What determines the Lerner index? The proper interpretation of inverse elasticity rule

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Abstract

Inverse elasticity rule is one of the best known equations in economics, but its interpretations are often erroneous and lacking. They posit that the price is set closer to the cost when the demand is more elastic with regard to price. Such interpretation, by focusing only on demand elasticity, ignores other important determinants of price-cost margin (i.e. Lerner index). It also fails to identify and distinguish the different effects of many features of demand and cost. This paper addresses these problems by showing that Lerner index is affected by the cost side as much as by the demand side of the market. The two sides interact with each other to determine Lerner index together. The important features are the (relative) heights of the demand and the marginal cost curves as well as their (relative) slopes. These results lead to more direct prediction of Lerner index and proper interpretation of the inverse elasticity rule.

Keywords: inverse elasticity rule, Lerner index, price discrimination, Ramsey pricing

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1 Introduction

One of the most famous equations in microeconomics is so called 'inverse elasticity rule', which maintains that the margin between price and marginal cost in a monopoly equilibrium is equal to the inverse of the price elasticity of the demand. That is,

$$\frac{p-MC}{p} = \frac{1}{\varepsilon} \quad (1)$$

where p and MC are the monopoly price and the marginal cost, respectively. ε is the price elasticity of demand. The left-hand side of the equation measures how far the monopoly market outcome deviates from that of the competitive market and is known as Lerner index. A competitive market obtains $p = MC$ at the market as well as at the individual firm level, which guarantees efficient resource allocation. So the more p diverges from MC, the farther is the monopoly outcome from the competitive outcome. The inverse elasticity rule means that the Lerner index is equal to the inverse of the price elasticity of demand.

The inverse elasticity rule also applies to third degree price discrimination, requiring that the same equation holds in each of the markets that the monopolist faces:

$$\frac{p_i-MC}{p_i} = \frac{1}{\varepsilon_i} \quad (2)$$

where the subscript i is for each market. Since production is pooled together, marginal cost is common to all markets and hence has no subscript. This extension is relatively simple and straightforward, but one of its variations has gained importance in its own right in the area of monopoly regulation. When a monopolist serving multiple markets is regulated to have zero total profit, what is the most efficient way to set the prices in the different markets? The answer to this question is another famous equation known as 'Ramsey pricing'. It states

$$\frac{p_i-MC_i}{p_i} = \alpha\frac{1}{\varepsilon_i} \quad (3)$$

where α is constant across different markets. Since the monopolist here may or may not have different marginal cost across different markets, equation (3) has subscript i for marginal cost but otherwise is almost the same as equation (2) except for the parameter α. Since α is identical across different markets, the Lerner index is again inversely related to the demand elasticity.

All these equations are derived from the first order conditions of the optimization problems. Inverse elasticity rule is to maximize monopoly profit, and Ramsey pricing is to maximize the social surplus subject to the zero profit constraint. However, what made them famous is their economic interpretation or rather 'misinterpretation'. Most economists, from undergraduate students to Ph.Ds, are used to the interpretation, "The more price sensitive the demand is, the lower is the monopoly price set relative to the marginal cost." This interpretation is frequently followed by more kind explanation that, when the demand is more elastic, the monopolist is afraid of sharp decline in demand when it increases the price. So they refrain from setting the price too high. This explanation is so simple and at the same time intuitively appealing that it earned wide recognition in economics. It even appears in many best-selling economics textbooks. For instance, an industrial organization textbook by Belleflamme and Peitz explicitly specifies the direction of causality from demand elasticity to Lerner index. Its "Lesson 2.3" reads, "A profit maximizing monopolist increases its markup as demand becomes less price elastic."[1] H. Varian's *Microeconomic Analysis* presents the inverse elasticity results for third degree price discrimination (i.e., Equation (2) in the above) with the following explanation. "Hence, the market with the more elastic demand – the market that is more price sensitive – is charged the lower price."[2]

However, these interpretations of the inverse elasticity rule are misleading and sometimes erroneous. It gives the impression that price elasticity is an exogenous feature of the demand, and that this demand side characteristic alone determines the Lerner index. But, as any student of economics knows, demand elasticity is not exogenous but is itself determined endogenously depending on where on the demand curve the equilibrium is obtained. Though some demand curves have constant price elasticity, they are

[1] Belleflamme, P, and M. Peitz (2010), p 27.
[2] Varian, H. (1992), p 249.

exceptions.[3] For any demand curve that has vertical and horizontal intercepts, the price elasticity can be anywhere from infinity to zero depending on where the elasticity is measured. Generally, the elasticity is high toward the upper-left end of the demand curve and is low toward the lower-right end. This means that, instead of the monopoly price being determined by the price elasticity, the price and the price elasticity are co-determined as a result of the monopolist's choice of a point upon the demand curve that maximizes the profit.

One of the negative effects of the 'demand-centric' interpretation of the inverse elasticity rule is that it distracts attention from other determinants of Lerner index. For example, almost no mention is made on the roles the cost side plays in determining Lerner index. It is true that demand conditions determine Lerner index if supply conditions (i.e., cost conditions) are equal. But this does not justify the general statement that demand conditions determine Lerner index. Representing demand conditions with only price elasticity is not desirable either. As mentioned before, price elasticity is an endogenous variable determined by many factors such as the price, the quantity, and the slope of the demand curve. So it is not straightforward to determine which demand is more or less price elastic. It is better to describe demand conditions with easily identifiable exogenous features such as the height and the slope of the demand curve.

This paper addresses these problems by showing that Lerner index is affected by the features of the cost side as much as by those of the demand side of the market. The two sides interact with each other to determine Lerner index together. The important features are the (relative) heights of the demand and the marginal cost curves as well as their (relative) slopes. These results shed more light on so-far under-appreciated determinants of Lerner index and leads to more accurate interpretation of the inverse elasticity rule. It should be, "As a result of the interaction of the demand and the cost sides, when the final monopoly solution is obtained on the demand curve where the demand elasticity high, the Lerner index is low." The same interpretation applies to Ramsey pricing.

This paper is organized as follows. Section 2 investigates the determinants of Lerner index in a monopoly market without price discrimination, and section 3 extends the discussion into third degree price discrimination.

[3] Demand functions with constant price elasticity of ε have the specific form of $Q = Ap^{-\varepsilon}$.

Section 4 deals with Ramsey pricing. Section 5 concludes the paper.

2 Monopoly without Price Discrimination

Throughout this paper, we assume that both demand curve and marginal cost curve are linear. This is primarily for the sake of simplicity but also has the advantage of defining the features of demand and cost conditions by two distinct parameters: their heights (i.e., their vertical intercepts) and slopes. So the demand side is represented by $P = a - bQ$, and the marginal cost is represented by $MC = c + dQ$. When necessary, the coefficients are marked with subscripts for different markets.

2.1 Case I: Constant Marginal Cost

We start with the simplest case where the monopolist does not engage in price discrimination and the marginal cost is constant: $MC = c$ $(c \leq a)$. The first order condition for profit maximization is that marginal revenue($a - 2bQ$) equals marginal cost(c), and the equilibrium quantity and price are $Q = \frac{a-c}{2b}$ and $p = \frac{a+c}{2}$, respectively. Then, the Lerner index(*LI*) is,

$$LI = \frac{a-c}{a+c} = \frac{\frac{a}{c}-1}{\frac{a}{c}+1} \tag{4}$$

Equation (4) immediately leads to the following proposition without the need for proof.

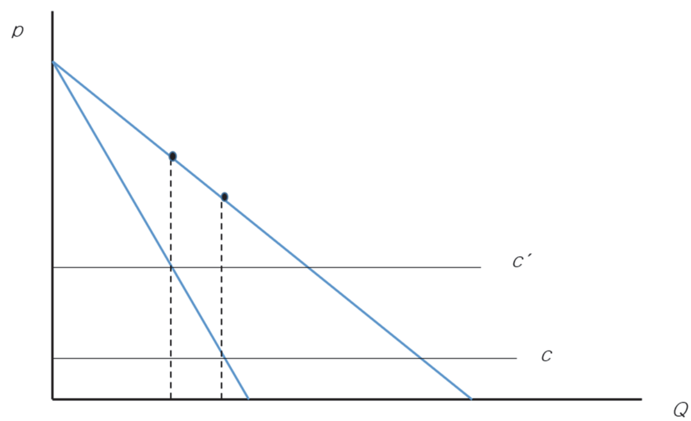
Proposition 1

When the marginal cost is constant (c), the Lerner index(*LI*) is solely determined by the ratio of the vertical intercepts of the demand curve and the marginal cost (i.e., $\frac{a}{c}$). It increases with $\frac{a}{c}$. As $\frac{a}{c}$ increases from its minimum value of 1(i.e., $c = a$) to its maximum of ∞, the Lerner index increases from zero to 1.

It is noteworthy that the slope of the demand curve does not have any role in determining the Lerner index. Instead, it is how high the marginal cost is relative to the starting price of demand that determines the index. When the marginal cost is relatively high, the monopoly price is set low relative to the marginal cost. Figure 1 provides the intuition behind this result. When the marginal cost is relatively high(c′), it pushes the equilibrium point up along the demand curve toward the upper end where the price elasticity is high and the gap between price and marginal revenue curve is narrow. Hence *LI* is low. When marginal cost is constant, its relative height alone determines how close the equilibrium is obtained toward the upper end or the lower end. The slope of the demand curve does not matter at all.

Considering that the slope is usually associated with the sensitivity of demand to price, this outcome seems to totally negate the relevance of price sensitivity in Lerner index. However, demand elasticity is also affected by the height of demand curve. In a linear demand curve, the higher the vertical intercept is, the lower is the price elasticity 'measured at a certain price level' regardless of the slope of the demand curve. So the price sensitivity still matters. Our result clarifies which aspect of the demand plays the key role.

Figure 1. Monopoly solution with constant marginal cost



2.2 Case 2. Increasing Marginal Cost

Now we allow the marginal cost curve to have a slope d: $MC = c + dQ$. The first order condition for profit maximization is $a - 2bQ = c + dQ$, and the equilibrium quantity and price for the monopolist are $Q = \frac{a-c}{2b+d}$ and $p = \frac{ab+ad+bc}{2b+d}$, respectively. Again, it takes simple arithmetic to show

$$LI = \frac{ab-bc}{a(b+d)+bc} = \frac{\frac{a}{c}-1}{\frac{a}{c}(1+\frac{d}{b})+1} \quad (5)$$

Equations (4) and (5) are almost the same except that the ratio of the slopes of the marginal cost curve and the demand curve ($\frac{d}{b}$) appears in the denominator. So, the slopes of the two curves matters as well as their starting heights. Now it is two ratios, $\frac{a}{c}$ and $\frac{d}{b}$, that jointly determine *LI*.

Proposition 2
When the demand curve and the marginal cost curve are $P = a - bQ$ and $MC = c + dQ$, respectively, *LI* increases with $\frac{a}{c}$. As $\frac{a}{c}$ increases from 1 (i.e., $c = a$) to ∞, *LI* increases from 0 to $\frac{b}{b+d}$.

Proof: The derivative of *LI* in Equation (5) with respect to $\frac{a}{c}$ is $\frac{2+\frac{d}{b}}{\left\{\frac{a}{c}(1+\frac{d}{b})+1\right\}^2}$.
Since b and d are both positive, the derivative is positive and so *LI* increases with $\frac{a}{c}$. For the second part of the proposition, $\lim_{\frac{a}{c}\to\infty} \frac{\frac{a}{c}-1}{\frac{a}{c}(1+\frac{d}{b})+1} = \lim_{\frac{c}{a}\to 0} \frac{1-\frac{c}{a}}{(1+\frac{d}{b})+\frac{c}{a}} = \frac{1}{1+\frac{d}{b}} = \frac{b}{b+d}$. *QED*

Proposition 2 is simple extension of *Proposition 1*. The only difference is that *LI* does not reach the maximum value of 1 even when $\mathrm{c} = 0$ as long as $d > 0$. This is because marginal cost becomes positive when $q > 0$ thanks to the positive slope d. The higher the marginal cost starts, the higher up goes the equilibrium point along the demand curve and the smaller *LI* becomes.

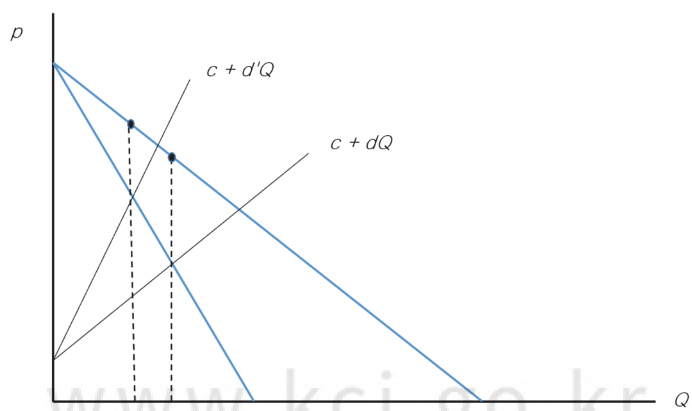
Proposition 3
When the demand curve and the marginal cost curve are $P = a - bQ$

and $MC = c + dQ$, respectively, LI decrease with $\frac{d}{b}$. As $\frac{d}{b}$ increases from 0 to ∞, LI decreases from $\frac{a-c}{a+c}$ to zero.
Proof: The denominator of LI in Equation (5) increases with $\frac{d}{b}$, and so LI itself decreases with $\frac{d}{b}$. When $\frac{d}{b}$ goes to ∞, so does the denominator. And LI goes to 0. QED

Here, the slope of the demand curve matters, but so does the slope of the marginal cost curve. It is the 'ratio' of the two slopes that matters, and not the slope of either alone. When marginal cost curve is steep relative to demand curve, LI is low. Figure 2 shows an example in which the difference in the slope of the marginal cost curve causes difference in LI even with the same demand. When the marginal cost is steeper relative to the demand curve(d'), the cost increases faster than the demand price decreases with the quantity. So the equilibrium point on the demand curve is obtained closer to the vertical intercept where the price elasticity is high.

Propositions 2 and 3 help us interpret the inverse elasticity rule properly. LI is lower when the monopoly solution is obtained on the demand curve where price elasticity is higher. Since price elasticity is higher toward the upper end (i.e., the vertical intercept where the elasticity goes to ∞) and lower toward the lower end (i.e., the horizontal intercept, where the elasticity is 0), LI is low when marginal cost reaches marginal revenue soon as q increases. This happens when marginal cost starts high and increases fast relative to the demand curve.

Figure 2. Monopoly solution with increasing marginal cost



2.3 Does High Lerner Index Mean Higher Monopoly Profit?

Another misunderstanding of Lerner index is about its relationship with monopoly profit. When Lerner index is higher, is it always better for the monopolist? Since Lerner index is price-cost margin, it might seem that the answer to the question is yes. But this is not the case.

When the demand curve and the marginal cost curve are $P = a - bQ$ and $MC = c + dQ$ and assuming no fixed cost, the monopoly profit is

$$\pi = \frac{(a-c)^2}{4b+2d} . \tag{6}$$

Equation (6) is the area of the triangle shaped by the vertical axis, the marginal revenue curve and the marginal cost curve. Since the area under the marginal revenue curve is the total revenue, and the area under the marginal cost curve is the total variable cost, their difference is the monopoly profit (before the fixed cost). $(a - c)$ is the base of this triangle and $Q = \frac{a-c}{2b+d}$ is the height. So their product determines the area of the triangle, and hence, the monopoly profit.

Now let's examine how the determinants of *LI* and the monopoly profits are related. $(a - c)$ is positively related with both *LI* and monopoly profit, and *d* is negatively related with both. When demand price starts high relative to the cost, the price-cost margin is high and so is the monopoly profit. When the marginal cost increases slowly, equilibrium is obtained toward the lower end of the demand curve so that both *LI* and the quantity *Q* are high. The monopoly profit also increases. So far, the results are consistent with the 'yes' answer to the above question.

However, the slope of the demand curve *b* has opposite effects on *LI* and the monopoly profit. When the demand curve is steep, demand curve drops fast. This pushes the equilibrium point down toward the lower end of the demand curve and so increases *LI*. But this time, *Q* is small and so is the monopoly profit. This result contradicts the interpretation that insensitive demand is advantageous to the monopolist because the monopolist can charge high price relative to the cost.

In short, the most profitable market and cost conditions for a monopolist are, 1) demand price that starts high and comes down slowly with the quantity and 2) marginal cost that starts low and rises slowly with the quantity. With these two conditions, the gap between marginal revenue and marginal cost starts wide and narrows down very slowly with quantity, increasing the area of the triangle shaped by them.

3 Monopoly with Third Degree Price Discrimination

In this section, the monopolist can engage in third degree price discrimination. We assume two separate markets, market 1 and market 2, which are represented by demand curves $P_1 = a_1 - b_1Q_1$ and $P_2 = a_2 - b_2Q_2$, respectively. Production is not separate between the two markets, and so there is one marginal cost curve, $MC = c + dQ$.

It is well known that the first order condition for profit maximization is that the marginal revenues of both markets should be equal to the common marginal cost. This can be summarized as $MR_1 = MR_2 = MC$, in which the first equation ensures that the total production is optimally allocated between the two markets, and the second equation ensures that the total production is set to maximize the profit. Since $MR_i = p_i(1 - \frac{1}{\varepsilon_i})$, the first order conditions can be rearranged into inverse elasticity rules.

$$\frac{p_i - MC}{p_i} = \frac{1}{\varepsilon_i} \quad (i = 1, 2). \tag{7}$$

Since the marginal cost is common between the two markets, Equation (7) implies that, in equilibrium, the market with the higher price is the market with the lower price elasticity. This leads to the interpretation mentioned in the introduction that the market with more elastic demand – the market that is more price sensitive – is charged lower price. Since cost is common among different markets, it is obvious that the price differences should be affected only by differences in demand attributes. However, this interpretation fails to point out which exogenous attributes of demand matters. Neither does it explain what determines the average level of Lerner index.

To identify the true factors that determine the price difference between the two markets, we proceed in the following order. We first identify the factors that determined the overall production level and the corresponding marginal cost. And then we examine how this common marginal cost is differently reflected to the price in each market. Figure 3 shows this process in one go. The overall marginal revenue curve(MR_T) of the monopolist firm is derived by adding the two marginal revenue curves horizontally as shown in panel (C). The intersection of this overall marginal revenue curve and the common marginal cost curve determines the optimal total production Q^*. Let's denote the marginal cost at Q^* as MC^*. Then the optimal allocation of Q^* between the two markets is determined as shown in panels (A) and (B). In each market, optimal quantity Q_i^* is obtained by $MR_i = MC^*$, and the price is determined as the height of the demand curve at Q_i^*. The two prices at markets 1, 2 are generally different, and hence arises price discrimination. The important question is what determines their difference.

Proposition 4

In third degree price discrimination, the difference between the prices of the two markets is determined by the difference of the intercepts of the two demand curves. More specifically,

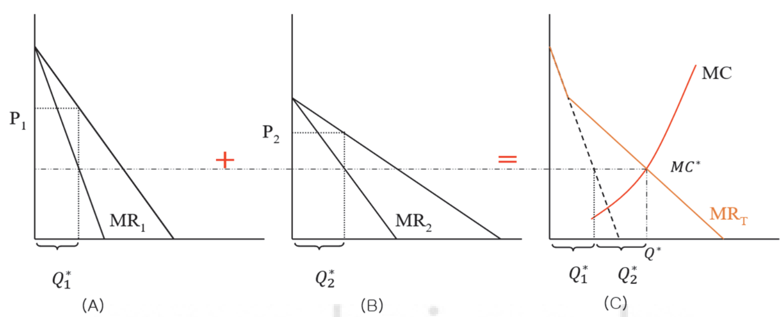
$$P_1 - P_2 = \frac{a_1 - a_2}{2} \tag{8}$$

Proof: Since the overall production Q^* and the corresponding marginal price MC^* are already determined at the entire firm level, it is as if marginal cost is constant at MC^* when we determine the optimal price of each market. Then, from the results of Section 2.1, we know

$$P_1 = \frac{a_1 + MC^*}{2}, \ P_2 = \frac{a_2 + MC^*}{2}. \tag{9}$$

So the difference between the two prices is as in Equation (8). *QED*

Figure 3. Monopoly with 3rd degree price discrimination



Notice that the slopes of the demand curves do not have any role in determining the price difference between the two markets regardless of the slope of the marginal cost curve. It is only the intercepts of the demand curves that determine which market ends up with higher price. This result seems to contradict the price-sensitivity interpretation of price discrimination if we associate price sensitivity only with the slope of demand curve. However, as mentioned in Section 2.1, the height (i.e. the vertical intercept) of the demand curve also affects price elasticity. In that sense, the price sensitivity interpretation is still correct, but it fails to pinpoint the very factor of demand that determines the price differences.

This result may help us explain the global pricing behavior of some monopolists. For instance, the companies selling luxury items, such as premium handbags, have strong monopoly brand power and can practice price discrimination among different markets of the whole world. They often set the prices higher in developing countries like China than in developed countries like the U.S. This pattern of pricing may seem irrational considering that the overall purchasing power toward luxury goods in rich countries should be greater than in developing countries. However, according to *Proposition* 4, it is not the overall demand of the market but how much the consumers at the top end of the market are willing to pay that determines which country is charged higher prices. If the new rich in the developing countries are more eager to show off their wealth with conspicuous consumption, they may be more willing to pay higher prices for luxury goods. Hence the higher monopoly price.

Note that *Proposition* 4 is only about the price difference and not about the price level, i.e., how high the average prices are compared with marginal cost. In other words, it just shows how prices are set differently once the overall marginal cost level MC^* is given. As can be seen from panels (A) and (B), once MC^* is set, setting price in each individual market is reduced to the problem of profit maximization with constant marginal cost. When the constant marginal cost is high relative to the intercept of the demand curve, price is set where price elasticity is high and Lerner index is low. The question of what determines the level of MC^* is addressed at the overall firm level shown in panel (C). MC^* is determined by the intersection of the monopolist's overall marginal revenue curve MR_T and marginal cost curve. Here the problem is similar to the problem of a single market monopolist with increasing marginal cost curve. When the overall marginal cost starts high relative to the overall marginal revenue curve, and when the marginal cost curve rises fast relative to the rate at which the overall marginal revenue declines, the marginal cost curve meets the overall marginal revenue curve higher up close to the upper end of MR_T curve. The resulting MC^* is set high relative to the two demands curves underlying the overall marginal revenue curve. Here, the slopes of the demand curves play a role. When the demand curves are relatively flat, the overall marginal revenue curve also becomes flat so that MC^* is set high. Notice that the slope of each demand curve affects final outcome only through its effect on the overall marginal revenue curve, which in turn affects the price level of both markets. This is why the slopes of demand curves do not affect the price difference between the two markets.

Going back to the luxury market example, the price in the U.S. market may be lower than in China since its highest willingness to pay is relatively low. However, the big increase in the U.S. demand as the price comes down helps the overall marginal revenue remain high as sales quantity increases, resulting in higher total production level and higher marginal cost. The high marginal cost pushes up the prices in both markets. Also as discussed in Section 2.3, although the price is higher in China, the high volume of sales from the U.S. may have greater contribution to the total profit.

4 Ramsey Pricing

Ramsey pricing is a solution to a different problem from monopoly profit maximization but the end result is another inverse elasticity rule very similar to the solution of third degree price discrimination. When regulating a monopoly with the marginal cost lower than the average cost, marginal cost pricing cannot guarantee full recovery of the cost. Then how should the regulator set the prices to maximize the social surplus while incurring neither profit nor loss for the monopolist? Especially when the monopolist can divide the entire market into multiple submarkets, how should it set the prices in those markets? Ramsey pricing is the answer to this question.

Let $P_i(Q_i)$ denote the demand curve of market i. Then the problem is set as,

$$\max_{Q_i} \sum_{i=1}^{m} \int_0^{Q_i} P_i(x_i)dx_i - C\left(\sum_{i=1}^{m} Q_i\right) \tag{10}$$

$$s.t. \ \sum_{i=1}^{m} P_i(Q_i) \cdot Q_i - C\left(\sum_{i=1}^{m} Q_i\right) = 0.$$

The objective is to maximize the social surplus, and the constraint is zero profit for the monopolist. Note that, as mentioned in the introduction, cost doesn't have to be the function of total production but can have a more general form of $C(Q_1, Q_2, Q_3, \ \dots \ Q_m)$. However, since our main interest is in the comparison of the prices across different submarkets, we focus on the case where the cost is common across submarkets.

The first order conditions are

$$P_i - MC + \lambda\left(P_i + \frac{dP_i}{dQ_i} - MC\right) = 0 \tag{11}$$

where λ is the Lagrange multiplier and MC is the common marginal cost. Substituting $\varepsilon_i = -\frac{dQ_i}{dP_i}\frac{P_i}{Q_i}$ and rearranging, we get

$$\frac{P_i - MC}{P_i} = \frac{\lambda}{1+\lambda}\frac{1}{\varepsilon_i}\ . \tag{12}$$

When the zero-profit constraint is not binding, $\lambda = 0$ and Equation (12) simply becomes the competitive solution of $P_i = MC$, i.e., marginal cost pricing. When the constraint is binding ($\lambda > 0$), however, Equation (12) becomes a kind of inverse elasticity rule similar to solution of the third degree price discrimination. Since $\frac{\lambda}{1+\lambda} < 1$ when $\lambda > 0$, the price-marginal cost margin is smaller than $\frac{1}{\varepsilon_i}$. But it is still proportional to $\frac{1}{\varepsilon_i}$ since λ is common across different markets. Equation (12) is called Ramsey pricing.

The inverse elasticity result of Ramsey pricing again leads to ambiguous interpretation in the same way as with the monopoly solutions. That is, "The regulator should set the price low in the market with high price elasticity of demand, and vice versa." This interpretation was often applied to implementation of regulation of actual monopoly industries. For example, when an electricity market is under monopoly of a large utility company, the rates are regulated to ensure exact recovery of the total cost. The demand for electricity can be separated into multiple subgroups such as residential demand, commercial demand, and industrial demand. Ramsey pricing was widely accepted as a guiding principle for setting electricity rates for these different submarkets. And more often than not, it is interpreted as dictating that the rate should be set higher in the market with lower price elasticity and vice versa. Again, price elasticity of demand is not exogenously given but is determined together with other endogenous variables such as the price and the quantity. When the solution is obtained on the demand curve where price elasticity is high, then the price is set low relative to the marginal cost.[4]

To find out how the prices are set differently according to Ramsey pricing, we proceed with an example of two submarkets, markets 1 and 2, with linear demand curves $P_1 = a_1 - b_1Q_1$ and $P_2 = a_2 - b_2Q_2$, respectively. We also assume that the overall marginal cost is constant at c. Then the first order conditions become

$$a_i - b_iQ_i - c + \lambda(a_i - 2b_iQ_i - c) = 0 \ (i = 1,2) \quad (13)$$

Equation (13) can be solved for the optimal quantities, $Q_i^* = \frac{a_i - c}{2b_i} \cdot \frac{2+2\lambda}{1+2\lambda}$ (i = 1,2). Compared with the unrestricted monopolist solution of $Q_i^m = \frac{a_i - c}{2b_i}$, Q_i^* is greater than Q_i^m by the same proportion of $\frac{2+2\lambda}{1+2\lambda} > 1$. When the zero profit constraint is not binding so that $\lambda = 0$, $\frac{2+2\lambda}{1+2\lambda}$ obtains its maximum value of 2 and Q_i^* becomes $\frac{a_i - c}{b_i}$, which is the competitive market outcome of $P_i = MC$. As the constraint becomes more binding, λ increases and $\frac{2+2\lambda}{1+2\lambda}$ approaches the minimum value of 1. Then Q_i^* approaches the monopoly solution of Q_i^m. Substituting Q_i^* in the demand equation, we get

$$P_i^* = a_i - \frac{a_i - c}{2}\frac{2+2\lambda}{1+2\lambda} \ (\mathrm{i} = 1,2). \quad (14)$$

From Equation (14) follows the next proposition without the need for proof.

Proposition 5

In Ramsey pricing, the difference between the prices of the two markets

[4] For an example of the role that Ramsey pricing principle plays in the discussion of electricity rates, see S. Bigerna and C. Bollino (2016), pp 603-612.

is determined by the difference of the intercepts of the two demand curves, a_1 and a_2, and the strength of the zero profit constraint, λ. More specifically,

$$P_1 - P_2 = (a_1 - a_2)\frac{\lambda}{1+2\lambda}\ . \qquad (15)$$

Proposition 5 is similar to *Proposition* 4 in third degree price discrimination, but there is a crucial difference. While the price difference in the latter was solely determined by the difference of the two intercepts, in Ramsey pricing λ plays the role of reducing the difference as the zero profit constraint becomes less binding. When λ is zero, there is no price difference because all the prices converge to the marginal cost. With $\lambda > 0$, the prices diverge from the marginal cost and from each other. But the only feature of the demand curves that affects the difference is the intercepts, or the starting prices of the demand. Here again, as with third price discrimination, the slopes of the demand curves play no part in determining the price difference.

5 Concluding Remarks

As much as Lerner index is recognized as a measure of deviation of market performance from competitive outcome, it is important to have proper understanding of its determinants. Thereby we can predict under what market conditions a monopoly problem becomes more serious. Current explanations, based on the inverse elasticity rule, have been both erroneous and lacking.

This paper starts with more rigorous interpretation of the inverse elasticity rule, emphasizing the fact that price elasticity of demand is not an exogenous feature of demand but endogenously determined in the final equilibrium. Then it shows the true exogenous market conditions that determine the Lerner index. Especially the paper establishes that the cost side of the market has as much influence on Lerner index as the demand side, a fact that has been mostly neglected so far. It also shows which demand aspects play key roles.

It is possible that price elasticity is exogenous and so it alone determines Lerner index when price elasticity is constant. However, such demand curves have very special shape of a hyperbola with neither vertical nor horizontal intercept. Demand curves with vertical and horizontal intercepts are much more frequently used in academic discussions and classroom teachings. So at least for the sake of consistency, we need to provide proper interpretation of the inverse elasticity rule for such demand curves. This also points to the need for empirical exploration of the overall shapes of real world demand curves beyond point estimation of price elasticity at current price and quantity.

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