General Log-Likelihood Ratio Expression and Its Implementation Algorithm for Gray-Coded QAM Signals

Ki Seol Kim, Kwangmin Hyun, Chang Wahn Yu, Youn Ok Park, Dongweon Yoon, and Sang Kyu Park

A simple and general bit log-likelihood ratio (LLR) expression is provided for Gray-coded rectangular quadrature amplitude modulation (R-QAM) signals. The characteristics of Gray code mapping such as symmetries and repeated formats of the bit assignment in a symbol among bit groups are applied effectively for the simplification of the LLR expression. In order to reduce the complexity of the max-log-MAP algorithm for LLR calculation, we replace the mathematical max or min function of the conventional LLR expression with simple arithmetic functions. In addition, we propose an implementation algorithm of this expression. Because the proposed expression is very simple and constructive with some parameters reflecting the characteristic of the Gray code mapping result, it can easily be implemented, providing an efficient symbol de-mapping structure for various wireless applications.

Keywords: LLR, soft metric, QAM

Manuscript received Aug. 30, 2005; revised Feb. 26, 2006.

Ki Seol Kim (phone: + 82 2 2220 0365, email: kiseol@ihanyang.ac.kr), Dongweon Yoon (phone: +82 2 2220 0362, email: dwyoon@hanyang.ac.kr), and Sang Kyu Park (email: skpark@hanyang.ac.kr) are with the Department of Electronics and Computer Engineering, Hanyang University, Seoul, Korea.

Kwangmin Hyun (email: kamihyun@wonju.ac.kr) is with the Department of Information and Communication Engineering, Wonju National College, Gangwon-do, Korea.

Chang Wahn Yu (email: ychang@etri.re.kr) and Youn Ok Park (email: parkyo@etri.re.kr) are with Mobile Telecommunication Research Division, ETRI, Daejeon, Korea.

I. Introduction

Quadrature amplitude modulation (QAM) is an attractive technique to achieve an improved high-rate transmission over wireless links without increasing bandwidth. For this reason, QAM is strongly recommended as a prospective modulation scheme for wireless communication systems, such as 3G and 4G mobile communication systems, wireless LAN, and digital video broadcasting. To function satisfactorily in a wireless link, however, QAM communication systems require a high signal-to-noise ratio (SNR) to combat the harsh wireless environment. In order to overcome this drawback, iterative decoding schemes with turbo or turbo-like codes such as the low density product code (LDPC) are being considered. Several works [1]-[3] have been done on adopting Turbo trellis coded modulation (TTCM). However, the TTCM system requires a specific Turbo codec corresponding to the Turbo code of the TTCM system.

The notably good performance of iterative decoding suggests that there is promise in combining Turbo or Turbo-like codes with a well-structured binary decoder for multi-level modulated signals in order to simultaneously obtain large coding gains and high bandwidth efficiency [4]-[11].

Although binary iterative decoding schemes provide coding gain with M-ary modulated signals, they require calculation of the bitwise metric. Performing this calculation with a conventional algorithm such as log MAP and max-log-MAP, however, is very tedious work. In order to reduce the complexity of the bit metric calculation, several methods [5]-[13] have been proposed for Gray coded signals, such as the pragmatic approach, the log likelihood ratio (LLR) approach, and others. But these approaches presented either specific soft metric algorithm for each corresponding modulated signal or approximated expression.

In this paper, we present a simple and general expression of an LLR based on the observation of the max-log-MAP algorithm for a rectangular QAM (R-QAM) signal. In addition, we suggest an implementation algorithm of the proposed expression. The remainder of this paper is organized as follows. Section II includes the system model with R-QAM and Gray code mapping. Section III surveys a conventional LLR expression and a method of obtaining the bit LLR for R-QAM signals. Section IV presents a simple and generally applicable bitwise LLR expression based on the Max-Log-MAP algorithm and shows its operation in detail. Section V discusses the numerical results of the provided LLR expression to verify its validity, and the implementation of the suggested algorithm. Section VI summarizes our conclusions.

II. System Model

The modulated arbitrary rectangular Gray coded QAM signal is assumed to be transmitted over an additive white Gaussian noise (AWGN) channel. In an $N\times L$ rectangular QAM, $\log_2(N\cdot L)$ bits of a serial information stream are mapped onto a 2-dimensional signal constellation using Gray coding, where N is the number of signal constellations on the in-phase axis and L is on the in-phase axis. Among the grouped information of $T = \log_2(N\cdot L)$ bits constituting code word $C = (c_0, c_1, \cdots, c_{T-1})$, the $K = \log_2 N$ bits constituting codeword $C_I = (b_{I,0}, b_{I,1}, \cdots, b_{I,K-1})$ are mapped onto the in-phase channel, whose amplitude A_I is selected over the set of $\{\pm d_I, \pm 3d_I, \cdots, \pm (N-1)d_I\}$. Similarly, the $X = \log_2 L$ bits constituting codeword $C_Q = (b_{Q,0}, b_{Q,1}, \cdots, b_{Q,X-1})$ are mapped onto the quadrature channel, whose amplitude A_J is selected over the set of $\{\pm d_Q, \pm 3d_Q, \cdots, \pm (L-1)d_Q\}$. Note that d_I and d_Q can be different without any loss of generality. The R-QAM signal can be divided into two independent Gray coded pulse amplitude modulation (PAM) signals, in-phase and quadrature components, and the two PAM signals have identical signal characteristics except for the rotation of the axis. Hence, we first consider a one-dimensional PAM signal with equidistant symbols, and then expand to an R-QAM signal. Figure 1 illustrates an 8-PAM constellation with its decision regions, the definition of the bit levels and the bit groups where the signal points in the constellation are assigned a perfect one-dimensional Gray code [12], [14]. When we consider only the bit b_0 level, it becomes the 2-PAM constellation.

The received R-QAM signal hypothesis can be expressed as

$$H_i: \quad z = \alpha s + n, \tag{1}$$

where α is a channel gain; complex received symbol $z = z_I + jz_Q$; complex transmitted symbol $s = s_I + js_Q$; and complex AWGN $n = n_I + jn_Q$ with zero mean and variance σ^2 per dimension. The in-phase component s_I of the transmitted symbol s belongs to a set of constellation points, $s_I \in \{S_{-N/2}, \ldots, S_{-1}, S_1, \ldots, S_{N/2}\}$, and the symbol $s_I = f(b_0, b_2, \ldots, b_{K-1})$, where $f(\cdot)$ is the Gray code mapping function with K bits. Figure 1 shows a signal space of N-PAM when $N = 8$. The signal space of the quadrature component s_Q of the transmitted symbol s can be expressed exactly the same as that of the in-phase component except for the number of signal points in the signal space. The character of the channel model depends on the probabilistic characteristics of the channel gain α. For example, if the channel gain is time-invariant, the channel is AWGN.

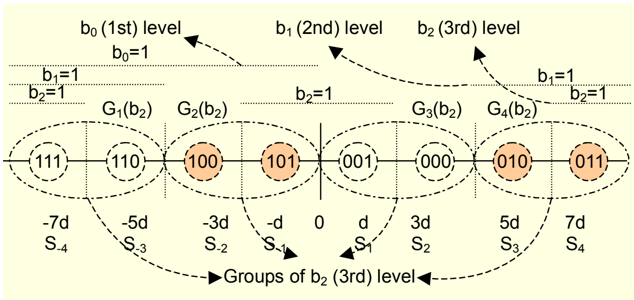


Fig. 1. Gray coded 8-PAM signal constellation.

III. N-PAM Log-Likelihood Ratio Calculation

1. Ordering of Citations

For an AWGN process (α=1), when perfect channel knowledge is available, an N-PAM LLR test of the received symbol as in [5] is given by

$$LLR_I(b_k) = \ln \sum_{A\in\{s:b_k=1\}} \exp\left(-\frac{(z-A)^2}{2\sigma^2}\right) - \ln \sum_{B\in\{s:b_k=-1\}} \exp\left(-\frac{(z-B)^2}{2\sigma^2}\right). \tag{2}$$

When we adapt the approximation $\ln \sum_j \exp(-a_j) \approx \max(-a_j) = \min(a_j)$ in [14] to simplify the calculation process of (2), the LLR (2) is rewritten as

$$\begin{aligned} LLR_I(b_k) &\approx \frac{1}{2\sigma^2}\left[\min_{B\in\{s:b_k=-1\}} |z-B|^2 - \min_{A\in\{s:b_k=+1\}} |z-A|^2\right] \\ &= \frac{1}{2\sigma^2}\left[\min_{B\in\{s:b_k=-1\}} (B^2-2Bz) - \min_{A\in\{s:b_k=+1\}} (A^2-2Az)\right]. \end{aligned} \tag{3}$$

When the specific symbols A and B for the mathematical min function in (3) are selected, (3) can be reduced to

$$
\begin{aligned}
LLR_I(b_k) &= \frac{1}{2\sigma^2}\left[(B-A)(B+A-2z)\right] \\
&= \frac{2}{\sigma^2}\left\{\left(\frac{B-A}{2}\right)\left[\left(\frac{B+A}{2}-D\right)-(z-D)\right]\right\},
\end{aligned} \tag{4}
$$

where D is an arbitrary value.

Conventionally, the LLR in (3) is calculated through case-by-case and region-by-region tests over the signal space to find the appropriate symbol values A and B for the mathematical min function in (3), resulting in (4). If we consider a stationary channel, we can normalize the LLR (3) and (4) with respect to constant $2/\sigma^2$ as

$$
\Lambda_I(b_k) = \frac{\sigma^2}{2} LLR_I(b_k)\,. \tag{5}
$$

Using (5), the bit b_0 LLR of the 2-PAM signal in Fig. 2(a) is obtained by

$$
\Lambda_I(b_0) = [(d^2-2dz)-(d^2+2dz)]/4 = -dz\,, \tag{6}
$$

for all z.

However, such calculations become very tedious work with the increase of the modulation order, due to the large number of cases and symbol regions that must be considered to find the minimum value over the entire signal space.

For example, consider calculating the bitwise LLR of the 8-PAM symbol referred to in Fig. 1. The bit of interest b_0 is the first bit of the binary 3-tuple $\{b_0, b_1, b_2\}$, which is a set of a symbol's bit group constituting one symbol for 8-PAM. We define the hypotheses, P_0 and P_1, of the bit of interest b_0 where the bit b_0 has a value of -1 (or 0) and +1 (or 1), respectively. As we receive a symbol on the S_1 region with the bit b_0 assigned a value of 0, we can easily acquire the hypothesis P_0 of the first min(・) term in (3). In order to calculate the second min(・) term in (3), however, we should find the minimum value of the opposite hypothesis P_1 after region-by-region examination among the four possible regions $\{S_{-4}, S_{-3}, S_{-2}, S_{-1}\}$, the regions in which the first bit b_0 is valued 1. In a similar way, as we receive a symbol on the region S_{-1} with the bit b_0 valued 1, we have to search the minimum value of P_0 among the results for the regions $\{S_1, S_2, S_3, S_4\}$. It is also necessary to follow the same complex process in order to get the bitwise LLRs of the other bits in the symbol.

In particular, let us assume that the received symbol value is placed in $0 < z_I \le 2d$, S_1, on an 8-PAM signal space as shown in Fig. 1, and we are trying to get the LLR of the first bit b_0. As this received symbol is placed on the bit b_0 region valued 0, we can easily decide the hypothesis P_0. However, we need to find the minimum value among the hypotheses P_1 of interest over $\{S_{-4}, S_{-3}, S_{-2}, S_{-1}\}$ using (3) as

$$
\begin{aligned}
\Lambda_I(b_0) &= \{(S_1) - \min\left(S_{-1}, S_{-2}, S_{-3}, S_{-4}\right)\}/4 \\
&= \{(d^2-2dz)-(d^2+2dz)\}/4 = -dz,
\end{aligned}
$$

where all elements of the set $\{S_{-4}, S_{-3}, S_{-2}, S_{-1}\}$ have the bit of interest b_0 valued at 1.

In the same way, the bitwise LLRs on each symbol region in Fig.1 are evaluated as

$$
\Lambda_I(b_0) = \begin{cases}
-dz, & 0 < z \le 2d \\
2d(d-z), & 2d < z \le 4d \\
3d(2d-z), & 4d < z \le 6d \\
4d(3d-z), & z > 6d \\
-dz, & -2d < z \le 0 \\
-2d(d+z), & -2d < z \le -4d \\
-3d(2d+z), & -4d < z \le -6d \\
-4d(3d+z), & z > -6d
\end{cases}
$$

$$
\Lambda_I(b_1) = \begin{cases}
-2d(3d-z), & 0 < z \le 2d \\
-d(4d-z), & 2d < z \le 4d \\
-d(4d-z), & 4d < z \le 6d \\
-2d(5d-z), & z > 6d \\
-2d(3d+z), & -2d < z \le 0 \\
-d(4d+z), & -4d < z \le -2d \\
-d(4d+z), & -6d < z \le -4d \\
-2d(5d+z), & z \le -6d
\end{cases} \tag{7}
$$

$$
\Lambda_I(b_2) = \begin{cases}
d(2d-z), & 0 < z \le 2d \\
d(2d-z), & 2d < z \le 4d \\
-d(6d-z), & 4d < z \le 6d \\
-d(6d-z), & z > 6d \\
d(2d+z), & -2d < z \le 0 \\
d(2d+z), & -4d < z \le -2d \\
-d(6d+z), & -6d < z \le -4d \\
-d(6d+z), & z \le -6d\,.
\end{cases}
$$

IV. General LLR Expression for R-QAM Signals

If we observe the result of the Gray code mapping rules and expand the LLR expression of the 2-PAM in (6) to that of the higher order PAM (in-phase channel or quadrature channel of the R-QAM signal), we can effectively establish a simple and general expression of a bitwise LLR by searching only the confined region, not over the entire signal space as in section III. In this section, we consider only the in-phase component of $8 \times L$ R-QAM, the 8-PAM constellation.

According to the Gray mapping rule, two or more consecutive symbols make a paired bit group {1, 0} or {0, 1} for the *k*-th bit of interest of the binary *K*-tuple, and we can define two important terms for the next step of the LLR calculation. First, the group can be classified as either a mirror-image or isomorphic group, seen in the bit-arranged form of the 2-PAM in Fig. 2(a). Second, each group can be identified as an axis-shifted version of 2-PAM where the amount of the shift is the distance from the decision line to the origin of the signal space.

For the bit b_2 of the 3-tuple, there are 4 groups, $G_1(b_2)=\{S_{-4}, S_{-3}\}=\{1, 0\}$, $G_2(b_2)=\{S_{-2}, S_{-1}\}=\{0, 1\}$, $G_3(b_2)=\{S_1, S_2\}=\{1, 0\}$, and $G_4(b_2)=\{S_3, S_4\}=\{0, 1\}$. Among the four groups, $G_1(b_2)$ and $G_3(b_2)$ are of the isomorphic groups, whereas $G_2(b_2)$ and $G_4(b_2)$ are mirror-image groups for the 2-PAM bit arrangement, the shaded groups in Fig. 2 (b). Each group has a different amount of shift from the bit decision line to the absolute coordinate origin. For example, group $G_1(b_2)$ is a version of 2-PAM axis-shifted to the extent of -6*d*, and group $G_4(b_2)$ is shifted to the extent of +6*d*, as shown in Fig. 2(c). Figure 2 illustrates the relationship between 2-PAM and each group of 8-PAM, as well as the detailed b_2 LLR process for groups $G_1(b_2)$ and $G_4(b_2)$ on the 8-PAM signal space. For the bit b_1 of a binary 3-tuple, there are two paired bit groups, the isomorphic group $G_1(b_1)=\{S_{-4}, S_{-3}, S_{-2}, S_{-1}\}=\{1, 1, 0, 0\}$ and mirror-image group $G_2(b_1)=\{S_1, S_2, S_3, S_4\}=\{0, 0, 1, 1\}$ as shown in Fig. 2 (b). The amounts of the shifts of the two groups, $G_1(b_1)$ and $G_2(b_1)$, are +4*d* and -4*d*, respectively.

When a received symbol is placed on a specific symbol region on the signal space, the search region can be confined within the same group. In order to obtain the bitwise LLR, we need to choose the minimum value of the opposite hypothesis (1 or 0) to the hypothesis of the bit of interest (0 or 1) for the received symbol. The limited search region is due to the most likely error caused by noise that involves the detection of the erroneous amplitude, which is located in the very next region to the transmitted symbol region in all probability. For this reason, we can easily decide that the region for the minimum opposite hypothesis to that of the bit of interest is always the nearest symbol region that has the opposite bit value. Also, to directly adapt the 2-PAM LLR expression (6) for the group of each bit in the symbol codeword, the decision line of the group of interest is moved to the axis of the origin (absolute zero) as shown in Fig. 2(c).

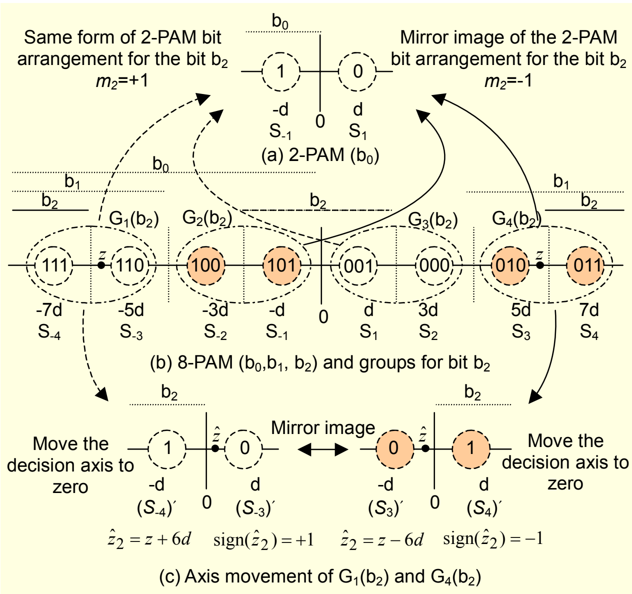


Fig. 2. Bit allocation relationship between 2-PAM and groups of 8-PAM, and the detailed LLR process for the bit b_2 of the groups $G_1(b_2)$ and $G_4(b_2)$ on the 8-PAM signal space.

Considering the previously defined terms and the widely spread results in (7), we can come to some specific conclusions straightforwardly.

1) The sign of the bitwise LLR is affected by the bit arrangement form parameter m_k and by the sign of the changed value ($\hat{z}$) resulted from the axis movement of the received value (z), as shown in Fig. 2 (c).
2) The LLR parameters are also related to the minimum and maximum distances, the relationship between the boundaries of the relevant symbol region, and the decision line of the bit group of interest.
3) The LLR is also sensitive to axis movement.

Considering the previously stated description and (4), the bitwise LLR for the *k*-th bit of the in-phase component of an R-QAM signal can be straightforwardly written as

$$\Lambda_I(b_k) = G_{\hat{z},k} \times m_k \times d_{\max,k} \times \{d_{\min,k} - |\hat{z}_k|\}\ , \qquad (8)$$

where the parameters are defined as follows:

$\hat{z}_k$: Axis shifted value of the in-phase component z_I to the distance D_k (from the decision line of the bit group of interest to the absolute origin).

m_k: Mirrored group indication. If the group is mirrored for the 2-PAM, m_k is -1; if not, m_k is +1.

$G_{\hat{z},k}$: Sign of the compensated received value $\hat{z}_k$.

$d_{\min,k}$: Absolute value of the minimum distance from the *k*-th bit decision line of the group of interest to the received symbol region after scaling with 2.

$d_{\max,k}$: Absolute value of the maximum distance from the *k*-th bit decision line of the group of interest to the received symbol region, which can be obtained from $d_{\max,k} = d_{\min,k} + d$ after scaling with 2.

In the same way as (8), the LLR $\Lambda_Q(b_x)$ for the quadrature component of R-QAM can also be expressed as

$$\Lambda_Q(b_x) = G_{\hat{z},x} \times m_x \times d_{\max,x} \times \{d_{\min,x} - |\hat{z}_x|\} \qquad (9)$$

with the quadrature component z_Q of the received R-QAM symbol value for the x-th bit of the quadrature codeword C_Q.

Note that the minimum and maximum distances can easily be calculated only by using the received value z and the prior information of modulation. For example, the minimum and maximum distances for the bit b_1 in the symbol of 8-PAM in Fig. 2(b) are as follows.

1) If the received symbol is in the region S_{-1}, S_{-4}, S_1, or S_4,

$$d_{\min,1} = \begin{Bmatrix} |-4d-(-2d)|/2, & \text{for region } S_{-1} \\ |-4d-(-6d)|/2, & \text{for region } S_{-4} \\ |4d-2d|/2, & \text{for region } S_1 \\ |4d-6d|/2, & \text{for region } S_4 \end{Bmatrix} = d,$$

$$d_{\max,1} = d_{\min,1} + d = 2d.$$

2) If the received symbol is in the region S_{-2}, S_{-3}, S_2, or S_3,

$$d_{\min,1} = \begin{Bmatrix} |-4d-(-4d)|/2, & \text{for region } S_{-2} \\ |-4d-(-4d)|/2, & \text{for region } S_{-3} \\ |4d-4d|/2, & \text{for region } S_2 \\ |4d-4d|/2, & \text{for region } S_3 \end{Bmatrix} = 0,$$

$$d_{\max,1} = d_{\min,1} + d = d.$$

To verify the effectiveness of the presented bitwise LLR expression of (8), we use it to calculate the LLRs for an 8-PAM. If we assume the received 8-PAM symbol is positioned in the region S_1, $0 < z \le 2d$ for the LLR of the bit of interest b_1, we can evaluate the required parameters in (8) as

$$S_1: \hat{z}_1 = z - 4d,\ G_1 = -1,\ m_1 = -1,\ d_{\min,1} = d,\ d_{\max,1} = 2d. \quad (10)$$

Then, we calculate the LLR of the bit b_1 in the region by substituting (10) into (8):

$$\Lambda_I(b_1) = -1 \cdot -1 \cdot 2d\left[d - |z - 4d|\right] = -2d(3d - z). \quad (11)$$

In a similar way, as we receive a symbol on the S_2 region, $-4d < z \le -2d$, for the LLR of the same bit b_1 of interest, the parameters of (8) become

$$S_2:\ \hat{z}_1 = z + 4d,\ G_1 = +1,\ m_1 = +1,\ d_{\min,1} = 0,\ d_{\max,1} = d. \quad (12)$$

By substituting (12) into (8), we have

$$\Lambda_I(b_1) = +1 \cdot +1 \cdot d\left[0 - |z + 4d|\right] = -d(4d + z). \quad (13)$$

The LLR results (11) and (13) of the bit of interest b_1 are the same as the conventional results in (7) on the same symbol region. Similarly, other bitwise LLRs of the 8-PAM signal for each bit of the received symbol regions can also be easily evaluated with (8). Thus, we can confirm that the results of the presented expression (8) are all exactly the same as the results of the conventional case-by-case, region-by-region calculation method of the Max-Log-MAP algorithm. This is because we induced the proposed expression from (3).

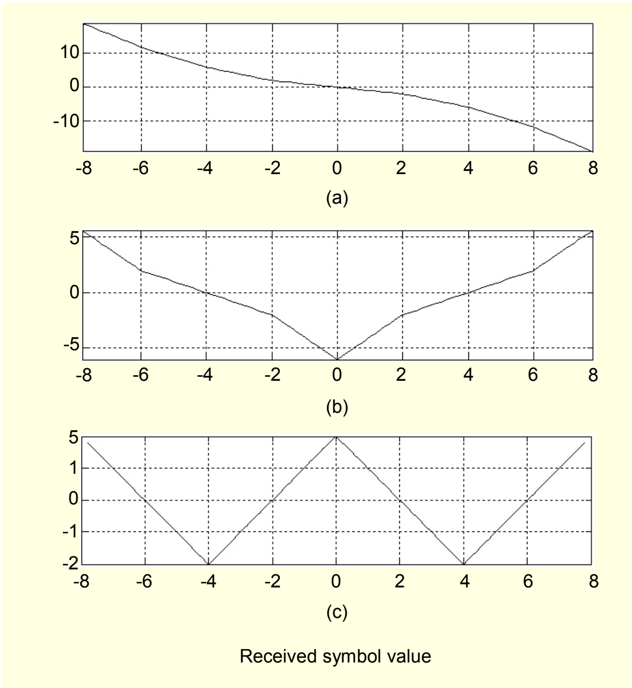


Fig. 3. Bitwise LLR conversion of a received 8-PAM symbol: (a) LLR of bit b_0, (b) LLR of bit b_1, and (c) LLR of bit b_2.

Figure 3 shows the computer-simulated bitwise LLR curves of a 3-tuple $\{b_0, b_1, b_2\}$ of an 8-PAM symbol using the presented expression (8). Even though (8) looks like a linear function of the received signal amplitude, the plot is not piecewise linear. This is because the parameters in (8) are not constant, but are variable values depending on the received symbol region, as shown in examples from (10) to (13).

Note that if we combine the LLR expression (8) for the in-phase component and the expression for the quadrature version with the different number of symbols on each axis, we can directly obtain the LLRs for R-QAM signals.

V. Implementation of the Bitwise LLR

In the previous section, we presented a simple expression to calculate the bitwise LLR for $(N \times L)$ R-QAM signals. To apply the proposed expression to an R-QAM signal, as a practical example, we need to implement $(K + X)$ bitwise LLR calculation blocks. Figure 4 shows the in-phase LLR calculation block for the received in-phase component with K-bit LLR blocks of an N-PAM signal, where the sign(·) function takes the sign (+1 or -1) from the input value. The LLR calculation block for the quadrature component has the

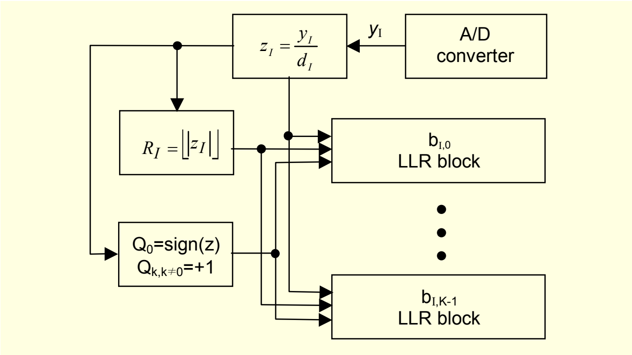


Fig. 4. LLR calculation block diagram for the in-phase component of an (N×L) R-QAM signal.

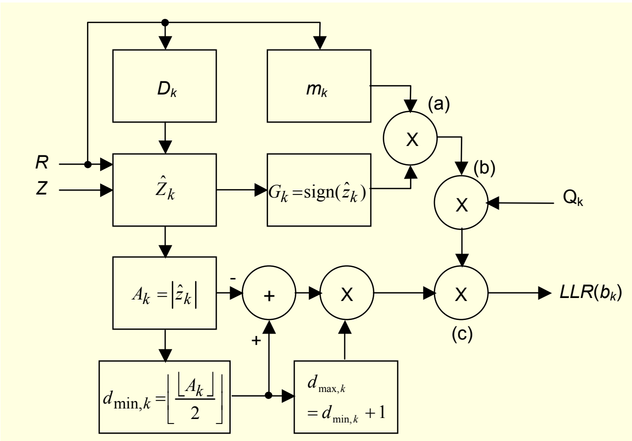


Fig. 5. Detail block diagram of the bitwise LLR calculation in Fig. 4.

same structure as in Fig. 4, except that it uses X-bit LLR blocks instead of K-bit LLR blocks. In Fig. 4, when the symbol value is received, we first need to limit the received value to the maximum energy, and then scale the limited value with the reference energy d to simplify value handling.

The mathematically floored value $R = (r_0, r_1, \ldots, r_{K-1})_2$ offers simple integer operation at each bitwise LLR block, where $(\ldots)_2$ is a binary value representation, r_0 is the most significant bit (MSB), and r_{K-1} is the least significant bit (LSB). These values are commonly applied for all bitwise LLR blocks.

Figure 5 shows details of the bitwise LLR block from Fig. 4. Note that in Fig. 5, the two multipliers (a) and (b) can be replaced with much simpler logical operations such as a bitwise Exclusive OR (XOR) with the sign inputs m_k, G_k, and Q_k. The multiplier (c) is a sign converter of the input depending on the result of the multiplier (b).

The parameters in Fig. 5 can be easily obtained with the value R through arithmetic operations with digital logic operations as follows.

1) The distance D_k of each bit group for axis-shift: From Fig. 2, we can observe that the distances D_k of the bit b_0 group and the bit b_1 group are always fixed as 0 and 4 respectively,

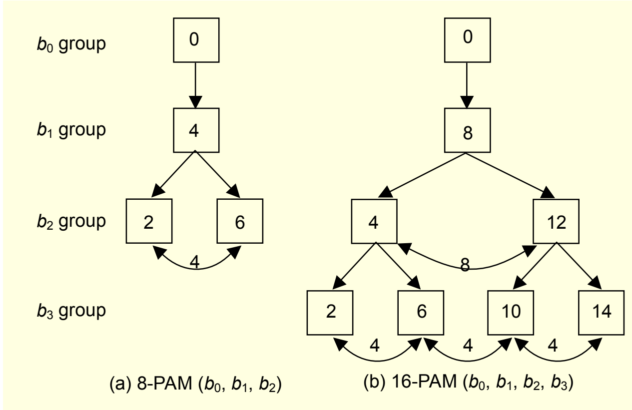


Fig. 6. Tree diagram of the axis-shift distance for each bit group of 8-PAM and 16-PAM.

regardless of the received value for an 8-PAM signal. In other words, the bit b_0 group does not need to shift the axis, and the bit b_1 groups always do the axis-shift to the right or left by an amount of 4.

If we received a 16-PAM signal, the distances for the bit b_1 groups become 8 as in Fig. 6(b). But that of the other groups should be found with the value R depending on the received symbol region. The relationships of the distances between the bit groups of each of the bit levels are given in Fig. 6. Figure 6 shows the distance tree diagram for the axis-shift of each bit group. Considering the distance of each group for axis-shift, we can find a rule for the distance D_k with the mathematically floored value R as:

$$\begin{aligned} D_0 &= 0, \\ D_1 &= 2^{(K-1)}, \\ D_2 &= \{(r_0)_2 \times 2 + 1\} \times 2^{(K-2)}, \\ D_k &= \{(r_0 \cdots r_{k-2})_2 \times 2 + 1\} \times 2^{(K-k)}, \quad k = 3, \cdots, K-1. \end{aligned} \tag{14}$$

2) Mirrored group indication m_k: Extract the bit placement form parameter from the value R. The indication value m_k is +1 or -1, which can be represented by the 1-bit binary value 0 or 1. Because the bit b_0 group always shows the isomorphic one as the 2-PAM bit arrangement, as in Fig. 2, m_0 is always 0 (+1). The bit b_1 groups are always mirrored, thus m_1 is 1 (-1). For other bit groups, we consider only the positive (left-half) plane after taking the absolute value of the received value. This is possible because all bit assignments except the bits b_0 and b_1 are symmetric about a line between the left-half and right-half planes, a consequence of Gray mapping. The m_2 of the bits b_2 groups is the bit value r_0 of the value R, and the m_3 of the bit b_3 groups is the bit value r_1. Thus, we can find another rule for the indication value m_k as

$$\begin{aligned} m_0 &= 0\,(+1), \\ m_1 &= 1\,(-1), \\ m_k &= r_{k-2}\,(0 \text{ or } 1), \quad k = 2, 3, \cdots, K-1. \end{aligned} \tag{15}$$

3) Calculate the boundary conditions $d_{min,k}$ and $d_{max,k}$ of each symbol region with the compensated value $\hat{z}_k$: Using the calculated distance D_k, we move the decision axis of the group of interest to the absolute zero axis and compensate the received value to the amount of the shifted distance D_k. Then, we evaluate the boundary conditions with the axis-shifted value as in Fig. 5. For the calculation process of the boundary condition $d_{min,k}$, we use the mathematical floor function and division by 2, which can be easily implemented with the bit truncation and the logical left-shift operation. In order to obtain the other boundary condition $d_{max,k}$, an arithmetic addition is required with the symbol distance. Here, we consider the symbols on the transmitted constellation to be placed at equidistance $2d$.

In order to implement the proposed expression for the in-phase component with Figs. 4 and 5, we assume that the format of the sampled value y_I is a floating-point. This sampled value is the received symbol value of the N-PAM signal in Fig. 1 by an analog-to-digital converter. Using this sampled floating-point value, we can calculate the bitwise LLR in (8) as follows:

(a) Take the absolute value $|z_I|$ of the scaled value z_I of the sampled value y_I with d_I in Fig. 4.

(b) After passing through a floor function, the result value $R_I = (r_0, r_1, r_2)_2$ can be expressed by the binary form of a decimal value from 0 to N, where r_0 is the MSB in Fig. 4.

(c) Calculate the distance D_k for an axis-shift with the value R_I of (b). For example, we can calculate this distance for the 8-PAM in Fig. 1, the in-phase component of an $(8\times L)$ R-QAM signal, as follows: For bit b_0, $D_0 = 0$; for bit b_1, $D_1 = 2^{K-1} = 4$; and for bit b_2, $D_2 = \{(r_0)_2\times 2+1\}\times 2^{K-2} = 2$ or 6, where $K=3=\log_2 N$ in Fig. 5.

(d) Decide the bit-arranged form parameter m_k (0 or 1) with the result R of (b). For bit b_0, $m_0 = 0$; for bit b_1, $m_1 = 1$; and for bit b_2, $m_2 = r_0$. The parameter m_k is a bit value 0 or 1 (+1 or -1) to be used in the binary operation in Fig. 5.

(e) Take the sign bit Q_0 for only bit b_0 valued -1 or +1 from the sampled value z in Fig. 4. In the case of the other bit b_k, $k \neq 0$, $Q_k = +1$. This parameter can be also represented with a single binary bit value (0 or 1) for a simple binary logic operation.

(f) Calculate the compensated value $\hat{z}_k$ by using the results of (a), (c), and (e) in Figs. 4 and 5 as $\hat{z}_k = |z| - C_k$.

(g) Take the sign bit $G_k = Q_k \times \mathrm{sign}(\hat{z}_k)$ valued -1 or +1 of the result of (f) in Fig. 5. This parameter can be represented with a single binary bit value (0 or 1) for simple binary logic operation.

(h) Calculate the minimum distance from the result of (f) as in Fig. 5.

(i) Calculate the maximum distance using the result of (h) and the known symbol distance as in Fig. 5.

(j) Finally, calculate the bitwise LLR of bit b_k by using the results of (d), (g), and (h) in Fig. 5.

As an example, we define a Gray coded 64-QAM signal space with transmitted symbols with the codeword $C = (c_0, c_1, c_2, c_3, c_4, c_5) = (b_{I,0}, b_{I,1}, b_{I,2}, b_{Q,0}, b_{Q,1}, b_{Q,2})$, and assume the received symbols are located at y_1 and y_2 as shown in Fig. 7. The received symbols y_1 and y_2 can be separated into in-phase and quadrature components (i, q) as $y_1 = (-7.8d, 3.5d)$ and $y_2 = (-3.5d, 4.5d)$. Using the proposed expressions (8) and (9) and their implementation algorithm, we finally obtain the bitwise LLR results of these received symbols as in Table 1. Table 1 shows each result of the processing steps for the received values y_1 and y_2.

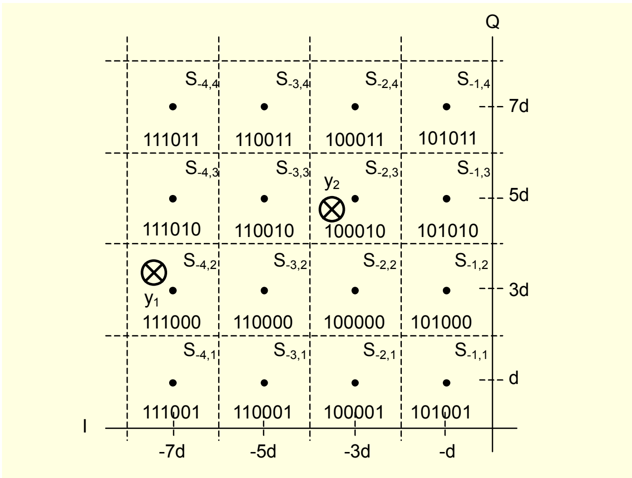


Fig. 7. Partial constellation of a Gray-coded 64-QAM.

Table 1. Bit LLR calculation examples of y_1 and y_2 for 8-PAM signals.

Steps	$y_1 = (-7.8d, 3.5d)$						$y_2 = (-3.5d, 4.5d)$					
	$i = -7.8d$			$q = 3.5d$			$i = -3.5d$			$q = 4.5d$		
	$b_{I,0}$	$b_{I,1}$	$b_{I,2}$	$b_{Q,0}$	$b_{Q,1}$	$b_{Q,2}$	$b_{I,0}$	$b_{I,1}$	$b_{I,2}$	$b_{Q,0}$	$b_{Q,1}$	$b_{Q,2}$
$\|z\|$	7.8			3.5			3.5			4.5		
R	$(111)_2$			$(011)_2$			$(011)_2$			$(100)_2$		
C_k	0	4	6	0	4	2	0	4	2	0	4	6
m_k	+1	-1	-1	+1	-1	+1	+1	-1	+1	+1	-1	-1
Q_k	-1	+1	+1	+1	+1	+1	-1	+1	+1	+1	+1	+1
$\hat{z}_k$	7.8	3.8	1.8	3.5	-0.5	1.5	3.5	-0.5	1.5	4.5	0.5	-1.5
G_k	-1	+1	+1	+1	-1	+1	-1	-1	+1	+1	+1	-1
$d_{min,k}$	3	1	0	1	0	0	1	0	0	2	0	0
$d_{max,k}$	4	2	1	2	1	1	2	1	1	3	1	1
LLR	19.2	5.6	1.8	-5.0	-0.5	-1.5	5.0	-0.5	-1.5	-7.5	0.5	-1.5
Decision	1	1	1	0	0	0	1	0	0	0	1	0

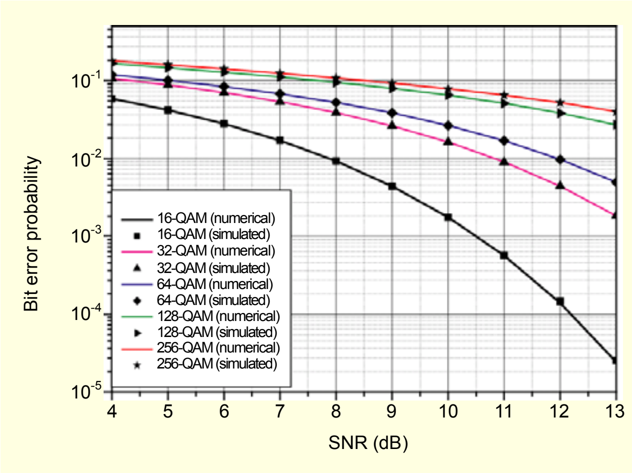


Fig. 8. Uncoded BER performance for different modulation orders.

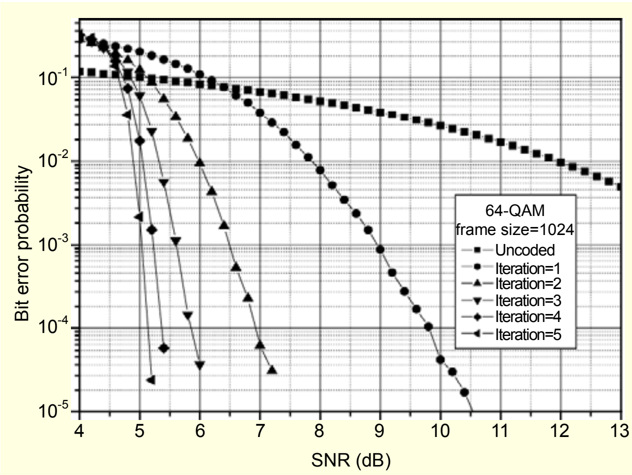


Fig. 9. BER performance for different iterations of a turbo decoder using the proposed algorithm (64-QAM, N=8, L=8).

Figures 8 and 9 show the BER performance versus SNR for different modulation orders of R-QAM and for different iterations of a Turbo decoder with the proposed LLR calculation algorithm, respectively. As shown in Fig. 8, the numerically calculated (solid line) results are almost the same as the simulated (symbol only) results.

In Fig. 9, as a practical example, we plot the BER versus SNR (E_b/N_0) for an uncoded result and the first five iterations of decoding results for a Turbo coded 64-QAM (N=6, L=8) signal over an AWGN channel. In this application, we used a random interleaver and two systematic convolutional encoders with feedback with memory 2. From this figure, we observe that for a BER of 10^{-3} the SNR is equal to 8.9 dB at iteration = 1, 6.5 dB at iteration = 2, 5.6 dB at iteration = 3, 5.2 dB at iteration = 4, and 5.05 dB at iteration = 5.

Implementation of a conventional bit metric generating function block with the Log-MAP or the Max-Log-MAP algorithm has very high structural complexity because of the logarithm calculation or Max operation. In order to reduce the complexity and the time latency of the conventional structure in practice, the highly complex functions are replaced with a look-up table (LUT) with pre-calculated values, even though this expedient introduces a certain amount of the performance degradation because of quantization errors. The proposed expression (8) in this paper, however, requires only basic mathematical functions such as multiplier and adder functions.

For the practical applications of certain function blocks, we need to take into account that power consumption, performance impact, semiconductor layout size, and many other areas are trade-offs for an efficient system design. Moreover, for an adaptive modulation and coding (AMC) scheme or a software radio (SR) system, the universality and flexibility of the structure can be considered more important than other issues. Taking all this into consideration, we submit that the proposed method is a promising alternative to conventional methods for reconfigurable systems such as AMC and SR.

VI. Conclusions

In this paper we provided a general and simple constructive expression for bitwise LLR calculation based on the unique mapping rules of arbitrary rectangular Gray coded QAM signals. Gray mapping creates the symmetric and repeated formats for the '0' and '1' assignment of the symbol codeword. The results of the proposed bitwise LLR expression are exactly the same as the conventional case-by-case, region-by-region calculation method of the Max-Log-MAP algorithm. Further, the parameters for the proposed LLR algorithm, such as the received symbol, the modulation order, and the mapping rule are easily extracted from the prior information at the receiver-side. We also suggested an implementation algorithm for the proposed expression. Thus, it can be easily applied for various applications with iterative decoding, such as a soft symbol de-mapper for R-QAM/N-PAM signals as an input to a Turbo or LDPC decoder.

References

[1] P. Roberson and T. Worz, "A Novel Bandwidth Efficient Coding Scheme Employing Turbo Codes," *ICC 1996*, June 1996, pp. 962-967.

[2] S. Benedetto, D. Divsalar, G. Montorsi, and F. Pollara, "Parallel Concatenated Trellis Coded Modulation," *ICC 1996*, June 1996, pp. 974-978.

[3] C. Fragouli and R. Wesel, "Turbo-Encoder Design for Symbol-Interleaved Parallel Concatenated Trellis-Coded Modulation,"

IEEE Trans. on Comm., vol. 49, no. 3, Mar. 2001, pp. 425-435.

[4] W. T. Webb and L. Hanzo, *Modern Quadrature Amplitude Modulation: Principles and Applications for Fixed and Wireless Channels*, IEEE Press, New York, 1994.

[5] R. M. Pyndiah, “Near-Optimum Decoding of Product Codes: Block Turbo Codes,” *IEEE Trans. on Comm.*, vol. 43, no. 8, Aug. 1995, pp. 2360-2369.

[6] R. Pyndiah, A. Picart, and A. Glavieux, “Performance of Block Turbo Coded 16-QAM and 64-QAM Modulations,” *GLOBECOM 1995*, Nov. 1995, pp. 1039-1043.

[7] J. Yuan, B. Vucetic, and W. Feng, “Turbo-Coded M-QAM for Fading Channels,” *IEE Electronics Lett.*, vol. 36, Issue 18, Aug. 2000, pp. 1562-1563.

[8] R. M. Pyndiah, "Near-Optimum Decoding of Product Codes: Block Turbo Codes," *IEEE Trans. Comm.*, vol. 43, Aug. 1995, pp. 2360-2369.

[9] S. Le Goff, A. Glavieux, and C. Berrou, “Turbo-Codes and High Spectral Efficiency Modulation,” *ICC 1994*, May 1994, p. 648.

[10] C. C. Wang, “A Bandwidth-Efficient Binary Turbo Coded Waveform Using QAM Signaling,” *ICCCS 2002*, June 2002, pp. 37-41.

[11] M. S. Raju, A. Ramesh, and A. Chockalingam, “BER Analysis of QAM with Transmit Diversity in Rayleigh Fading Channels,” *GLOBECOM 2003*, Dec. 2003, pp. 641-645.

[12] J. K. Kwon, S. Park, and D. K. Sung, “Log-Likelihood Ratio (LLR) Conversion Schemes in Orthogonal Code Hopping Multiplexing,” *IEEE Comm. Lett.*, vol. 7, Issue 3, Mar. 2003, pp. 104–106.

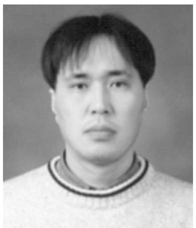
[13] A. J. Viterbi, J. K. Wolf, E. Zehavi, and R. Padovani, “A Pragmatic Approach to Trellis-Coded Modulation,” *IEEE Comm. Magazine*, vol. 27, no. 7, July 1989, pp. 11-19.

[14] K. Cho and D. Yoon, “On the General BER Expression of One- and Two-Dimensional Amplitude Modulations,” *IEEE Trans. on Comm.,* vol. 50, Issue 7, July 2002, pp. 1074-1080.

[15] A. J. Viterbi, “An Intuitive Justification and a Simplified Implementation of the MAP Decoder for Convolutional Codes,” *IEEE J. on Sel. Areas in Comm.*, vol. 16, no. 2, 1998, pp. 260-264.

Ki Seol Kim received the BS and MS degrees in electronic communication engineering from Hanyang University, Seoul, in 1982 and 1994, respectively. He is currently a PhD candidate in the division of Electronics and Computer Engineering at Hanyang University. He worked for LG Group and Korea Electric Power Network Company from 1982 to 1995, where he was in charge of the R&D for telecommunication systems and system design. Since 1995, he has been working for Seokyo Telecommunication Co., Ltd. as a CEO. His research interests are in the area of communications theory, image processing, video compression, optic transmissions, and signal processing for digital wireless.

Kwangmin Hyun received the BS, MS, and PhD degrees in electronics and computer engineering from Hanyang University, Seoul, Korea, in 1989, 1995, and 2004. In 1989, he joined the Radio Communications System Laboratory, Daewoo Telecom Inc., Korea. From 1995 to 1996, he worked at the Institute of Advanced Engineering (IAE) and from 1997 to 2001 at KMW, Inc., Korea. In 2005, he joined the faculty of the Information and Communication Engineering Department at Wonju National College. His research interests are in the areas of communications theory, communication systems, and signal processing for digital wireless systems.



Chang Wahn Yu received the BS and MS degrees in electrical engineering from Ajou University in 1996 and 1998, respectively. He joined ETRI in 1999 and is currently a Senior Member of the Research Staff in Mobile Telecommunication Research Division. He is engaged in research for WiBro communication systems.

Youn Ok Park has lead the wireless broadband (WiBro) modem development activities within ETRI since 2003. He manages the HPi modem algorithm and architecture including channel coding and WiBro standards. From February 1987 to December 1997, he served as a senior researcher on the NAIS Project and developed three kinds of middle range super computers, TiCOM II, TiCOMIII, and TiCOM IV. From July 1997 to December 1998, he served as a senior researcher on base station modem development project using the Smart Antenna System. From January 1999 to December 1999, he developed a synchronous wide band CDMA systems mobile station modem. From January 2000 to December 2001, he developed a mobile station modem, which was used in WCDMA systems compliant to 3GPP. From January 2002 to December 2002, he developed two kinds of Mobile/Base station modems for the 4th Generation Mobile system using OFDM and QAM CDMA. From January 2003 to December 2005, he developed the WiBro modem including base station and mobile station compliant to the standards of TTA PG302 & IEEE802.16e. Before joining ETRI, he had one year of research experience in Samsung Electronics Complex Research Center. He has presented more than 20 technical papers in IEEK and other conferences. He received the BS degree in electronics engineering from Hanyang University, Seoul, Korea in 1986, and the MS degree in computer engineering from Chungnam National University, Daejeon, Korea in 1997, where since 2001 he has been in the information engineering PhD program at Chungnam National University.

Dongweon Yoon received the BS (summa cum laude), MS and PhD degrees in electronic communications engineering from Hanyang University, Seoul, Korea, in 1989, 1992 and 1995. From March 1995 to August 1997, he was an Assistant Professor in the Division of Electronic and Information Engineering of Dongseo University, Pusan, Korea. From September 1997 to February 2004, he was an Associate Professor in the Division of Information and Communications Engineering of Daejeon University, Daejeon, Korea. Since March 2004, he has been on the faculty of Hanyang University, Seoul, Korea, where he is now an Associate Professor in the Department of Electronics and Computer Engineering. He has twice been an Invited Researcher at ETRI, Daejeon, Korea (February to Decmeber 1997, November 2002 to December 2005). He was a Visiting Professor at Pennsylvania State University, University Park, Pennsylvania, for the academic year 2001 to 2002. He has served as a consultant for a number of companies and given many lectures on the topics of digital communications and wireless communications. His research interests include new modulation techniques, accurate performance evaluations, digital communications theory and system, spread spectrum communications, wireless communications, and ad-hoc networks.

Sang Kyu Park received the BS degree from Seoul National University, Korea, in 1974, the MS degree from Duke University, USA, in 1980, and the PhD degree from the University of Michigan, USA, in 1987, all in electrical engineering. From July 1976 to October 1978, he was a Research Engineer at the Agency for Defense Development, Korea. From August 1990 to August 1991, he was a Visiting Scholar at the University of Southern California, USA. Since March 1987, he has been with the Department of Electronics and Computer Engineering at Hanyang University, Korea, where he is currently a Professor. His research interests are in the areas of communications theory, wireless communications, mobile communications, spread spectrum communications, and secure communications.