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Determination of Plasma Flow Velocity by Mach Probe and Triple Probe with Correction by Laser-Induced Fluorescence in Unmagnetized Plasmas

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Plasma flow velocity was measured by Mach probe (MP) and laser-induced fluorescence (LIF) methods in unmagnetized plasmas with supersonic ion beams. Since the ion gyro-radius was much larger than the probe radius, unmagnetized Mach probe theory was used to determine plasma flow in argon RF plasma with a weak magnetic field (<200 G). In order to determine flow velocities, the Mach probe is calibrated via LIF in the absence of the ion beam, where existing probe theories may be valid although they use different geometries (sphere and plane) and analyzing tools [particle-in-cell (PIC) and kinetic models]. For the comparison of the average plasma flow velocities by MP and LIF, the supersonic ion beam velocity was measured by LIF and then incorporated into a simple formula for average plasma velocity with provisions for background plasma density and beam-corrected electron temperature (T_e) measured by a triple probe. [DOI: 10.1143/JJAP.45.5945]

KEYWORDS: plasma flow velocity, Mach probe, triple probe, laser induced fluorescence, LIF, unmagnetized plasma

1. Introduction

Despite the progress of edge physics in fusion research, flow measurements near X-points including $\mathbf{E} \times \mathbf{B}$ shear velocity and supersonic flow are still under debate.^{1,2)} There is interest in determining the ion velocities in plasma processing and space propulsion systems to support analysis and improve relevant processes. Although several probe theories for unmagnetized flowing plasmas are available, none of them is prevalent, and there is room for improved determination of flow velocity from Mach probe (MP) measurements. An MP is a combination of two directional electric probes separated by an insulator that collect ions moving in opposite directions.

Laser-induced fluorescence (LIF) may provide measurements of the ion velocity distribution function (IVDF) in plasmas, yet there are restrictions on LIF applications; for example, measurement of hydrogen ions is not possible, and it is also not easy even for hydrogen atoms because the longest wavelength to the ground state of hydrogen is in the range of vacuum ultraviolet (121.6 nm).³⁾ The (2 + 1)photon-LIF, which measures hydrogen atoms using 243 and 486 nm wavelengths, is applicable only over a hydrogen atom density of 10^{12} cm⁻³.⁴) To apply the MP to space propulsion devices such as variable specific impulse magnetoplasma rocket (VASIMR),⁵⁾ or gas dynamic mirror (GDM) fusion propulsion,⁶⁾ where the application of LIF is not easy for hydrogen unmagnetized plasmas at the exhaust, calibration of the MP with the LIF in some known plasma would be helpful. For the Ar ion velocity distribution function, three LIF schemes are available.⁷⁾ Hence, it is possible to calibrate the unmagnetized MP theory with the LIF method.

While recent results from Hutchinson⁸⁾ for a spherical probe using a particle-in-cell (PIC) code and previous result from Chung⁹⁾ for a planar probe using a kinetic analysis produce similar ion velocities in unmagnetized plasmas at

low ion temperatures, further verifications of Mach probe results by additional means are warranted. Comparison of the velocity deduced by the MP with that of the LIF has been tried by Gulick¹⁰⁾ in a weakly magnetized electron cyclotron resonance (ECR) plasma. However, there has been no comparison in unmagnetized plasmas. Oksuz and Hershkowitz¹¹⁾ have reported a new method to derive ion flow velocity from electron saturation rather than ion saturation current in unmagnetized plasma, yet this would benefit from being calibrated with a known flow velocity such as one measured via LIF.

A new theory is introduced in §2 for the determination of electron temperature with an ion beam, especially a supersonic ion beam. With modifications of existing triple probe (TP) theory, electron temperature is estimated in §2. Then in §3 MP results are calibrated with LIF in the absence of the ion beam. Ion beam velocity determined by the MP and compared with the LIF is described in §4. Conclusions are given in §5.

2. Temperature Correction: Triple Probe

Figure 1 shows the UCI (University of California at Irvine) device composed of inductively coupled plasma (ICP) source and an ion beam source. For the main diagnostics, a versatile electric probe and LIF systems are installed.¹²⁾ The electric probe system is composed of a TP and an MP, which are shown in Fig. 2. Figure 3 shows the circuits of the TP and the MP. For LIF measurement, we used the quartet transition pumped from the $3d^4G_{7/2}$ metastable ion to the $4p^4D_{5/2}$ excited state with a wavelength of 668.61 nm (in vacuum), and we measured the fluorescence signals at a wavelength of 442.72 nm emitted when the $4p^4D_{5/2}$ exited state decays to the $4s^4D_{3/2}$ ground state.⁷

Argon plasma was produced in the UCI device at a pressure of 1 mTorr and with a plasma source power of 30 watts at f = 103.4 MHz applied to the RF coil. The MP probe is attached at the center port (A) and drift velocity of background plasma, which was generated by the RF coil, was measured at that position. The MP probe consists of two tantalum tips, 1 mm wide and 8 mm long, and the two tips

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Fig. 1. UCI test chamber (MP: Mach probe, TP: triple probe, LIF: laser-induced fluorescence, and PMT: photo-multiplier tube).



Fig. 2. Geometry of a versatile electric probe system: (a) side view and (b) top view. The MP is composed of probes P4 and P5; the TP is composed of probes P1, P2, and P3.

are separated with a ceramic tube.

To determine the speed of an ion beam, one needs to know the temperatures and densities of the plasma and ion beam. Although triple probe theory for Maxwellian plasmas is well established,¹³⁾ a triple probe analysis in plasmas with ion beams has not been attempted. With the inclusion of an ion beam, equations for the collection of currents at the probe tips (Fig. 3) are modified as follows:

$$-I_{1} = J_{e}S\exp(eV_{1}/k_{B}T_{e}) + J_{i}(V_{1})S + J_{b}(V_{1})S_{x}, \quad (1)$$

$$I_2 = J_e S \exp(eV_2/k_B T_e) + J_i(V_2)S + J_b(V_2)S_x, \quad (2)$$

 $I_3 = J_e S \exp(eV_3/k_B T_e) + J_i(V_3)S + J_b(V_3)S_x, \quad (3)$

where *S* is the probe collection area, S_x is the probe crosssection where the supersonic ion beam impinges, k_B is the Boltzmann constant, T_e is the electron temperature, $V_{1,2,3}$ are potentials of the three probes with respect to the plasma potential, and J_e , J_i , and J_b are electron saturation, ion, and ion beam current density, respectively. Here

$$J_{e} = n_{e}e\sqrt{k_{B}T_{e}/2\pi m_{e}},$$

$$J_{i}(V) = n_{i}e\sqrt{k_{B}T_{e}/m_{i}}g(V),$$

$$J_{b}(V) = en_{b}\sqrt{2E_{0}/m_{i}}f(V),$$
(4)

where E_0 is the beam energy, g(V) is a function of ion current, and f(V) is a function of beam current change by the probe potential with respect to the plasma potential. Conventional equations for TP analysis are given without the beam terms $J_b(V_1, V_2, V_3)$ in eqs. (1)–(3).

With the introduction of the ion beam into the plasma, abnormal behavior of the TP was observed, i.e., the apparent distortion of the electron temperature is shown in Fig. 4(a). Although the current at each probe is changed by the presence of the ion beam, the direct display system,¹³ which is given by letting $I_2 = 0$, can be used to determine electron temperature with compensation of the ion beam effects. The beam currents at the probe tips cancel each other if the beam currents have similar magnitude $[J_b(V_1) \approx J_b(V_2) \approx J_b(V_2)]$. However, the results of the TP data showed unexpected profiles as shown in Fig. 4(a) for electron temperature and Fig. 4(b) for plasma density, which is deduced by

$$n = I/(0.61 eS \sqrt{T_{\rm e}/M_{\rm i}}),$$
 (5)

where I is the current passing through resistor R with the collection area S.

If the beam currents at each probe tip are not the same, the





Fig. 3. Circuits of (a) triple probe and (b) Mach probe. V_{d2} = voltage difference between floating probe (P2) and positive probe (P1), R is the resistance for the ion saturation current deduction (I), V_{d3} = voltage difference between V_1 and V_3 , which is negatively biased. Two probes of the Mach probe (MP1 = P4, MP2 = P5) separated by an insulator collect ion saturation currents: MP1 is for the upstream current and MP2 is for the downstream. R_1 and R_2 are resistors for the current detection, and α_1 and α_2 are the conversion factors for each probe due to data acquisition circuits.

TP relation of electron temperature is given by letting $I_2 = 0$, $I = I_1 = I_3$ and dividing [eq. (2) minus eq. (1)] by [eq. (3) minus eq. (1)] to get:

$$\frac{I + [J_{\rm b}(V_1) - J_{\rm b}(V_2)S_{\rm x}]}{2I + [J_{\rm b}(V_1) - J_{\rm b}(V_3)S_{\rm x}]} = \frac{1 - \exp(eV_{\rm d2}/k_{\rm B}T_{\rm e})}{1 - \exp(eV_{\rm d3}/k_{\rm B}T_{\rm e})},$$
 (6)

where $V_{d2} = V_2 - V_1$, $I = I_1 = I_3$, $V_{d3} = V_3 - V_1$, and as assumed in original TP theory.¹³⁾ The denominator of the right term in eq. (6) can be regarded as 1 because V_{d3} has a large negative value $[\exp(eV_{d3}/k_BT_e) \approx 0]$. To determine electron temperature from eq. (6), one must know current density of each beam at the probe tips, while the electron temperature in the absence of the ion beam can be calculated from $\exp(eV_{d3}/k_BT_e) \approx 0$ as

$$k_{\rm B}T_{\rm e}/e = -V_{\rm d2}/0.693.$$
 (7)

Because there is no big difference between probe voltages V_1 and V_2 , one can expect $J_b(V_1) \approx J_b(V_2)$, while there is some difference between $J_b(V_1)$ and $J_b(V_3)$ because the voltage difference of V_1 and V_3 (= V_{d3}) is about 200 V.



Fig. 4. Uncorrected TP results: (a) electron temperature profile and (b) electron density profile.

Since not only the ion beams are cold but also ion beam– electron, and ion beam–ion collision rates are small, such as 0.25 Hz (ion beam–electron collision) and <10 µHz (counter-streaming ion–ion collision), it can be assumed that there is little change in the following parameters: the electron temperature, electron density, background ion distribution, ion beam distribution, and plasma potential. The beam current caused by the ion beam, $[J_b(V_1) - J_b(V_3)]S_x$, is easily obtained as $\Delta I = [[I_1(\text{with beam}) - I_1(\text{without} \text{beam})] - [I_3(\text{with beam}) - I_3(\text{without beam})]]$. Then the electron temperature with the ion beam is calculated as

$$T_{\rm e} = -eV_{\rm d2}/\ln[1 - I/(2I + \Delta I)]. \tag{8}$$

Figures 4(a) and 4(b) show the electron temperature and density calculated by applying conventional TP theory, while Fig. 5 is a new temperature profile corrected using eq. (8). Since this analysis is very simple, the effect of the beam on the triple probe theory deserves further development.

3. Velocity Calibration without Ion Beam: Mach Probe vs Laser-Induced Fluorescence

An MP is composed of two separated directional probes with strongly negative biased potential; one collects the ion saturation current with the plasma flow (facing upstream) and the other collects ion current moving against the plasma flow (downstream). With plasma flow, these two currents



Fig. 5. Corrected electron temperature profile.

show asymmetry, producing a measured ratio (R_m) of saturation currents which is greater than one:

$$R_{\rm m} = J_{\rm up}/J_{\rm down} \ge 1. \tag{9}$$

From theories of ion collection, one can relate R_m as follows: $R_m = \exp(kM)$, where k is a conversion factor and the M is the Mach number defined by the following: $M = v_f / \sqrt{ZT_eM_i}$, where v_f is the plasma flow velocity. The k factor is given as 1.29 and 1.34 by Chung⁹⁾ and Hutchinson,⁸⁾ respectively. Hence the flow velocity determined as a non-dimensional form (Mach number) is given as

$$M = k^{-1} \ln[J_{\rm up}/J_{\rm down}] = k^{-1} \ln[R_{\rm m}].$$
(10)

In order to obtain an accurate ratio of ion saturation current densities in the upstream and downstream directions, the MP must be rotated 180° as shown in previous work.¹⁴⁾ Rotating can reduce error from probe area differences and circuit differences of upstream and down stream probes. From the schematic drawing of the circuit for the Mach probe, Fig. 3(b), the resultant voltage from each probe is given as $V_1 = \alpha_1 I_1 R_1$ and $V_2 = \alpha_2 I_2 R_2$, respectively, where α_1 and α_2 are the calibration factors due to data acquisition circuits such as BNC cables and isolation amplifiers. Then, the ratio of upstream to downstream current densities is given as

$$R_{\rm m} = \frac{J_{\rm up}}{J_{\rm down}} = \frac{I_1/A_1}{I_2/A_2} = \frac{V_1/\alpha_1 R_1 A_1}{V_2/\alpha_2 R_2 A_2} = \frac{V_1}{V_2} \frac{\alpha_2}{\alpha_1} \frac{R_2 A_2}{R_1 A_1}, \quad (11)$$

where the area and resistance of each probe is usually same. If one knows the calibration factors of each probe, which are ideally very close to unity and are to be defined before or after the experiment, one can obtain the ratio of current densities from the direct measurement of voltages to determine of the Mach number.

Figure 6 shows the Ar IVDF in the absence of the supersonic ion beams by the LIF system. From the velocity distribution of ions measured by LIF, the background argon plasma ion thermal velocity and temperature were measured as 340 m/s and 0.05 eV, respectively. Background plasma drift velocity observed by LIF was 179 ± 17 m/s toward the ion beam source, which is located opposite from the plasma source shown in Fig. 1, while the MP indicated 222 ± 67 m/s by applying the PIC model [$R = \exp(kM)$, k = 1.34 for $T_i \leq 3T_e$]⁸ for unmagnetized plasmas and as



Fig. 6. LIF measurements of plasma flow without the ion beam; MP indicates a flow about $222\pm67\,m/s$ by a PIC code^{8)} and $230\pm70\,m/s$ by a kinetic code.⁹⁾

 230 ± 70 m/s by the unmagnetized kinetic model [k = 1.29for $T_{\rm i} = 0.01T_{\rm e}$ by linear extrapolation of the adjacent values of k: k = 1.2 and 1.0 for $T_i/T_e = 0.2$ and 1.0, respectively, yet it might be larger if one would take nonlinear extrapolation including k = 0.9 for $T_i/T_e = 2.0.$] to eq. (10).9) Here we use the unmagnetized MP theories since the ion gyro-radius is much bigger than the probe size $(\rho_i/a > 2)$. The LIF data are well fit to the results of these models within experimental errors. We note that a k factor of 1.66 would fit the Mach probe data to the LIF-measured drift speed. Such a small drift velocity does not provide an MP calibration point for much faster ion drifts; it would be useful to calibrate the MP by the LIF for higher velocities, especially near the ion sound velocity. However, in our experiments the velocity of the background plasma could not be varied much more than an order of magnitude. Thus, we arranged an energetic ion beam injection into the background plasma to generate fast plasma ion flow. The flow velocity of the ion beam and background plasma was measured by MP. However, the presence of an ion beam distorted TP data, as mentioned before.

4. Velocity Measurement with Supersonic Ion Beams

After calibrating the MP with the slowly-drifting background plasma, we measured the flow velocity of the ion beam. The experiment was done at port B (Fig. 1). In this circumstance the background plasma had a drift velocity (measured via LIF) of 179 m/s towards the ion beam source (Veeco/IonTech commercial 3 cm rf argon ion beam). Ion beams were produced with controlled energies ranging from $70-370 \,\text{eV}$.

To determine flow velocity, one could apply existing theories if the average plasma flow were subsonic, since the MP, which collects ions of the background plasma and the ion beam, produces the average flow velocity of the plasma. Figures 7(a) and 7(b) show the downstream and upstream ion saturation currents collected by Mach probes in terms of position. The upstream ion saturation current at the beam center decreased as beam energy increased for $V_b \leq 100$ volt and increased for $V_b > 100$, while the downstream current showed a monotonic decrease.



Fig. 7. Ion saturation currents of Mach probe: (a) ion saturation current collected by downstream probe, and (b) ion saturation current collected by upstream probe.

In order to calculate the average flow velocity of plasma with the ion beam present, one must know densities and flow velocities of the beam and plasma. Since the electric probe basically measures the particle flux, one can assume that a flow velocity determined by the MP shows the averaged velocity of the ion beam with the background plasma, and it would be presented as

$$\langle v \rangle = (n_{\rm p}v_{\rm p} + n_{\rm b}v_{\rm b})/(n_{\rm p} + n_{\rm b}),$$
 (12)

where n_p and v_p are density and velocity of background plasma without the ion beam, and n_b and v_b are beam density and velocity. Actually the MP is for the measurement of the drift velocity expressed as in a shifted Maxwellian distribution, but not for the thermal velocity as in a Maxwellian distribution, yet it can be used as a diagnostic tool even for the measurement of bulk flow velocity.

Without background plasma, the velocities of ion beams are measured by the LIF system, and densities are measured by a Faraday cup. These velocities of the ion beams are calibrated in terms of bias voltage (V_b) applied to the ion beam source, which is the same as the kinetic velocities of the ion beams with beam bias energy of eV_b . During the experiment with background plasmas, one can know the beam velocity in terms of the bias voltage to the ion beam source, since the velocity of the ion beam is little changed with low density background plasmas. These are also inferred from the ion energy analyzer. Both methods give



Fig. 8. Ion beam density profile translated in terms of beam center measured by Faraday cup at pressure $= 3 \times 10^{-4}$ torr, rf frequency = 18.75 MHz, and rf power = 80 W: (a) beam current density, and (b) beam density.

almost the same value for $v_{\rm b}$. For example, when the observed ion beam energy is 370 eV, the velocity of the ion beam measured by LIF is about 4.2×10^4 m/s. Without the ion beam, the background plasma density, electron temperature, and flow velocity are $n_{\rm p} = 7.5 \times 10^9$ cm⁻³ and $T_{\rm e} = 7.6$ eV by TP, and $v_{\rm pf} = -179$ m/s ("–" sign means the plasma flows toward the ion beam source) by LIF.

Figures 8(a) and 8(b) show the ion beam current density and ion beam number density measured with Faraday cup without background plasma when the 80 W RF power was applied to the ion beam source with 370 eV of beam energy. The velocity of the plasmas with ion beams, $\langle v \rangle$, is calculated with the assumptions of no strong collisional effects between background plasma and supersonic ion beams using eq. (12) with the following parameters: the velocity of the plasma without the ion beam, v_p , determined by Mach probe measurements using eq. (12); the ion beam velocity, v_b , with ion beam energy (eV_b), which is also calibrated by LIF; plasma density, n_p , is measured by TP, and ion beam density, n_b , is given by ion beam current.

Figure 9(a) shows the ratio of upstream ion saturation current to downstream ion saturation current density with ion beam energy (E_0): 70 < E_0 < 370 eV. With background



Fig. 9. (a) Ratio of upstream to downstream ion current density at the center position of the beam in terms of beam energy (E_0) . (b) the measured flow velocity by MP at the beam center and calculated average flow velocity of the plasma and ion beam. The "–" flow velocity means the plasma flows toward the ion beam source and "+" means flows toward the RF-coil (or the ICP plasma source).

plasmas, from the measurement of the current densities with the MP, the velocity of plasmas with the ion beams is determined using eq. (12), in which we have used two calibration factors of k: k = 1.32 as an average of kinetic and PIC models, and 1.66 as the calibrated one by LIF method when there is no ion beam. These calculated ones (or indirectly measured using LIF) and deduced ones (or measured using the MP) with two values of k are shown in Fig. 9(b). The "-" flow velocity means the plasma flows toward the ion beam source and "+" means flows toward the ICP plasma source. The MP gives flow velocities similar to the LIF results for ion beam energies up to 170 eV, while some divergence develops for higher ion beam energies. This discrepancy of flow velocities by MP and LIF for larger ion beam energy ($E_0 > 170 \,\text{eV}$) may be due to: (1) invalidity of Mach probe theory for unmagnetized flowing plasma with supersonic ion beams; (2) error in density calculations which do not include interaction of the ion beam with the background plasma. Because the MP was calibrated as k =1.66 for the background plasma alone as already discussed in Y.-S. CHOI et al.

§3, if we use a k of 1.66, the discrepancy is reduced as shown in Fig. 9; however, further investigation of the discrepancy is warranted.

5. Conclusions

Plasma flow velocities are measured by a MP using existing probe theories (kinetic and PIC models) for the unmagnetized plasmas and are compared to those from the laser-induced fluorescence method with supersonic ion beams. In order to determine the flow velocities, we calibrated the MP via LIF in the absence of the ion beam, where the LIF data are well fit to those models within experimental errors. For the comparison of the average plasma flow velocities by MP and LIF, the supersonic ion beam speed was measured by LIF and incorporated into a simple formula of average plasma speed. During this process, one must know the background plasma density and electron temperature, which were inferred from a triple probe measurement. With the calibration for velocity and electron temperature, we determined the average flow velocities from LIF and compared them with those from MP. Agreement was found generally across the ion beam energies tested, with closer agreement at the lower beam energies. Future work on MP calibration with LIF should include data with calibration such as various magnetic fields, subsonic flow velocities near M = 1, and controlled plasma flow as well as ion beam energy.

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