Stepped θ-QAM for high-order modulation

Jaeyoon Lee*, Heejin Hong* and Dongweon Yoon✉

M-ary stepped θ-QAM (θ-QAM) for $M = 2^{2l}$, $l \geq 3$ based on square QAM is proposed. The signal constellation of stepped θ-QAM and the coordinates of the symbols generalised for modulation order M are presented. Then analyse the symbol error rate (SER) performance, and the optimum value of θ in terms of minimum SER for stepped θ-QAM in additive white Gaussian noise and fading channels. Through computer simulations, the theoretical results are validated and show that stepped θ-QAM offers better error performance than θ-QAM.

Introduction: It is well known that cross QAM (CQAM) based on rectangular QAM (RQAM) has lower peak-to-average power ratio and better symbol error performance than RQAM [1]. Recently, θ-QAM based on CQAM was proposed, and the error performance and the optimum constellation for θ-QAM were analysed in additive white Gaussian noise (AWGN) and fading channels [2, 3]. It was shown that the symbol error rate (SER) of θ-QAM, when θ is 60°, is lower than that of CQAM. These results imply that the construction process of signal constellation, i.e. RQAM → CQAM → θ-QAM, can guarantee better transmission power efficiency in a communication system.

In this Letter, we turn our attention to the construction of signal constellation based on SQAM and propose stepped θ-QAM. We first apply the signal constellation construction method of CQAM from RQAM presented in [1], moving the last columns of the signal points at each far left and far right to the top and bottom rows, to SQAM. The constructed signal constellation has a step shape rather than a cross shape; hence, it is called stepped QAM [4]. We then construct the signal constellation, hereafter, we call it stepped θ-QAM, with a triangular lattice structure, by applying angle θ as in [2] to stepped QAM. We analyse the exact SER, and the optimum θ in terms of minimum SER of stepped QAM in AWGN and fading channels, and through computer simulations, we validate the theoretical results.

Stepped θ-QAM: M-ary stepped θ-QAM for $M = 2^{2l}$, $l \geq 3$ is constructed by applying a parameter θ to a step-shaped signal constellation, stepped QAM, constructed by moving the upper $2^{2l}/32$ signal points and lower $2^{2l}/32$ signal points of $2^l/8$ columns on the far left and far right, to the top and bottom rows in M-ary SQAM. Fig. 1 shows the signal constellation construction method of M-ary stepped θ-QAM when $M = 64$, as an example. First, signal points in the far left and far right columns are moved: the two topmost signal points and the two lowest signal points (shown as blank circles in Fig. 1*a*) of each end column are moved to the top and bottom rows such that the signal constellation becomes step-shaped as shown in Fig. 1*a*. Subsequently, by applying a parameter θ to the step-shaped signal constellation, stepped θ-QAM is constructed as shown in Fig. 1*b*.

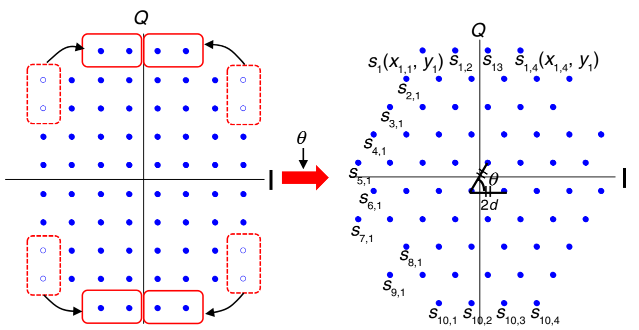

Fig. 1 *Construction of 64-ary stepped θ-QAM signal constellation*

a Stepped QAM
b Stepped θ-QAM

Extending this construction method to a higher-order modulation, the signal constellations of 256- and 1024-ary stepped θ-QAM can be formed as shown in Fig. 2.

The nth signal point on the mth row, $s_{m,n}$, in the signal constellation is denoted by a coordinate pair $(x_{m,n}, y_m)$, as shown in Fig. 1*b*, to represent each signal point of stepped θ-QAM, where $x_{m,n}$ is an in-phase value of the nth signal point on the mth row and y_m is a quadrature value of the mth row. By generalising the coordinate according to modulation order M, $(x_{m,n}, y_m)$ for $M = 2^{2l}$, $l \geq 3$ can be expressed as

$$(x_{m,n}, y_m) = \Big(\Big[2(n-1) + 1 - \sqrt{M}\Big]d + [2 \bmod (m, 2) - 1]\alpha, \; -\operatorname{sgn}\big(\log_2 M - 7\big) \Big[2(m-1) + 1 - \frac{5}{4}\sqrt{M}\Big]\beta\Big) \quad (1)$$

where $m = 1, 2, 3, \ldots, 5\sqrt{M}/4$, $n = 1 + c\sqrt{M}/8, 2 + c\sqrt{M}/8, 3 + c\sqrt{M}/8, \ldots, \sqrt{M} - c\sqrt{M}/8$, and $c = \lfloor 2|-2m + 5\sqrt{M}/4 + 1|/\sqrt{M} \rfloor$; $\alpha = 2d\cos\theta$; $\beta = 2d\sin\theta$; mod(·) denotes the modulo operator; sgn(a) denotes the signum function, and $2d$ is the Euclidean distance between two adjacent signal points.

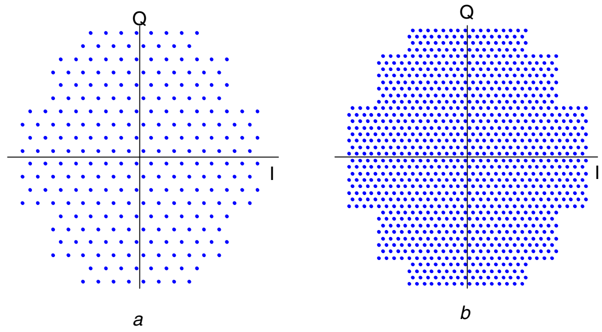

Fig. 2 *M-ary stepped θ-QAM signal constellations*

a $M = 256$
b $M = 1024$

By using (1), the average energy per symbol E_s of stepped θ-QAM can straightforwardly be calculated as follows:

$$\begin{aligned} E_{\text{avg}} &= \frac{1}{M}\sum_{i=1}^{M}\left(x_i^2 + y_i^2\right) \\ &= \frac{95}{384}M^2d^2 + \frac{1}{6}Md^2 - Md^2\cos\theta + Md^2\cos^2\theta + \frac{3}{32}M^2d^2\sin^2\theta \\ &\quad - \frac{1}{8}Md^2\sin^2\theta + \frac{125}{3}2^{(3\sqrt{M}/8)-1}\sqrt{M}d^2\sin^2\theta - \frac{5}{3}2^{(\sqrt{M}/8)-1} \\ &\quad \sqrt{M}d^2\sin^2\theta \end{aligned} \quad (2)$$

Table 1 shows the average energy per symbol of SQAM, θ-QAM and stepped θ-QAM when $\theta = 60°$. From Table 1, we can see that stepped θ-QAM has lower average symbol energy than SQAM and θ-QAM.

Table 1: Comparison of average symbol energy

Modulation type	Modulation order			
	64	256	1024	4096
SQAM	$42\ d^2$	$170\ d^2$	$682\ d^2$	$2730\ d^2$
θ-QAM (when $\theta = 60°$)	$37\ d^2$	$149\ d^2$	$597\ d^2$	$2389\ d^2$
Stepped θ-QAM (when $\theta = 60°$)	$35.6\ d^2$	$143.5\ d^2$	$575\ d^2$	$2301\ d^2$

Numerical and simulation results: A closed-form expression of the SER for M-ary stepped θ-QAM can be obtained for a given M, but unfortunately a generalised closed-form expression of the SER for M cannot be derived. This is because as M increases, the decision regions with new polygonal shapes appear. In this section, we analyse the SER of M-ary stepped θ-QAM using the method presented in [3], where the exact SER is calculated by the error probability of each decision region of the closed region (eq. (6) in [3]) and the open region (eq. (7) in [3]). As an example, for $M = 64$, the SER of 64-ary stepped θ-QAM can be obtained by

$$\begin{aligned}P_{\text{SER\_64-ary}} =& \frac{1}{64}\sum_{i=1}^{6}\{N_i(1-P_{R_i})\}\\ =&1-a_1Q\left(-k,-k;\frac{1}{2}\right)-a_2Q\left(k,k;\frac{1}{2}\right)\\ &+a_3Q\left(-k,k;-\frac{1}{2}\right)-a_4Q\left(k,k;-\frac{1}{2}\right)\\ &+a_5Q\left(-k,\sqrt{3}k;-\frac{\sqrt{3}}{2}\right)-a_5Q\left(-\sqrt{3}k,-k;\frac{\sqrt{3}}{2}\right)\\ &+a_5Q\left(-k,\sqrt{3}k;-\frac{1}{2}\right)\end{aligned} \quad (3)$$

where $k=\sqrt{8\gamma_s/285}$ and $\gamma_s=E_s/N_0$ denotes the symbol energy-to-noise spectral density ratio; the coefficients are $a_1=15/16$, $a_2=9/16$, $a_3=99/32$, $a_4=1/8$, and $a_5=1/16$. Analogously, the SER of M-ary stepped θ-QAM can be obtained for other values of M.

Fig. 3 shows the SER of M-ary SQAM, θ-QAM, and stepped θ-QAM for $M=2^{2l}$ $l=3, 4, 5, 6$ when $\theta=60°$ in AWGN channel. As shown in Fig. 3, stepped θ-QAM has a lower SER than SQAM and θ-QAM. This is because stepped θ-QAM has lower average energy per symbol than SQAM and θ-QAM, as can be seen in Table 1.

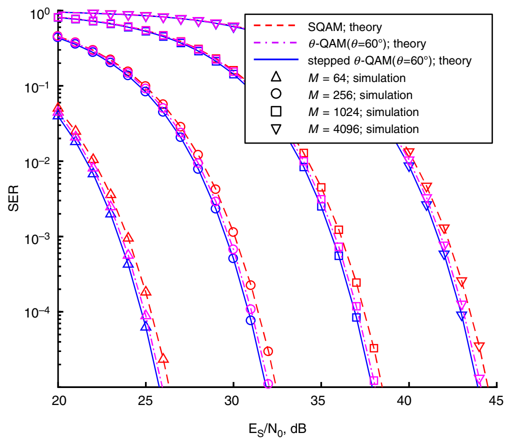

Fig. 3 *SER of SQAM, -QAM and stepped -QAM in AWGN channel*

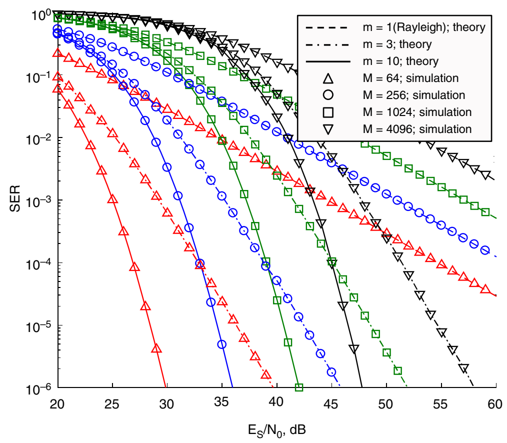

Fig. 4 *SER of stepped θ-QAM in Nakagami-m fading channel when $\theta=60°$*

The SER expression for M-ary stepped θ-QAM, e.g. the SER expression of (3) for 64-ary stepped θ-QAM, in AWGN channel, can be straightforwardly extended to Nakagami-m fading channels [3]. Fig. 4 shows the SER of M-ary stepped θ-QAM in a Nakagami-m fading channel when $m=1$, 3, and 10, where there is a perfect match between the theoretical and simulation results.

We depict the optimum angles of stepped θ-QAM in terms of minimum SER for various values of E_s/N_0 in Fig. 5. Fig. 5*a* shows the optimum angles of M-ary stepped θ-QAM in AWGN channel. From Fig. 5*a* we see that as E_s/N_0 increases, the optimum angle increases and converges to 60° regardless of the modulation order M. This is because, when the optimum angle is 60° ($\theta=60°$), the average energy per symbol, for a fixed Euclidean distance 2d between two adjacent signal points, is the minimum. Fig. 5*b* depicts the optimum angles of 64-ary stepped θ-QAM in terms of minimum SER according to E_s/N_0 and fading severity m in Nakagami-m fading channels. From Fig. 5*b*, we can see that, in Nakagami-m fading channels (except for $m=\infty$) the optimum angle is 60° ($\theta=60°$) for low E_s/N_0, whereas it varies with fading severity m for high E_s/N_0. Notably, the optimum angle tends to gradually converge to 60° regardless of m as E_s/N_0 increases.

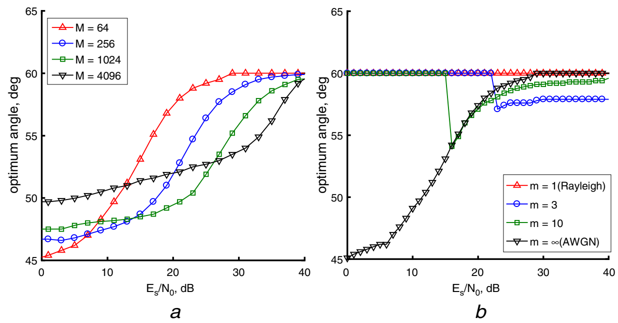

Fig. 5 *Optimum angles*
a M-ary stepped θ-QAM in AWGN channel
b 64-ary stepped θ-QAM in various fading channels

Conclusion: We proposed M-ary stepped θ-QAM based on M-ary SQAM for $M=2^{2l}$, $l\geq 3$, and presented the signal constellation and the coordinates of the symbols generalised for the modulation order M. We then analysed SER and the optimum values of θ in terms of minimum SER in AWGN and fading channels.

The optimum angle varies with SNR. For AWGN channel, as SNR increases, the optimum angle converges to 60°. In Nakagami-m fading channels (except for $m=\infty$) the optimum angle is 60° for low SNR, whereas it varies with fading severity m for high SNR and tends to gradually converge to 60° regardless of m as SNR ever increases.

The proposed M-ary stepped θ-QAM is a most promising high-order modulation scheme to transmit very large amounts of data in wireless communication and broadcasting systems. Future work is required to analyse the optimum bit mapping and bit error performance of stepped θ-QAM.

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One or more of the Figures in this Letter are available in colour online.

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References

1 Vitthaladevuni, P.K., Alouini, M.S., and Kieffer, J.C.: 'Exact BER computation for cross QAM constellations', *IEEE Trans. Wirel. Commun.*, 2005, **4**, (6), pp. 3039–3050
2 Pappi, K.N., Lioumpas, A.S., and Karagiannidis, G.K.: 'θ-QAM: a parametric quadrature amplitude modulation family and its performance in AWGN and fading channels', *IEEE Trans. Commun.*, 2010, **58**, (4), pp. 1014–1019
3 Lee, J., Yoon, D., and Cho, K.: 'Error performance analysis of M-ary θ-QAM', *IEEE Trans. Veh. Technol.*, 2012, **61**, (3), pp. 1423–1427
4 Awadalla, A., Wu, K.T., and Yevick, D.: 'Comparison between square and stepped-square QAM for optical transmission'. Optical Fiber Communication Conf., California, USA, 2015