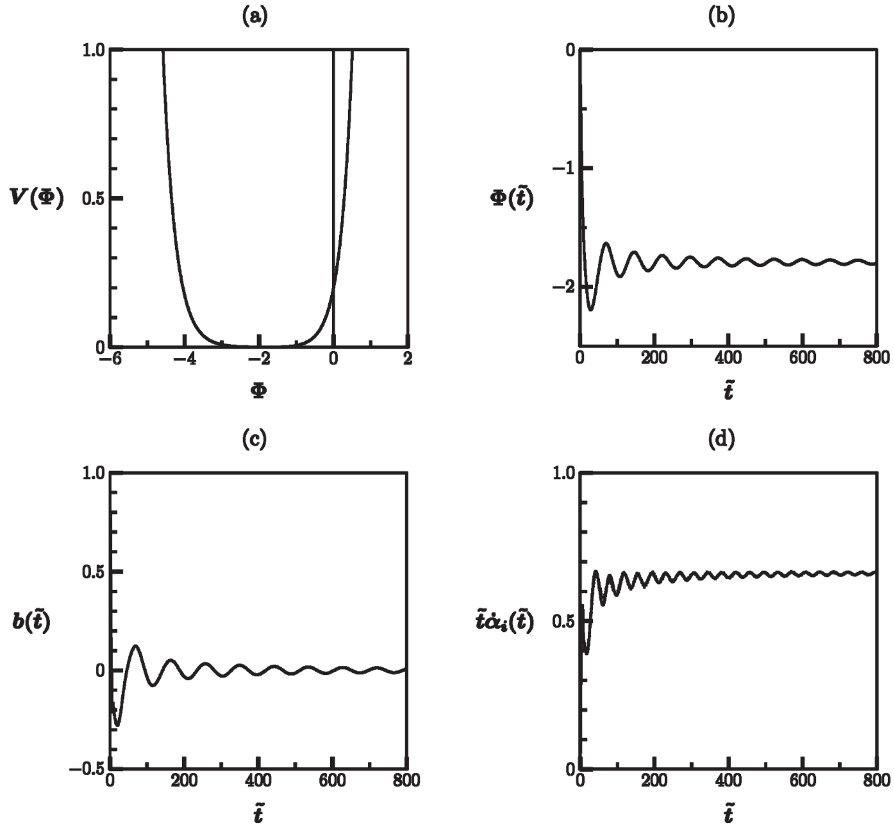

FIG. 8. (a) The potential $V(\Phi)$ for $\lambda = -0.1$. The minimum of the potential is at $\Phi_m = -\ln 6$ with $V(\Phi_m) = 0$. (b), (c) and (d) show the numerical solutions of $\Phi(\tilde{t})$, $b(\tilde{t})$ and $\dot{\alpha}_i(\tilde{t})$, respectively, for $\lambda = -0.1$ and initial conditions $\Phi_0 = 0$, $\dot{\Phi}_0 = 0$ and $b_0 = 1$. $\Phi(\tilde{t})$, $b(\tilde{t})$, and $\tilde{t}\dot{\alpha}_i(\tilde{t})$ approach Φ_m, 0, and 2/3, respectively.

$$\mu^2 = \frac{1}{5}\left(-\frac{5}{3}\lambda\right)^6, \tag{48}$$

where $\mu^2 = m_{\rm F}^2/m_B^2$. Then the minimum is located at

$$\Phi_{\rm F} = \ln\left(-\frac{5}{3}\lambda\right). \tag{49}$$

The shape of this fine-tuned potential for $\lambda = -0.1$ is shown in Fig. 8. Now the Eqs. (35)–(39) are modified accordingly

$$\ddot{\Phi} + (\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3)\dot{\Phi} = -2\rho_B - \rho_\Lambda - \frac{1}{2}\rho_b - \tilde{\rho}_b + \frac{3}{2}\rho_{\rm F}, \tag{50}$$

$$\ddot{\alpha}_1 + \dot{\alpha}_1(\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3) = \rho_\Lambda + \rho_b + \rho_{\rm F}, \tag{51}$$

$$\ddot{\alpha}_3 + \dot{\alpha}_3(\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3) = 2\rho_B + \rho_\Lambda + \tilde{\rho}_b + \rho_{\rm F}, \tag{52}$$

where $\rho_{\rm F} = \frac{1}{2}\mu^2 e^{-3\Phi}$, while the Eq. (34) for b is not changed.

With the stabilizing potential for the dilaton, the cosmological evolution is completely changed. The dilaton and the antisymmetric tensor field rapidly come to the oscillation about the potential minimum $\Phi = \Phi_{\rm F}$ and $b = 0$. Oscillating Φ and B fields behave like ordinary matter satisfying the equation of state $p = 0$ [7]. The universe becomes isotropic and matter dominated. The numerical solutions for the stabilizing dilaton potential (47) with $\lambda = -0.1$ are plotted in Fig. 8. One can see the oscillation of Φ and b, and that both $\dot{\alpha}_1$ and $\dot{\alpha}_3$ approach to $2/3t$, indicating the matter domination. Damping of Φ and b oscillations is due to the expansion of the universe.

VI. CONCLUSIONS

We investigated cosmology of a four-dimensional low energy effective theory arising from the NS-NS sector of string theory with a D-brane which contains the dynamical degrees of freedom such as the gravity, the dilaton, and the antisymmetric tensor field of second rank, coupling to the gauge field strength living on the brane. The dynamics of the system crucially depends on the curvature Λ and the brane tension m_B through which the dilaton obtains a potential of the form; $\Lambda e^{2\Phi} + \frac{1}{2}m_B^2 e^{3\Phi}$. Here, the latter becomes the effective mass of the antisymmetric tensor field (B-matter).

In terms of the homogeneous solution in flat spacetime, we first showed how the dilaton $\Phi(t)$ and the nonvanishing magnetic component of the tensor field $B(t)$, in one direction evolve in time. For positive Λ, one finds that the dilaton runs away to negative infinity, and $B(t)$ reaches a constant value at late time. When Λ is negative, one can see that the dilaton is stabilized at a finite value and then the dilaton (and also $B(t)$) shows an oscillatory behavior around that value, or runs away again depending on the choice of the initial conditions. Such a dilaton stabilization, however, produces a negative effective cosmological constant, and thus leads to a collapsing universe in string cosmology.

When $B(t)$ is turned on, the universe undergoes an anisotropic expansion described by the Bianchi type-I cosmology. We found the attractor solutions showing the overall features of general solutions and confirmed it through numerical analysis. The dilaton $\Phi(t)$ decreases and settles to a logarithmic decrease in time to negative infinity. When the brane tension term dominates, the anisotropy is sustained. If there is a positive curvature term, it dominates finally over the brane tension term as the dilaton rolls down to negative infinity. Then the expansion of the universe turns to be isotropic and linear in time. Accordingly, $B(t)$ decreases inversely proportional to time.

There have been various proposals to stabilize the dilaton. In order to study the dynamics of the *B*-matter and the stabilized dilaton system, we adopted an option of generating a dilaton mass term of the form $m_{\mathrm{F}}^2 e^{-3\Phi}$. Then the potential for the dilaton has a global minimum and the cosmological constant of our Universe can be fine-tuned to a desired value with negative Λ. While the dilaton evolved to a stabilized value, B shows an oscillatory matterlike behavior, and the universe expands as in the usual matter-dominated era recovering the isotropy. The obtained result is consistent with that of Ref. [7].

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