

Greenberger-Horne-Zeilinger nonlocality in arbitrary even dimensions

Jinhyoung Lee,^{1,2} Seung-Woo Lee,² and M. S. Kim³

¹*Department of Physics, Hanyang University, Seoul 133-791, Korea*

²*Quantum Photonic Science Research Center, Hanyang University, Seoul 133-791, Korea*

³*School of Mathematics and Physics, The Queen's University, Belfast, BT7 1NN, United Kingdom*

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We generalize Greenberger-Horne-Zeilinger (GHZ) nonlocality to every even-dimensional and odd-partite system. For the purpose we employ concurrent observables that are incompatible and nevertheless have a common eigenstate. It is remarkable that a tripartite system can exhibit the genuinely high-dimensional GHZ nonlocality.

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I. INTRODUCTION

Quantum nonlocality is one of the most profound virtues inherent in quantum mechanics and it is a fundamental resource for quantum-information processing. Quantum nonlocality is implied by the Bell theorem that quantum mechanics conflicts with any local realistic theories. In a bipartite system, Bell constructed a statistical inequality that all local realistic theories should satisfy but quantum mechanics can violate [1]. Later, Greenberger, Horne, and Zeilinger (GHZ) proved the Bell theorem without any inequalities in a tripartite system [2]. The so-called GHZ state that exhibits the GHZ nonlocality has been employed as a quantum channel for quantum key distribution [3] and quantum secret sharing [4]. For complex tasks of scalable quantum computation and quantum error correction, the nature of multipartite entanglement and its nonlocality test have become important issues.

During the early period, discussions on nonlocality were centered on two-dimensional systems such as spins and polarizations. However, while most physical systems are defined in higher-dimensional Hilbert spaces, only little is known about the higher-dimensional multipartite systems. Bell's inequality has been generalized to an arbitrary-dimensional bipartite system [5]. Very recently, Bell's inequality was discussed for a three-dimensional tripartite system [6]. In order to generalize GHZ nonlocality to an arbitrary even-dimensional system, Żukowski and Kaszlikowski suggested an experiment using optical elements such as multipoint beam splitters and phase shifters [7]. Their work was compensated by Cerf *et al.* with Mermin's formulation, which emphasizes relations between a set of operators [8,9]. The work by both groups requires N subsystems, with $N=d+1$, to exhibit the d -dimensional GHZ nonlocality for d an even integer. GHZ-like nonlocality with statistical expectations was argued for a d -dimensional d -partite system [10]. On the other hand, the original GHZ nonlocality requires only three subsystems in two-dimensional Hilbert space. Here an extremely important question arises. Is there no such nonlocality test without inequalities for a d -dimensional N -partite system where N is independent of d ? We answer this question in this paper.

Żukowski and Kaszlikowski [7] and Cerf *et al.* [8] started their arguments from compatible composite observables which led the discussion to the local complementary observ-

ables. This approach is derived from GHZ's original study. However, we go back to the argument on physical reality by Einstein, Podolsky, and Rosen (EPR) [11]. We will show that this leads to a generalization of GHZ nonlocality which is used for a nonlocality test in a multidimensional multipartite system *without statistical inequalities*.

In this paper, to prove the Bell theorem without inequalities, we employ concurrent observables, which are mutually incompatible but still have their common eigenstate. The concurrent observables are chosen such that the common eigenstate is a "generalized GHZ state." Their local observables are shown to be involved in elements of physical reality according to EPR's criterion [11]. It is proved that our generalization is genuinely multidimensional. We emphasize that this work first shows that a *tripartite* system suffices for the genuinely d -dimensional GHZ nonlocality with d an even integer. We discuss its extension to a multipartite system.

II. CONCURRENT OBSERVABLES AND ELEMENTS OF PHYSICAL REALITY

EPR [11] proposed as a sufficient condition for recognizing an element of physical reality: "If, without in any way disturbing a system, we can predict with certainty the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity." Elements of physical reality accompanied with Einstein's locality play an essential role in nonlocality. They have been investigated by finding compatible observables and their common eigenstate for a given composite system. For instance, in the EPR-Bohm paradox of two spin-1/2 particles [12], Bohm considered the set of commuting observables $\{\hat{\sigma}_a \otimes \hat{\sigma}_a | a=x, y, z\}$ and their simultaneous eigenstate $|\psi\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$. Finding the compatible observables has been regarded as a crucial step to recognize the elements of physical reality. The approach of the compatible observables (for elements of physical reality) can be extended when one faithfully follows EPR's criterion. The extension is one of the key points in generalizing GHZ nonlocality.

For a quantum system of $d (>2)$ dimension, there are some incompatible observables which nevertheless have a common eigenstate [14]. The observables whose common eigenstate is equal to a given system state are called *concur-*

rent observables. The measurement results for the concurrent observables can simultaneously be specified as long as the quantum system is prepared in their common eigenstate. Note that compatible observables are concurrent observables if the quantum system is prepared in one of their common eigenstates. Following EPR's criterion, one can involve concurrent observables (more precisely, their local observables) in elements of physical reality. Here it is crucial that the system state is an eigenstate of the composite observables. For instance, a composite system of subsystems A and B is prepared in a quantum state $|\psi\rangle$ and the two subsystems are separated at a long distance. The state $|\psi\rangle$ is chosen such that it is an eigenstate of a composite observable $\hat{X} \otimes \hat{Y}$: $\hat{X} \otimes \hat{Y}|\psi\rangle = \lambda|\psi\rangle$ where λ is the corresponding eigenvalue. Suppose that the variable X for the subsystem A is measured and its outcome is x (one of the eigenvalues of \hat{X}). One can predict with certainty the value of Y , i.e., $y = \lambda/x$, for the subsystem B . Assuming Einstein's locality, as the two systems are separated by a long distance, the measurement performed on the subsystem A can instantaneously cause no real change in the subsystem B . Thus, the variable Y is predetermined *before the measurement* and it is an element of physical reality according to EPR's criterion. Similarly, the variable X is also an element of physical reality.

It is in general difficult to find all concurrent observables. Instead, a particular set of them is easily found once symmetries are known for a given quantum state. Suppose that a quantum state $|\psi\rangle$ of a given system is an eigenstate of an observable \hat{X} with the eigenvalue λ : $\hat{X}|\psi\rangle = \lambda|\psi\rangle$. Let G be a group of symmetry operations such that each operation $g \in G$ is represented by some unitary operator $\hat{V}(g)$ that leaves the quantum state invariant, i.e., $\hat{V}(g)|\psi\rangle = |\psi\rangle$. Then the quantum state $|\psi\rangle$ is the common eigenstate, with the same eigenvalue λ , of the composite observables $\hat{X}(g) = \hat{V}(g)\hat{X}\hat{V}^\dagger(g)$:

$$\hat{X}(g)|\psi\rangle = \hat{V}(g)\hat{X}\hat{V}^\dagger(g)|\psi\rangle = \lambda|\psi\rangle. \quad (1)$$

Here the form of the unitary operator $\hat{V}(g)$ was not conditioned. However, in order to discuss elements of physical reality, we require that such a unitary operator should be in the form of the tensor product of local unitary operators. For instance, $\hat{V}(g) = \hat{U}_1(g) \otimes \hat{U}_2(g)$ for a bipartite system.

Consider a tripartite system of A , B , and C . Each subsystem is of d dimensions, hence called a qudit. The composite system is assumed to be in a state

$$|\psi\rangle = \frac{1}{\sqrt{d}} \sum_{n=0}^{d-1} |n, n, n\rangle, \quad (2)$$

where $\{|n\rangle\}$ is a complete orthonormal basis set. The state $|\psi\rangle$ is conventionally called a generalized GHZ state. Let us consider a unitary operator in the form of

$$\hat{V} = \hat{U}_1 \otimes \hat{U}_2 \otimes \hat{U}_3, \quad (3)$$

where

$$\hat{U}_\alpha = \sum_{n=0}^{d-1} \omega^{f_\alpha(n)} |n\rangle\langle g(n)|. \quad (4)$$

Here $\omega = \exp(2\pi i/d)$ is a primitive d th root of unity over the complex field and $g(n)$ is a permutation map on the set $D = \{0, 1, \dots, d-1\}$. Note that \hat{U}_α reduces to a phase shift operator if $g(n)$ is an identity map. The unitary operator \hat{V} leaves $|\psi\rangle$ invariant if

$$f_1(n) + f_2(n) + f_3(n) \equiv 0 \pmod{d}, \quad (5)$$

for each $n \in D$. The expression of " $x \equiv y \pmod{d}$ " implies that $x - y$ is congruent to zero modulo d throughout the paper.

III. GENERALIZED GHZ NONLOCALITY

A. Tripartite system

Suppose that three observers, say, Alice, Bob, and Charlie are mutually separated at a long distance and they will perform their measurements on the qudits A , B , and C , respectively. Each observer is allowed to choose one of two variables X and Y . The choice is made by deciding local parameters in each measuring device. Each variable takes, as its value, an element in the set of order d , $S = \{1, \omega, \dots, \omega^{d-1}\}$. The elements of S are the d th roots of unity over the complex field.

In quantum mechanics, an *orthogonal* measurement is described by a complete set of orthonormal basis vectors $\{|n\rangle_p\}$, where the subscript p denotes the set of parameters in the measuring device. Distinguishing the measurement outcomes can be indicated by a set of values, called eigenvalues. As the variable X or Y takes a value of $\omega^n \in S$, let the set of eigenvalues be the set S such that the operator is represented by $\hat{X} = \sum_{n=0}^{d-1} \omega^n |n\rangle_{xx}\langle n|$. Similarly, $\hat{Y} = \sum_{n=0}^{d-1} \omega^n |n\rangle_{yy}\langle n|$. In this representation the "observable" operator \hat{X} or \hat{Y} is unitary [8]. Each measurement described is nondegenerate with all distinct eigenvalues, and hence is called a maximal test [14]. We note that such a unitary representation induces mathematical simplifications without altering any physical results.

Consider the observable operator \hat{X} that we obtain by applying a quantum Fourier transformation \hat{Q} on the reference observable $\hat{Z} = \sum_n \omega^n |n\rangle\langle n|$. For each eigenvalue ω^n , the eigenvector of \hat{X} is thus given by

$$|n\rangle_x = \hat{Q}|n\rangle = \frac{1}{\sqrt{d}} \sum_{m=0}^{d-1} \omega^{nm} |m\rangle. \quad (6)$$

The observable \hat{X} can be represented in terms of the reference basis set $\{|n\rangle\}$ by

$$\hat{X} = \sum_{n=0}^{d-1} |n\rangle\langle n+1|, \quad (7)$$

where we used the convention that $|n\rangle \equiv |n \pmod{d}\rangle$ and thus $|d\rangle \equiv |0\rangle$. The operator \hat{X} performs a periodic shift operation on a basis vector:

$$|n+1\rangle \rightarrow |n\rangle \quad \text{and} \quad |0\rangle \rightarrow |d-1\rangle. \quad (8)$$

Then, the generalized GHZ state $|\psi\rangle$ in Eq. (2) is the eigenstate of the composite observable $\hat{v}_0 = \hat{X} \otimes \hat{X} \otimes \hat{X}$ with the unit eigenvalue as

$$\hat{X} \otimes \hat{X} \otimes \hat{X} |\psi\rangle = \frac{1}{\sqrt{d}} \sum_{n=0}^{d-1} |n-1, n-1, n-1\rangle = |\psi\rangle. \quad (9)$$

We note that \hat{X} has the form of Eq. (4) as \hat{v}_0 is also a symmetry operator for which $|\psi\rangle$ remains invariant.

By using the symmetric operations (3) for the generalized GHZ state $|\psi\rangle$, we can construct other concurrent observables from the composite observable \hat{v}_0 . Such a typical unitary operator $\hat{V}_1 = \hat{U}_1 \otimes \hat{U}_2 \otimes \hat{U}_2$ where \hat{U}_α are given with $g(n) = n$, $f_1(n) = (d-1)n$, and $f_2(n) = n/2$ in Eq. (3). Note that \hat{V}_1 satisfies the condition (5) as $f_1(n) + 2f_2(n) = dn \equiv 0 \pmod{d}$ for all n and it leaves the state $|\psi\rangle$ invariant. The observable obtained is $\hat{v}_1 = \hat{V}_1 \hat{v}_0 \hat{V}_1^\dagger = \omega \hat{X} \otimes \hat{Y} \otimes \hat{Y}$. Here the observable operator \hat{Y} is of the measurement Y . For each eigenvalue ω^n , the eigenvector of \hat{Y} is given by applying a phase shift operation \hat{U}_2 on $|n\rangle_x$:

$$|n\rangle_y = \hat{U}_2 |n\rangle_x = \frac{1}{\sqrt{d}} \sum_{m=0}^{d-1} \omega^{(n+1/2)m} |m\rangle. \quad (10)$$

The operator \hat{Y} can be written similarly to Eq. (7) by

$$\hat{Y} = \omega^{-1/2} \left(\sum_{n=0}^{d-2} |n\rangle \langle n+1| - |d-1\rangle \langle 0| \right). \quad (11)$$

Contrary to \hat{X} , the operator \hat{Y} performs an antiperiodic shift operation with a phase shift $\omega^{-1/2}$:

$$|n+1\rangle \rightarrow \omega^{-1/2} |n\rangle \quad \text{and} \quad |0\rangle \rightarrow -\omega^{-1/2} |d-1\rangle. \quad (12)$$

In a similar manner, we obtain the other two concurrent observables $\hat{v}_2 = \omega \hat{Y} \otimes \hat{X} \otimes \hat{Y}$ and $\hat{v}_3 = \omega \hat{Y} \otimes \hat{Y} \otimes \hat{X}$ by applying $\hat{V}_2 = \hat{U}_2 \otimes \hat{U}_1 \otimes \hat{U}_2$ and $\hat{V}_3 = \hat{U}_2 \otimes \hat{U}_2 \otimes \hat{U}_1$, respectively. The three observables \hat{v}_i obtained respectively satisfy

$$\begin{aligned} \hat{X} \otimes \hat{Y} \otimes \hat{Y} |\psi\rangle &= \omega^{-1} |\psi\rangle, \\ \hat{Y} \otimes \hat{X} \otimes \hat{Y} |\psi\rangle &= \omega^{-1} |\psi\rangle, \\ \hat{Y} \otimes \hat{Y} \otimes \hat{X} |\psi\rangle &= \omega^{-1} |\psi\rangle. \end{aligned} \quad (13)$$

We note that the four concurrent observables \hat{v}_i have a common eigenstate of $|\psi\rangle$, even though they are mutually incompatible, i.e., $[\hat{v}_i, \hat{v}_j] \neq 0$ for $i \neq j$.

Quantum mechanics allows that the concurrent observables \hat{v}_i can simultaneously be specified as long as the composite system is prepared in the generalized GHZ state $|\psi\rangle$, as they satisfy Eqs. (9) and (13). In other words, all the composite measurements \hat{v}_i collapse the state $|\psi\rangle$ to itself and the order of the measurements does not affect the result.

Nevertheless, the value of the local variable X or Y for each qudit is revealed only by actually performing the measurement.

On the other hand, the local realistic description implies that the local variables X and Y are elements of physical reality and the values of the local variables X and Y are predetermined before the measurements, contrary to the quantum-mechanical description. We then attempt to assign values to the variables X_α and Y_α for each qudit α . This attempt converts Eqs. (13) to the algebraic equations that the predetermined variables X_α and Y_α must obey

$$\begin{aligned} X_A Y_B Y_C &= \omega^{-1}, \\ Y_A X_B Y_C &= \omega^{-1}, \\ Y_A Y_B X_C &= \omega^{-1}. \end{aligned} \quad (14)$$

By definition $X_\alpha = \omega^{x_\alpha}$ and $Y_\alpha = \omega^{y_\alpha}$, where x_α and y_α are integers, and the above equations can be rewritten in the simpler form of

$$\begin{aligned} x_A + y_B + y_C &\equiv -1 \pmod{d}, \\ y_A + x_B + y_C &\equiv -1 \pmod{d}, \\ y_A + y_B + x_C &\equiv -1 \pmod{d}. \end{aligned} \quad (15)$$

Summing these equations results in the ‘‘local realistic condition’’

$$x_A + x_B + x_C \equiv -2(y_A + y_B + y_C) - 3 \pmod{d}. \quad (16)$$

For an even integer d , the right-hand side of Eq. (16) is always an odd integer modulo d for arbitrary y_α . In other words, for even d , there exist no integer solutions of $y = y_A + y_B + y_C$ to the equation $2y + 3 \equiv 0 \pmod{d}$. This is in contradiction to the quantum expectation, from Eq. (9),

$$x_A + x_B + x_C \equiv 0 \pmod{d}. \quad (17)$$

Thus we prove the Bell theorem without statistical inequalities for an arbitrary even-dimensional tripartite system. For $d=2$, in particular, the observables \hat{X} and \hat{Y} respectively reduce $\hat{\sigma}_x$ and $\hat{\sigma}_y$ with $\omega = -1$ and the nonlocality is equivalent to that originally proposed by GHZ [2].

B. Genuine multidimensionality

One may try to extend the GHZ nonlocality in two dimensions [2] to higher dimensions by employing anticommuting observables [13]. However, such an extension is a *de facto* two-dimensional nonlocality [8]. It is because two anticommuting observables can always be represented by a direct sum of two-dimensional observables.

To confirm that the generalized GHZ nonlocality is genuinely d dimensional, we prove that it is impossible to represent the observables \hat{X} and \hat{Y} by a direct sum of any subdimensional observables. Suppose that the observable operator \hat{X} were block-diagonalizable by some similarity transformation \hat{S} such that $\hat{S}^{-1} \hat{X} \hat{S} = \hat{X}_1 \oplus \cdots \oplus \hat{X}_N$. Suppose further that \hat{S}

could simultaneously block-diagonalize the observable operator \hat{Y} : $\hat{S}^{-1}\hat{Y}\hat{S}=\hat{Y}_1\oplus\cdots\oplus\hat{Y}_N$. Here \hat{X}_i and \hat{Y}_i are observables of d_i dimensions with $\sum_i d_i=d$. Then, it should hold that $\text{Tr}\hat{X}_i\hat{Y}_j=0$ for $i\neq j$. In other words, there should exist some eigenvectors $|n\rangle_x$ of \hat{X} and $|m\rangle_y$ of \hat{Y} such that $0=\langle n|\hat{S}\hat{S}^{-1}|m\rangle_y=\langle n|m\rangle_y$. However, no such eigenvectors can exist because for every n and m

$$|\langle n|m\rangle_y|^2=\frac{1}{d^2\sin^2[(\pi/d)(m-n+1/2)]}>0. \quad (18)$$

Therefore the generalized GHZ nonlocality is genuinely d dimensional.

C. Extension to multipartite systems

The tripartite GHZ nonlocality can be extended to a general N -partite and d -dimensional system where N is an odd integer and d an even integer. This extension requires a set of $(N+1)$ concurrent observables, which includes $\hat{X}^{\otimes N}$, $\hat{X}\otimes\hat{Y}^{\otimes(N-1)}$, and its permutations, i.e.,

$$\begin{aligned} \hat{v}_0 &= \hat{X} \otimes \hat{X} \otimes \hat{X} \otimes \cdots \otimes \hat{X}, \\ \hat{v}_1 &= \hat{X} \otimes \hat{Y} \otimes \hat{Y} \otimes \cdots \otimes \hat{Y}, \\ \hat{v}_2 &= \hat{Y} \otimes \hat{X} \otimes \hat{Y} \otimes \cdots \otimes \hat{Y}, \\ \hat{v}_3 &= \hat{Y} \otimes \hat{Y} \otimes \hat{X} \otimes \cdots \otimes \hat{Y}, \\ &\vdots \\ \hat{v}_N &= \hat{Y} \otimes \hat{Y} \otimes \hat{Y} \otimes \cdots \otimes \hat{X}. \end{aligned} \quad (19)$$

Here \hat{X} is given in Eq. (7), while \hat{Y} is modified by generalizing the local unitary operator \hat{U}_2 with $f_2(n)=n/(N-1)$ from $f_2(n)=n/2$:

$$\hat{Y}=\omega^{-1/(N-1)}\left(\sum_{n=0}^{d-2}|n\rangle\langle n+1|+\omega^{d/(N-1)}|d-1\rangle\langle 0|\right). \quad (20)$$

The N -partite generalized GHZ state

$$|\psi\rangle=\frac{1}{\sqrt{d}}\sum_{n=0}^{d-1}|n,n,\dots,n\rangle \quad (21)$$

is a common eigenstate of all the concurrent observables with the eigenvalues 1 for \hat{v}_0 and ω^{-1} for the others \hat{v}_i . Following the same argument as for the tripartite GHZ nonlocality, we obtain the local realistic condition

$$\sum_{\alpha=1}^N x_\alpha \equiv -(N-1)\sum_{\alpha=1}^N y_\alpha - N \pmod{d}, \quad (22)$$

where x_α and y_α come from the variables $X_\alpha=\omega^{x_\alpha}$ and Y_α

$=\omega^{y_\alpha}$ for each qudit α . For each pair of odd N and even d , this is in contradiction to the quantum expectation, resulting from \hat{v}_0 ,

$$\sum_{\alpha=1}^N x_\alpha \equiv 0 \pmod{d}. \quad (23)$$

The pairs (N,d) of odd N and even d include a particular element of $(d+1,d)$. Our extension thus covers the previous works on d -dimensional $(d+1)$ -partite nonlocality [7,8].

In order to test the generalized GHZ nonlocality, one may consider an optical experiment using multipoint beam splitters and phase shifters, similar to that by Żukowski and Kaszlikowski [7]. It was shown that all unitary operators on a qudit can be implemented by a series of those linear optical devices [15]. Thus, one can implement the local measurement bases for \hat{X} and \hat{Y} by simply placing such optical devices before detectors.

IV. REMARKS

Our formulation of the generalized GHZ nonlocality is different from the conventional approaches. First, it employs concurrent observables instead of compatible observables. Second, it releases the condition of mutual complementarity between the local observables \hat{X} and \hat{Y} [17,16]. If the local observables \hat{X} and \hat{Y} were mutually complementary, their eigenvectors would satisfy

$$|\langle n|m\rangle_y|^2=\frac{1}{d}. \quad (24)$$

This is not the case as shown in Eq. (18). These differences enable a tripartite system to suffice for the higher-than-two-dimensional GHZ nonlocality, contrary to the previous works that demand a $(d+1)$ -partite system [7,8]. This work will encourage further study of the nonlocality for more general systems.

In summary, we presented a genuinely multidimensional and multipartite GHZ nonlocality. The proof of nonlocality was based on concurrent observables that are incompatible but still have a common eigenstate of the generalized GHZ state.

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