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Research Article

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Higher-order circular intuitionistic fuzzy time series forecasting methodology: Application of stock change index

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Abstract: This article presents a higher-order circular intuitionistic fuzzy time series forecasting method for predicting the stock change index, which is shown to be an improvement over traditional time series forecasting methods. The method is based on the principles of circular intuitionistic fuzzy set theory. It uses both positive and negative membership values and a circular radius to handle uncertainty and imprecision in the data. The circularity of the time series is also taken into consideration, leading to more accurate and robust forecasts. The higher-order forecasting capability of this method provides more comprehensive predictions compared to previous methods. One of the key challenges we face when using the amount featured as a case study in our article to project the future value of ratings is the influence of the stock market index. Through rigorous experiments and comparison with traditional time series forecasting methods, the results of the study demonstrate that the proposed higher-order circular intuitionistic fuzzy time series forecasting method is a superior approach for predicting the stock change index.

Keywords: fuzzy set, circular intuitionistic fuzzy sets, score function, higher-order time series forecasting

MSC 2020: 03B52, 90B50

1 Introduction

Decision-making can be defined as the process of selecting the best option from a collection of options based on a range of factors to achieve organizational goals [1]. The main research area in the complex decision-making problem of today, with several goals or circumstances, is multi-criteria decision making (MCDM). A number of MCDM techniques, such as printer selection [2], solar power plant [3], and others, have been developed to address decisions involving a range of conflicting criteria in ambiguous situations. These classic MCDM algorithms cannot handle ambiguous or inaccurate language judgments since precise numerical values are required. To better capture this ambiguity, intuitionistic fuzzy sets, neutrosophic sets, and spherical fuzzy sets

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(SFSs) [4] have been added to standard fuzzy sets in MCDM techniques. In this article, a proposed method is used to forecast the data, which will be helpful for MCDM in the future. Fuzzy set theory is one of the most impressive tools for addressing the problems of handling accurate and sufficient data for real-world decision-making due to the confusion of statistics. Zadeh created a fuzzy set to support her in decision-making when faced with challenges in daily life [5]. There is only one element with a membership degree that varies from 0 to 1 for each domain set component x in a fuzzy set. Fuzzy sets are subject to a number of limitations, such as the inability to display nonmembership. The intuitionistic fuzzy set (IFS), however, was created by Atanassov [6] to comprehend general information regarding membership degree. He defined the IFS using two lists, membership degree $V(p)$ and nonmembership degree $M(p)$, with the requirement that $0 \leq V(p) + M(p) \leq 1$. The IFS has been studied by several researchers and effectively used in a variety of real-world situations [7]. An interval-valued intuitionistic fuzzy set was analyzed by Nayagam et al. [8] following IFS. They are both a fuzzy set and an IFS tweak. They are commonly employed in a variety of decision-making circumstances [9,10].

Because they deal with situations that comprise particular attributes in which the sum of their membership and nonmembership degrees is greater than one, according to several real-life choice concepts. IFS would not be able to produce an appropriate outcome in this situation. The Pythagorean fuzzy set (PyFS) was recommended by Yager [11] as a solution to these problems. In a PyFS, the degree of membership of an element is determined by the Pythagorean theorem, which relates the length of the sides of a right triangle to the length of its hypotenuse. This allows for a more nuanced representation of uncertainty than traditional fuzzy sets, which only allow for binary membership values of either 0 or 1. We now want to determine the neutral membership independently. Cuong and Kreinovich [12] expressed the concept of a picture fuzzy set (PFS). He used three items with the limitation of PFS membership degree $V(p)$, neutral membership degree $K(p)$, and nonmembership degree $M(p)$, and $0 \leq V(p) + K(p) + M(p) \leq 1$. Weighted averaging operations for picture fuzzy sets were invented by Garg [13]. In the past, some of the researchers have looked into and used the PFS in numerous beneficial decision-making fields [14]. In practice, we encounter a variety of problems that PFS cannot solve, such as when $V(p) + K(p) + M(p) \geq 1$. Under these circumstances, PFS is unable to provide a good outcome. Ashraf et al. [15] suggest the SFSs, a variant of the PFS, based on these factors. The use of SFSs can lead to improved results in decision-making problems, as they allow for a more precise and accurate representation of uncertainty. Some authors [16] used SFS on t-conorms and t-norms. With the healthcare diagnostics [17], COVID-19 [18], and many more, SFS has become one of the most valuable fuzzy sets for decision-making. Ullah et al. [19] used this methodology in T-spherical fuzzy sets (T-SFSs) to solve a number of decision-making problems. A T-SFSs is a type of fuzzy set that extends the concept of SFS to handle uncertainty in higher-dimensional spaces. The implications of circular intuitionistic fuzzy sets (C-IFSs) in this article are the reason for the presence of a circular radius, which is not present in IFS.

Making predictions about upcoming events or patterns based on historical data and knowledge is the process of predicting. It is used in a variety of fields, including finance, economics, marketing, and weather prediction, among others. To solve problems that changed over time, the time series data technique was used. Song and Chissom's definition of time series in fuzzy time series [20] outlines the notion of fuzzy time series data or its techniques. Using fuzzy sets, Song and Chissom [21,22] projected the data. Following this, Kumar and Gangwar modify the Song and Chissom's method [23]. To estimate data in fuzzy theory, many scholars employ diverse strategies. The majority of them made use of IFSs. Kumar and Gangwar [24] and Joshi and Kumar [25] used IFSs to include the amount of uncertainty in fuzzy logical links and develop only a few temporal forecasting models [26]. Jiang et al. [27] derived the formula used to predict wind speed. Prior to this, the majority of the researchers used his proposed approach using data from the University of Alabama [28,29]. They compared the mistakes of each result with one another after determining the conclusion to determine the best planned technique.

There are several different definitions of fuzzy forecasting that have been proposed by researchers. Some of these include:

(1) Linguistic Forecasting: This definition views fuzzy forecasting as a method that uses linguistic variables to express uncertainty in the forecast. Linguistic variables are variables that can take on values from a predefined set of linguistic terms, such as “high,” “medium,” and “low” [30].

(2) Probabilistic Forecasting: This definition views fuzzy forecasting as a method that uses fuzzy sets to express the probability of events. Fuzzy sets are sets that allow for partial membership, meaning that an element can belong to a set to a certain degree [31].
(3) Interval Forecasting: This definition views fuzzy forecasting as a method that uses fuzzy intervals to express the uncertainty in the forecast. Fuzzy intervals are intervals that allow for partial membership, meaning that an element can belong to an interval to a certain degree [32].
(4) Hybrid Forecasting: This definition views fuzzy forecasting as a method that combines different forecasting techniques, such as regression analysis and time series analysis, with fuzzy set theory [33].

Cakir et al. [34] expanded on this notion in the C-IFS by including circles with centers $(j_A(x), \ell_A(x))$ rather than points. A circle with a radius of $0 \le r \le 1$ and a center coordinate of $(j_A(x), \ell_A(x))$ is used to represent each C-IFS component. In this C-IFS circle, the total number of membership values is limited to one. In other words, this theory explains the constituents of X inside the intuitionistic fuzzy using circles instead of dots and offers richer models for confused and contradictory information than that of the IFS notion. At $r = 0$ [35], the C-IFS is converted to an IFS. The C-IFS is distinguishable from the normal IFS when $r > 0$. As a complement to the C-IFS notion, decision-makers can create grades as a form of circular membership. Later, additional C-IFS research was carried out, and it was used to address the MCDM concerns [36,37]. Figure 1 shows the geometrical representation of C-IFSs.

Scientists used to make choices by rating and evaluating data. However, they are insufficient to estimate future values. Chen [38] began by using time series analysis to forecast enrollments. Kumar and Gangwar [39] proposed attempting to cope with forecasting by employing induced IFSs. Abhishekh et al. [40] employed this strategy on higher-order IFSs. Furthermore, if we want to check the radius of a circle in IFS, we are unable to find it. As a result, authors are thrilled to be able to meet this need. As a result, we must employ C-IFS to deal with this sort of issue. This is a transition to all predicting algorithms capable of handling any form of membership, and nonmembership includes circular radius. They are useful when we need to calculate the radius of IFS. The question arises: Why do we calculate the radius of any set? The answer is that after finding the radius, we know to check where the values of oversetting lie in this radius, which is helpful in observing our results. Hence, we implicate this technique in this article to solve any type of data for future prediction. IFS cannot handle this type of thing. We worked on filling this with circular intuitionistic fuzzy time series (C-IFTSs), which are useful for time series forecasting, particularly when the total of membership and nonmembership is less than or equal to one with a circular radius. We offer a forecasting strategy for higher-order C-IFTSs that has lower error rates than earlier studies in this work. As part of my proposed strategy, we used the example of the stock change index. The motivation for researching stock change indices is likely to be to better understand the dynamics of financial markets and to provide investors with useful information for making informed investment decisions. Stock change indices track the performance of a group of stocks and can serve

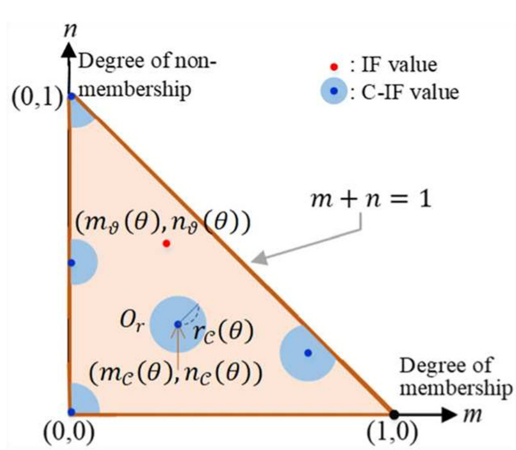

Figure 1: Graphical representation of C-IFSs.

as a barometer for the overall health of a given market or economy. By analyzing changes in these indices over time, researchers can gain insights into market trends, investor sentiment, and other factors that drive stock prices. In addition, the results of such research can inform investment strategies and help investors make more informed decisions about buying, holding, or selling stocks. This idea may be applied to a range of potential future predictions.

The rest of this article is organized as follows:

(1) C-IFS and fuzzy sets are stated as being utilized to connect the other portions of the article.
(2) Following that, we gave some definitions for implementing my proposed technique, as well as a circular intuitionistic membership, nonmembership, and radius number for determining the score value.
(3) Time-variant and time-invariant C-IFTSs concepts were provided.
(4) Following that, we showed a flowchart outlining how we would anticipate data using our suggested technique, as well as specific descriptions of the proposed method.
(5) To evaluate the outcomes, we applied this approach to the stock change index and computed the findings on tables.
(6) After that, we expanded on this work to a higher-order forecasting.
(7) Finally, we presented the article's conclusion.

2 Preliminaries

This section provides concise definitions of fuzzy sets, circular fuzzy sets, and time series analysis. They are important for linking to the following portion of this article.

Definition 2.1. The fuzzy set notion introduced by Zadeh is stated as follows: Let ξ be a set. Fuzzy set ξ in Ψ be characterized:

$$\xi = \{\langle o, \mu_\xi(o)\rangle | \ \forall o \in \Psi\},$$

where $\mu_q(o)$ is a membership function of fuzzy set $\xi, \mu_q(o) : \Psi \to [0, 1]$, and $\mu_\xi(o)$ gives the membership function degree o in ξ.

Definition 2.2. [41] Suppose a nonempty set Ψ. An intuitionistic fuzzy set ξ in Ψ is an attribute having the form $\xi = \{\langle o, \mu_\xi(o), \nu_\xi(o)\rangle; o \in \Psi\}$, where the function, $\mu_\xi(o), \nu_\xi(o) : \to [0, 1]$, respectively, define its membership and nonmembership degree and for every aspect $o \in \Psi$, $0 \le \mu_\xi(o) + \nu_\xi(o) \le 1$.

Definition 2.3. [42] Let Ψ be a universal set. A C-IFS ξ in Ψ is an attribute having the form:

$$\xi = \{\langle o, \mu_\xi(o), \nu_\xi(o); r\rangle | o \in \Psi\},$$

where

$$0 \le \mu_\xi(o) + \nu_\xi(o) \le 1, \tag{1}$$

and $r \in [0, 1]$ is the radius of a circle between each component $o \in \Psi$, which is called C-IFS, and the functions $\mu_\xi : \Psi \to [0, 1]$ and $\nu_\xi : \Psi \to [0, 1]$ represent the degree of membership and nonmembership, respectively, of the element $o \in \Psi$.

The level of uncertainty is calculated as follows:

$$\pi_\xi(o) = 1 - \mu_\xi(o) - \nu_\xi(o). \tag{2}$$

Definition 2.4. [43] The following is a definition of the operations that C-IFS entails. For every $\text{Å}, \ddot{E} \in$ C-IFS(Ψ),

(1) $\text{Å} \subseteq \ddot{E}$ iff $o \in \Psi$, $(\mu_{\text{Å}}(o) \le \mu_{\ddot{E}}(o)$ and $\nu_{\text{Å}}(o) \ge \nu_{\ddot{E}}(o))$;

(2) $Å = \ddot{E}$ iff $Å \subseteq \ddot{E}$ and $\ddot{E} \subseteq Å$;
(3) $Å^c = \{(o, \nu_Å(o), \mu_Å(o))\}$;

$$d(Å, \ddot{E}) = \sqrt{\frac{1}{2k}\sum_{j=1}^{k}(\mu_Å(o_j) - \mu_{\ddot{E}}(o_j))^2 + (\nu_Å(o_j) - \nu_{\ddot{E}}(o_j))^2 + (\pi_Å(o_j) - \pi_{\ddot{E}}(o_j))^2}, \tag{3}$$

$$d(Å, \ddot{E}) = \frac{1}{2}\left(\frac{r_Å - r_{\ddot{E}}}{\sqrt{2}} + \sqrt{\frac{1}{2k}\sum_{j=1}^{k}(\mu_Å(o_j) - \mu_{\ddot{E}}(o_j))^2 + (\nu_Å(o_j) - \nu_{\ddot{E}}(o_j))^2 + (\pi_Å(o_j) - \pi_{\ddot{E}}(o_j))^2}\right). \tag{4}$$

$d(Å, \ddot{E})$ is the standardized shortest distance between Å and $\ddot{E}$.

Definition 2.5. [44] Let $Å = ((\mu_Å(o), \nu_Å(o)); r_Å)$ and $\ddot{E} = ((\mu_{\ddot{E}}(o), \nu_{\ddot{E}}(o)); r_{\ddot{E}})$ be two C-IFS numbers that loop back on themselves. Since they provide the least and greatest volatility, respectively, the operations are based on the lowest and largest radii. A smaller radius indicates less ambiguity in C-IF pairings, while a bigger radius indicates more vagueness. The way they operate is as follows:
(1) $Å \cap_{\min} \ddot{E} = \{o, \min(\mu_Å(o), \nu_{\ddot{E}}(o)), \max(\mu_Å(o), \nu_{\ddot{E}}(o));\ \min(r_Å, r_{\ddot{E}}) | o \in \Psi\}$
(2) $Å \cap_{\max} \ddot{E} = \{o, \min(\mu_Å(o), \nu_{\ddot{E}}(o)), \max(\mu_Å(o), \nu_{\ddot{E}}(o));\ \max(r_Å, r_{\ddot{E}}) | o \in \Psi\}$
(3) $Å \cup_{\min} \ddot{E} = \{o, \max(\mu_Å(o), \nu_{\ddot{E}}(o)), \min(\mu_Å(o), \nu_{\ddot{E}}(o));\ \min(r_Å, r_{\ddot{E}}) | o \in \Psi\}$
(4) $Å \cup_{\min} \ddot{E} = \{o, \max(\mu_Å(o), \nu_{\ddot{E}}(o)), \min(\mu_Å(o), \nu_{\ddot{E}}(o));\ \max(r_Å, r_{\ddot{E}}) | o \in \Psi\}$
(5) $Å \oplus_{\min} \ddot{E} = \{o, \mu_Å(o) + \mu_{\ddot{E}}(o) - \mu_Å(o) \times \mu_{\ddot{E}}(o), \nu_Å(o) \times \nu_{\ddot{E}}(o);\ \min(r_Å, r_{\ddot{E}}) | o \in \Psi\}$
(6) $Å \oplus_{\max} \ddot{E} = \{a, \mu_Å(o) + \mu_{\ddot{E}}(a) - \mu_Å(o) \times \mu_{\ddot{E}}(o), \nu_Å(o) \times \nu_{\ddot{E}}(o);\ \max(r_Å, r_{\ddot{E}}) | o \in \Psi\}$.

Theorem 2.6. *For any $Å_{r_1}, \ddot{E}_{r_2} \in$ C-IFS (Ψ), where $r_1, r_2 \in [0, \sqrt{2}]$, the expressions* (4) *are defined metrics (distance).*

Proof. We must demonstrate that the formulas mentioned in expression (4) abide by the property of metric [45]. As previously demonstrated, the terms indicated in (3) are well-defined distances in IFS(Ψ).

Because it is clear from the definition of C-IFSs, $Å_{r_1} = \ddot{E}_{r_2}$ in C-IFS(Ψ) iff $Å = \ddot{E}$ in IFS(Ψ) and $r_1 = r_2$. But $Å = \ddot{E}$ in IFS(Ψ) iff $D(Å, \ddot{E}) = 0$. The sum of two nonnegative numbers is 0, and if both numbers are equal to 0, the first axiom for a distance is proven.

Because D is symmetric, the validity of the second axiom is clear. To demonstrate the third axiom's validity, let us take a third C-IFS $\ddot{I}_{r_3}$ and show that the triangle property

$$D(Å_{r_1}, \ddot{I}_{r_3}) \le D(Å_{r_1}, \ddot{E}_{r_2}) + D(\ddot{E}_{r_2}, \ddot{I}_{r_3}) \tag{5}$$

holds.

We know that distance without radius.

$$D(Å, \ddot{I}) \le D(Å, \ddot{E}) + D(\ddot{E}, \ddot{I}) \tag{6}$$

holds for the IFSs Å, $\ddot{E}$ and Ï.

From well-known inequality $|x| + |y| \ge |x + y|$ for three real numbers, it follows that

$$|r_3 - r_1| \le |r_2 - r_1| + |r_3 - r_2| \tag{7}$$

for all choices of ,$r_1, r_2, r_3 \in [0, \sqrt{2}]$. As a result, by adding both sides of the final two inequality expressions (6) and (7), the validity of (5), i.e., the third distance axiom, holds true. □

Definition 2.7. If $\vartheta(e)(e = 0, 1, 2, \ldots,)$ is a subset of L and the universe of discourse on which the circular intuitionistic fuzzy sets $f_k(e) = \langle \mu_\xi(o), \nu_\xi(o);\ r\rangle$ $(k = 1, 2, \ldots,)$ are defined, then $F(e) = f_1(o), f_2(o)\ldots$ is a collection of $f_k(e)$ in order to build the C-IFTSs on $\vartheta(e)$ $(e = 0, 1, 2, \ldots)$.

Definition 2.8. If circular intuitionistic logical relationship exist $L(e-1, e)$, then $V(e) = V(e-1) \times L(e-1, e)$ and $V(e)$ is caused by $V(e-1)$ a connection between $V(e)$ and $V(e-1)$ is indicated by $V(e-1) \to V(e)$.

Definition 2.9. Suppose $V(e)$ is caused $V(e-1)$ and symbolize by $V(e-1) \to V(e)$ sequentially, a circular intuitionistic relationship exists, which is between $V(e)$ and $V(e-1)$, which is represented as $V(e) = V(e-1) \times L(e-1, e)$, since L is a first-order model of $V(e)$. Then $V(e)$ is a time-invariant circular intuitionistic time series if $L(e-1, e)$ is independent of time e, $L(e, e-1) = L(e-1, e-2)$ for all e. Likewise, $V(e)$ is called a time-variant circular intuitionistic time series.

Definition 2.10. Suppose $V(e-1) = G_a$ and $(e) = G_b$, a circular intuitionistic logical relationship is described as $G_a \to G_b$, where G_a, G_b are the current and coming state of circular intuitionistic logical relation (C-ILRs). Since $V(e)$ is happened by more than one circular intuitionistic fuzzy set $V(e-n), V(e-n+1), \ldots V(e-1)$, then the circular intuitionistic fuzzy set is represented by $G_{a1}, G_{a2}, \ldots, G_{an} \to G_b$, where $V(e-n) = G_{a1}$, $V(e-n+1) = G_{a2}$. This kind of relationship is called a higher-order circular intuitionistic time series.

3 An algorithm of handling circular intuitionistic time series forecasting

There are three parts to our algorithm (A, B, and C). To cope with situations of this nature in C-IFTSs, we created a strategy. The first portion establishes the C-ILR and its groups; the second section employs the circular intuitionistic forecasting technique to determine the forecasted value of our issue; and the last section informs us of the approach's flaws. Figure 2 shows the flow chart of the proposed technique.

A. First-order C-IFTSs proposed method of forecasting The following will guide you through all the procedures necessary to establish a C-ILR and its groups using the score formula.

Step I: The universe of discourse is determined by time series data to the stated range Ψ. $\Psi = [A_{\min} - A_1, A_{\max} - A_2]$, where $A_{\min}$ and $A_{\max}$ are the smallest and largest data points in the time series data, respectively, and A_1 and A_2 are any reasonable positive values that can manage the data time series in its whole.

Step II: A subset of Ψ divides the discourse universe into intervals of similar length.

Step III: Compute the nth circular intuitionistic fuzzy membership and nonmembership value ρ_v according to the intervals that were erected in Step II.

$$\mu(\varrho;\ [\xi, o, \theta]) = \begin{cases} \dfrac{(\varrho - \xi)}{(o - \xi)} - \varepsilon, & \text{if } \xi < \varrho \leq o \\ \dfrac{(\theta - \varrho)}{(\theta - o)} - \varepsilon, & \text{if } o < \varrho \leq \theta \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

$$\nu(\varrho;\ [\xi, o, \theta]) = 1 - \mu(\varrho;\ [\xi, o, \theta]). \tag{9}$$

Step IV: Calculate the radius of a circular intuitionistic fuzzy set by using equations (10) and (11).

In an IFS N_i, let intuitionistic fuzzy pairs have the form $\{\langle c_{i,1}, d_{i,1}\rangle\langle c_{i,2}, d_{i,2}\rangle, \ldots\}$, where i is the number of IFS N_i each including λ_i. First, the arithmetic average of the intuitionistic fuzzy pairs is calculated as follows:

$$\langle \mu_{(N_i)}, \nu_{(N_i)}\rangle = \left\langle \frac{\Sigma_{j=1}^{\lambda_i} c_{i,j}}{\lambda_i}, \frac{\Sigma_{j=1}^{\lambda_i} d_{i,j}}{\lambda_i} \right\rangle, \tag{10}$$

where λ_i is the number of intuitionistic fuzzy pairs N_i.

The radius of the $\langle \mu_{(N_i)}, \nu_{(N_i)}\rangle$ is the highest of the Euclidian distances.

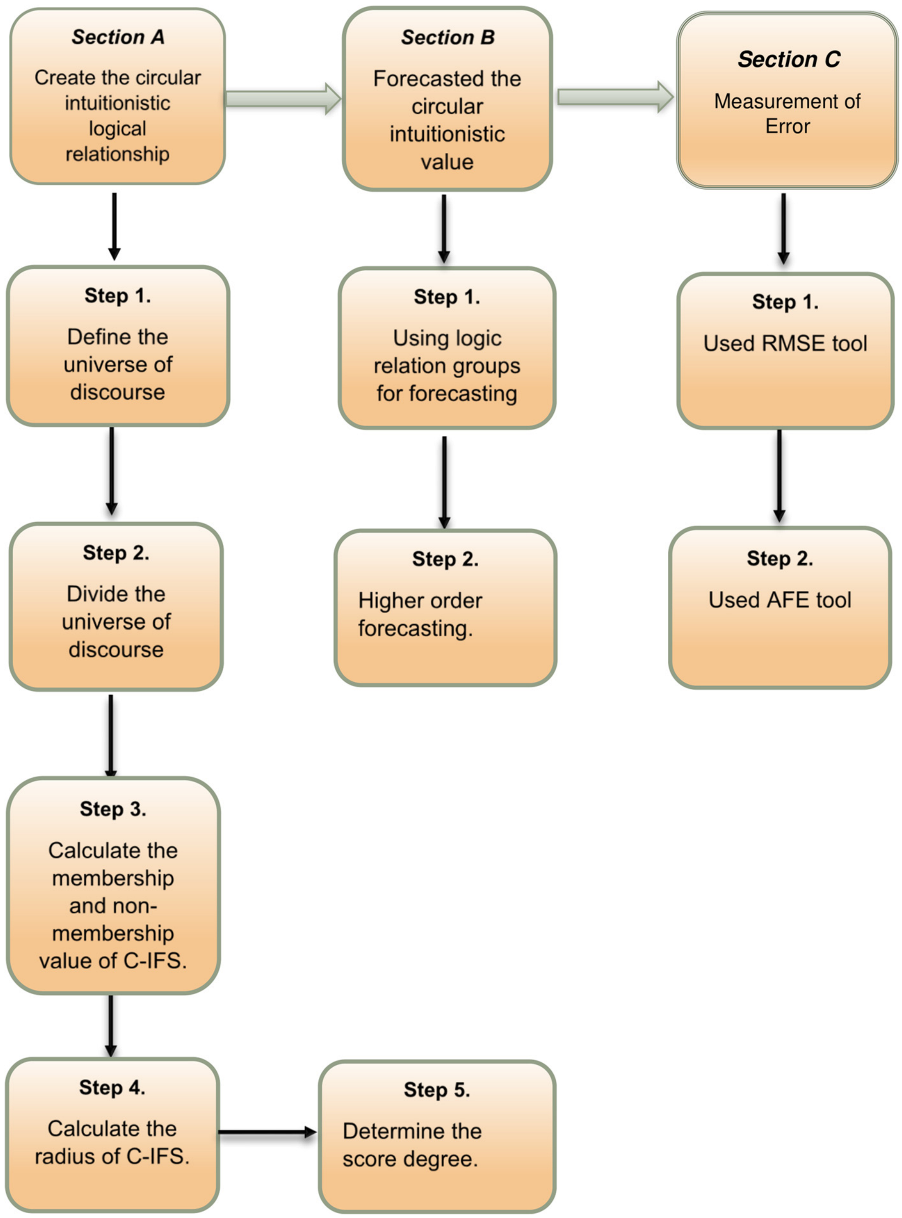

Figure 2: A flowchart that shows how the proposed method works.

$$r_i = \max_{1 \le j \le \lambda_i} \sqrt{(\mu_{(N_i)} - c_{i,j})^2 + (\nu_{(N_i)} - d_{i,j})^2}\,. \tag{11}$$

Step V: To calculate the score degree, utilize the following equation, and then choose the score degree that has the highest possible value [46]:

$$\zeta(s) = \frac{1}{3}(\mu(s) - \nu(s) + \sqrt{2}r(2p - 1)) \quad \text{where } \zeta(s) \in [-1, 1], \tag{12}$$

p is any value between 0 and 1.

Step VI: Construct the circular intuitionistic fuzzy logical relationships (C-IFLRs): If ρ_a is the C-IFS given of year y and ρ_b is the C-IFS coming year of $y + 1$, then the C-IFLRs are represented by $\rho_a \to \rho_b$. Furthermore, ρ_a is called the current state, and ρ_b is the coming next state.

Step VII: Depending on the C-IFLRs, create circular intuitionistic fuzzy logical relationship groups (C-IFLRGs).

B. Measure the forecasted value of circular intuitionistic fuzzy sets

The following is the technique that is used to determine the predicted values:

If circular intuitionistic value of data $\wp_a$ is not formed by any other circular intuitionistic values, then look forward to the C-IFLRG of the same value. If you are unable to determine the value that relies upon $\wp_a$, then the circular intuitionistic value will stay the same (null). If the circular intuitionistic value of data $\wp_a$ is generated by $\wp_b(\wp_b \to \wp_a)$, then see C-IFLRG of $\wp_b$.

- If C-IFLRGs of $\wp_b$ is vacant $(\wp_b \to \wp_b)$, then the value of forecasted is the center one of $\wp_b$.
- If C-IFLRGs of $\wp_b$ is one-to-one $(\wp_b \to \wp_a)$, then the value of forecasted is the center one value of $\wp_a$.
- If C-IFLRGs of $\wp_b$ is not one-to-one $(\wp_b \to \wp_{a1}, \wp_{a2}, \ldots, \wp_{an})$, then value of forecasted is the average of center one value of $\wp_{a1}, \wp_{a2}, \ldots, \wp_{an}$.

C. Measurement of error by using root mean square error (RMSE) and average forecasting error (AFE)

Techniques such as RMSE and AFE are utilized rather frequently to evaluate the precision of time series forecasting. The following definitions are applicable to all types of error measures.

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}(O_i - F_i)^2}{\tau}},$$

$$\text{Forecasting percentage error (fpe)} = \frac{|F_i - O_i|}{O_i} \times 100,$$

$$\text{AFE} = \frac{\sum(\text{fpe})}{\tau}.$$

In this case, the predicted and observed data of the time series are denoted by F_i and O_i, respectively, and τ is the number of observations contained in the time series. When the RMSE or AFE is lower, it indicates that the forecasting method is becoming more accurate.

4 Case study

The values of specific shares are used to derive a stock market index, which is a measurement of the worth of a portion of the stock market. This value is computed based on the index. It is a technique that investors employ to provide a description of the market and to compare the returns generated by various investments. An index provides a fast assessment of a market's condition. Active funds are a reduced method of investing that offers greater returns than most fund managers and aids investors in more consistently achieving their objectives. In this article, we picked the data from the stock change index (Taiwan).

Forecasting the change in the Taiwan stock exchange index can bring several benefits for investors and traders. Some of these benefits include:

- Improved decision-making: By having an accurate forecast, investors and traders can make informed decisions on when to buy or sell their investments, which can lead to better returns.
- Risk management: Accurate forecasting can help investors and traders manage their risk by allowing them to make informed decisions on when to enter or exit the market.
- Increased confidence: With a better understanding of the future progress of the stock change index, investors and traders can have increased confidence in their investment decisions, which can help them make more informed and profitable trades.
- Improved portfolio diversification: By forecasting the stock change index, investors and traders can better understand the potential impact of their investments on their overall portfolio, which can help them diversify their investments for better risk management.

4.1 Implementation of the proposed method of stock change index

It is important to note that stock market forecasts are not always accurate and that past performance does not guarantee future results. Therefore, investors and traders should always use a combination of forecasting tools and fundamental analysis when making investment decisions.

We apply the method using data from the stock index in 2001 and explain the findings step-by-step to make the process of understanding and verifying the suggested model as simple as possible. The following is a list of the steps:

Step I : The universe of discourse Ψ for the stock indexes for the year 2001 is defined as follows:

Universe of discourse $\Psi = [3{,}920, 5{,}600]$. By taking two proper positive numbers, $A_1 = 9.69$ and $A_2 = 48.76$. $A_{\min}$ and $A_{\max}$ are taken from the Table 1.

Step II: The universe of discourse is divided into 14 intervals by Ψ. $\hbar_v = [3{,}920 + (v-1)p,\ 3{,}920 + vp]$, where $v = 1, 2, 3, \ldots, 14$ and $p = 120$.

Step III: Following 14 C-IFTS, $\wp_v (v = 1, 2, 3, \ldots, 12)$ according to the interval $\hbar_v$, which are described in the universe of the discourse Ξ:

$\wp_v = [3{,}920 + (v-1)p,\ 3{,}920 + vp, 3{,}920 + (i+1)p]$ for $v = 1, 2, 3, \ldots, 13$, where $p = 120$.

$\wp_v = [3{,}920 + (v-1)p,\ 3{,}920 + vp, 3{,}920 + ip]$ for $v = 14$ where, $p = 120$.

By using equations (8) and (9), the grades of membership and nonmembership in circular intuitionistic fuzzy sets are calculated as follows: Suppose $\varepsilon = 0.001$.

$\wp_1$ = {(3929.69, 0.08, 0.92), (3998.48, 0.65, 0.35), (4080.51, 0.66, 0.34), (4082.92, 0.64, 0.36), (4158.15, 0.01, 0.99), (4135.03, 0.21, 0.79), (4123.78, 0.30, 0.70), (4136.54, 0.19, 0.81)}

$\wp_2$ = {(4080.51, 0.34, 0.66), (4082.92, 0.36, 0.64), (4158.15, 0.97, 0.03), (4135.03, 0.79, 0.21), (4123.78, 0.70, 0.30), (4172.63, 0.89, 0.11), (4136.54, 0.80, 0.20), (4277.70, 0.02, 0.98)}

$\wp_3$ = {(4172.63, 0.10, 0.90), (4277.70, 0.98, 0.02)}

$\wp_4$ = {(4403.59, 0.97, 0.03), (4446.62, 0.61, 0.39), (4455.80, 0.53, 0.47), (4450.02, 0.58, 0.42), (4159.08, 0.01, 0.99), (4447.58, 0.60, 0.40), (4465.83, 0.45, 0.55), (4441.12, 0.66, 0.34)}

$\wp_5$ = {(4403.59, 0.03, 0.97), (4446.62, 0.39, 0.61), (4548.63, 0.76, 0.53), (4455.80, 0.46, 0.54), (4533.37, 0.89, 0.11), (4450.02, 0.42, 0.58), (4519.08, 0.99, 0.01), (4608.32, 0.26, 0.74), (4580.33, 0.50, 0.50), (4447.58, 0.40, 0.60), (4465.83, 0.55, 0.45), (4441.61, 0.35, 0.65)}

$\wp_6$ = {(4548.63, 0.24, 0.76), (4533.37, 0.11, 0.89), (4608.32, 0.73, 0.27), (4580.33, 0.50, 0.50), (4646.61, 0.94, 0.06)}

Table 1: Fuzzy stock change index

Date	True value	C-Intuitionistic value	Date	True value	C-Intuitionistic value
01-11-2001	3929.69	$\wp_1$	03-12-2001	4646.61	$\wp_6$
02-11-2001	3998.48	$\wp_1$	04-12-2001	4766.43	$\wp_7$
05-11-2001	4080.51	$\wp_1$	05-12-2001	4924.56	$\wp_8$
06-11-2001	4082.92	$\wp_1$	06-12-2001	5208.86	$\wp_{11}$
07-11-2001	4158.15	$\wp_2$	07-12-2001	5333.93	$\wp_{12}$
08-11-2001	4135.03	$\wp_2$	10-12-2001	5321.28	$\wp_{12}$
09-11-2001	4123.78	$\wp_2$	11-12-2001	5273.97	$\wp_{11}$
12-11-2001	4172.63	$\wp_2$	12-12-2001	5539.31	$\wp_{13}$
13-11-2001	4136.54	$\wp_2$	13-12-2001	5407.54	$\wp_{12}$
14-11-2001	4277.70	$\wp_3$	14-12-2001	5486.73	$\wp_{13}$
15-11-2001	4403.59	$\wp_4$	17-12-2001	5456.15	$\wp_{13}$
16-11-2001	4446.62	$\wp_4$	18-12-2001	5329.19	$\wp_{12}$
19-11-2001	4548.63	$\wp_5$	19-12-2001	5221.96	$\wp_{11}$
20-11-2001	4455.80	$\wp_4$	20-12-2001	5309.10	$\wp_{12}$
21-11-2001	4533.37	$\wp_5$	21-12-2001	5109.24	$\wp_{10}$
22-11-2001	4450.02	$\wp_4$	24-12-2001	5164.73	$\wp_{10}$
23-11-2001	4519.08	$\wp_5$	25-12-2001	5372.81	$\wp_{12}$
26-11-2001	4608.32	$\wp_6$	26-12-2001	5392.43	$\wp_{12}$
27-11-2001	4580.33	$\wp_6$	27-12-2001	5332.98	$\wp_{12}$
28-11-2001	4447.58	$\wp_4$	28-12-2001	5398.28	$\wp_{12}$
29-11-2001	4465.83	$\wp_5$	31-12-2001	5551.24	$\wp_{14}$
30-11-2001	4441.12	$\wp_4$			

$\wp_7$ = {(4646.61, 0.05, 0.95), (4766.43, 0.95, 0.05)}
$\wp_8$ = {(4766.43, 0.05, 0.95), (4924.56, 0.63, 0.37)}
$\wp_9$ = {(4924.56, 0.37, 0.63), (5109.24, 0.09, 0.91)}
$\wp_{10}$ = {(5208.86, 0.26, 0.74), (5221.96, 0.15, 0.85), (5109.24, 0.91, 0.09), (5164.73, 0.63, 0.37)}
$\wp_{11}$ = {(5208.86, 0.74, 0.26), (5333.93, 0.22, 0.78), (5329.19, 0.26, 0.74), (5221.96, 0.85, 0.15), (5309.10, 0.42, 0.58), (5164.73, 0.37, 0.63), (5332.98, 0.22, 0.78), (5273.97, 0.72, 0.28), (5321.28, 0.32, 0.68)}
$\wp_{12}$ = {(5333.93, 0.78, 0.22), (5407.54, 0.60, 0.40), (5456.15, 0.20, 0.80), (5329.19, 0.74, 0.26), (5309.10, 0.57, 0.43), (5372.81, 0.89, 0.11), (5392.43, 0.73, 0.27), (5332.98, 0.77, 0.23), (5398.28, 0.68, 0.32), (5321.28, 0.68, 0.32), (5273.97, 0.28, 0.72)}
$\wp_{13}$ = {(5407.54, 0.40, 0.60), (5486.73, 0.94, 0.06), (5456.15, 0.80, 0.20), (5372.81, 0.11, 0.89), (5392.43, 0.27, 0.73), (5398.28, 0.32, 0.68), (5551.24, 0.41, 0.59), (5539.31, 0.50, 0.50)}
$\wp_{14}$ = {(5486.73, 0.06, 0.94), (5551.24, 0.59, 0.41), (5539.31, 0.49, 0.51)}

Step IV: Calculate the radius of circular intuitionistic fuzzy set by using equations (10) and (11).

$\wp_1$ = {(3929.69(0.08, 0.92; 0.37)), (3998.48(0.65, 0.35; 0.44)), (4080.51(0.66, 0.34; 0.45)), (4082.92(0.64, 0.36; 0.42)), (4158.15(0.01, 0.99; 0.47)), (4135.03(0.21, 0.79; 0.19)), (4123.78(0.30, 0.70; 0.06)), (4136.54(0.19, 0.81; 0.21))}
$\wp_2$ = {(4080.51(0.34, 0.66; 0.39)), (4082.92(0.36, 0.64; 0.36)), (4158.15(0.97, 0.03; 0.52)), (4135.03(0.79, 0.21; 0.26)), (4123.78(0.70, 0.30; 0.12)), (4172.63(0.89, 0.11; 0.40)), (4136.54(0.80, 0.20; 0.28)), (4277.70(0.02, 0.98; 0.84))}
$\wp_3$ = {(4172.63(0.10, 0.90; 0.62)), (4277.70(0.98, 0.02, 0.62))}
$\wp_4$ = {(4403.59(0.97, 0.03; 0.59)), (4446.62(0.61, 0.39; 0.08)), (4455.80(0.53, 0.47; 0.02)), (4450.02(0.58, 0.42; 0.04)), (4159.08(0.01, 0.99; 0.77)), (4447.58(0.60, 0.40; 0.07)), (4465.83(0.45, 0.55; 0.14)), (4441.12(0.66, 0.34 : 0.15))}
$\wp_5$ = {(4403.59(0.03, 0.97; 0.65)), (4446.62(0.39, 0.61; 0.14)), (4548.63(0.76, 0.53; 0.26)), (4455.80(0.46, 0.54; 0.04)), (4533.37(0.89, 0.11; 0.57)), (4450.02(0.42, 0.58; 0.10)), (4519.08(0.99, 0.01; 0.71)), (4608.32(0.26, 0.74; 0.32)), (4580.33(0.50, 0.50; 0.02)), (4447.58(0.40, 0.60 : 0.13)), (4465.83(0.55, 0.45; 0.09)), (4441.61(0.35, 0.65; 0.20))}
$\wp_6$ = {(4548.63(0.24, 0.76; 0.38)), (4533.37(0.11, 0.89; 0.56)), (4608.32(0.73, 0.27; 0.32)), (4580.33(0.50, 0.50; 0.01)), (4646.61(0.94, 0.06; 0.62))}
$\wp_7$ = {(4646.61(0.05, 0.95; 0.63)), (4766.43(0.95, 0.05; 0.63))}
$\wp_8$ = {(4766.43(0.05, 0.95; 0.41)), (4924.56(0.63, 0.37; 0.41))}
$\wp_9$ = {(4924.56(0.37, 0.63; 0.20)), (5109.24(0.09, 0.91; 0.20))}
$\wp_{10}$ = {(5208.86(0.26, 0.74; 0.32)), (5221.96(0.15, 0.85; 0.48)), (5109.24(0.91, 0.09; 0.60)), (5164.73(0.63, 0.37; 0.20))}
$\wp_{11}$ = {(5208.86(0.74, 0.26; 0.40)), (5333.93(0.22, 0.78; 0.34)), (5329.19(0.26, 0.74; 0.29)), (5221.96(0.85, 0.15; 0.55)), (5309.10(0.42, 0.58; 0.05)), (5164.73(0.37, 0.63; 0.12)), (5332.98(0.22, 0.78; 0.33)), (5273.97(0.72, 0.28; 0.37)), (5321.28(0.32, 0.68; 0.19))}

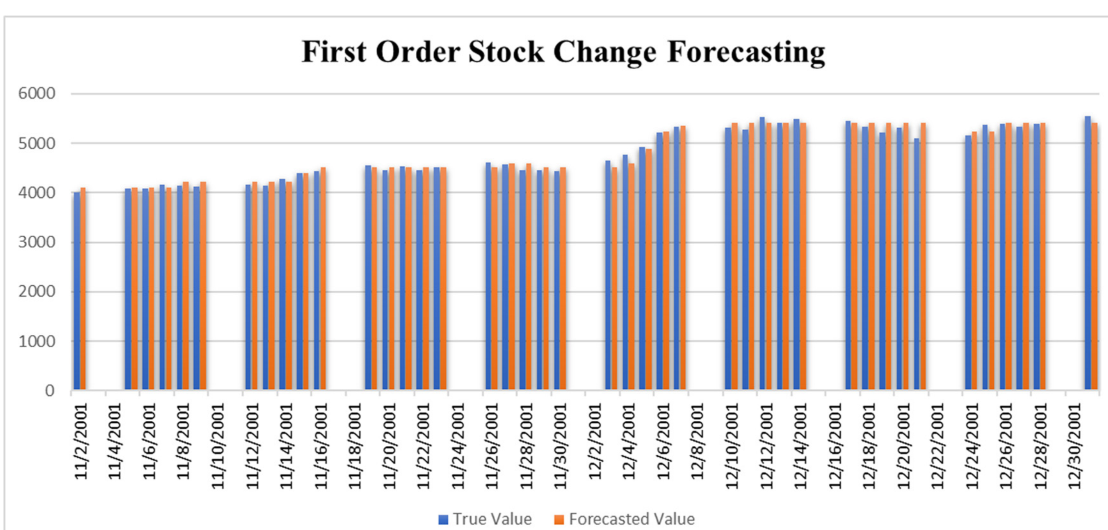

Figure 3: True value and observed value of order I.

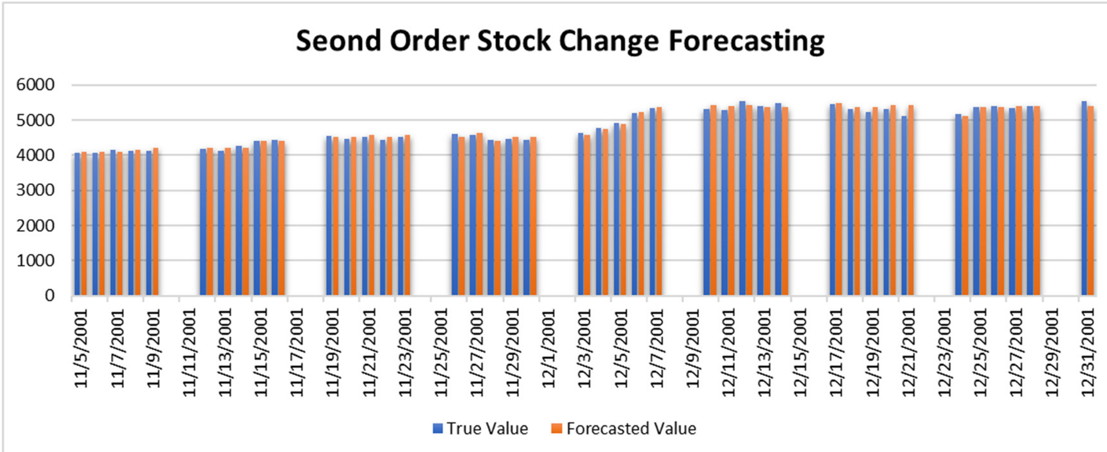

Figure 4: Graph of the actual and proposed value of order II.

$\wp_{12}$ = {(5333.93(0.78, 0.22; 0.13)), (5407.54(0.60, 0.40; 0.13)), (5456.15(0.20, 0.80; 0.70)), (5329.19(0.74, 0.26; 0.07)), (5309.10(0.57, 0.43; 0.17)), (5372.81(0.89, 0.11; 0.28)), (5392.43(0.73, 0.27; 0.05)), (5332.98(0.77, 0.23; 0.12)), (5398.28(0.68, 0.32; 0.02)), (5321.28(0.68, 0.32; 0.02)), (5273.97(0.28, 0.72; 0.58))}
$\wp_{13}$ = {(5407.54(0.40, 0.60; 0.10)), (5486.73(0.94, 0.06; 0.67)), (5456.15(0.80, 0.20; 0.47)), (5372.81(0.11, 0.89; 0.51)), (5392.43(0.27, 0.73; 0.28)), (5398.28(0.32, 0.68; 0.21)), (5551.24(0.41, 0.59; 0.09)), (5539.31(0.50, 0.50; 0.05))}
$\wp_{14}$ = {(5486.73(0.06, 0.94; 0.46)), (5551.24(0.59, 0.41; 0.30)), (5539.31(0.49, 0.51; 0.16))}

Step V: Use (12) in your computation to obtain the C-IFS score value.

The actual value of 4080.51 is between the two values of the circular intuitionistic membership and nonmembership of $\wp_1$ and $\wp_2$. Then, we determined the highest score degree of 4080.51 as follows: The degrees of membership, nonmembership, and the radii of these values are given in step IV. Suppose the value of p is 0.5.

$$\mu_{\wp_1}(4080.51) = 0.66, \quad \nu_{\wp_1}(4080.51) = 0.34, \quad r_{\wp_1}(4080.51) = 0.45.$$

Similarly,

$$\mu_{\wp_2}(4080.51) = 0.34, \quad \nu_{\wp_2}(4080.51) = 0.66, \quad r_{\wp_2}(4080.51) = 0.39.$$

We must calculate the degree of score 4080.51 $\wp_1$ and $\wp_2$ and then select the larger value.

$$\zeta_{\wp_1}(4080.51) = \frac{1}{3}(0.66 - 0.34 + \sqrt{2 \times 0.45}(2 \times 0.5 - 1)) = 0.11,$$

$$\zeta_{\wp_1}(4080.51) = \frac{1}{3}(0.34 - 0.66 + \sqrt{2 \times 0.39}(2 \times 0.5 - 1)) = -0.11.$$

Table 2: Circular intuitionistic logical relationships of order I

$\wp_1 \to \wp_1$	$\wp_1 \to \wp_1$	$\wp_1 \to \wp_1$	$\wp_1 \to \wp_2$	$\wp_2 \to \wp_2$	$\wp_2 \to \wp_2$	$\wp_2 \to \wp_2$
$\wp_2 \to \wp_2$	$\wp_2 \to \wp_3$	$\wp_3 \to \wp_4$	$\wp_4 \to \wp_4$	$\wp_4 \to \wp_5$	$\wp_5 \to \wp_4$	$\wp_4 \to \wp_5$
$\wp_5 \to \wp_4$	$\wp_4 \to \wp_5$	$\wp_5 \to \wp_6$	$\wp_6 \to \wp_6$	$\wp_6 \to \wp_4$	$\wp_4 \to \wp_5$	$\wp_5 \to \wp_4$
$\wp_4 \to \wp_6$	$\wp_6 \to \wp_7$	$\wp_7 \to \wp_8$	$\wp_8 \to \wp_{11}$	$\wp_{11} \to \wp_{12}$	$\wp_{12} \to \wp_{12}$	$\wp_{12} \to \wp_{11}$
$\wp_{11} \to \wp_{13}$	$\wp_{13} \to \wp_{12}$	$\wp_{12} \to \wp_{13}$	$\wp_{13} \to \wp_{13}$	$\wp_{13} \to \wp_{12}$	$\wp_{12} \to \wp_{11}$	$\wp_{11} \to \wp_{12}$
$\wp_{12} \to \wp_{10}$	$\wp_{10} \to \wp_{10}$	$\wp_{10} \to \wp_{12}$	$\wp_{12} \to \wp_{12}$	$\wp_{12} \to \wp_{12}$	$\wp_{12} \to \wp_{12}$	$\wp_{12} \to \wp_4$

Table 3: Circular intuitionistic logical relationship groups of order I

$\wp_1 \rightarrow \wp_1$	$\wp_1 \rightarrow \wp_1$	$\wp_1 \rightarrow \wp_1$	$\wp_1 \rightarrow \wp_2$					
$\wp_2 \rightarrow \wp_2$	$\wp_2 \rightarrow \wp_2$	$\wp_2 \rightarrow \wp_2$	$\wp_2 \rightarrow \wp_2$	$\wp_2 \rightarrow \wp_3$				
$\wp_3 \rightarrow \wp_4$								
$\wp_4 \rightarrow \wp_4$	$\wp_4 \rightarrow \wp_5$	$\wp_4 \rightarrow \wp_5$	$\wp_4 \rightarrow \wp_5$	$\wp_4 \rightarrow \wp_5$	$\wp_4 \rightarrow \wp_6$			
$\wp_5 \rightarrow \wp_4$	$\wp_5 \rightarrow \wp_4$	$\wp_5 \rightarrow \wp_6$	$\wp_5 \rightarrow \wp_4$					
$\wp_6 \rightarrow \wp_6$	$\wp_6 \rightarrow \wp_4$	$\wp_6 \rightarrow \wp_7$						
$\wp_7 \rightarrow \wp_8$								
$\wp_8 \rightarrow \wp_{11}$								
$\wp_{10} \rightarrow \wp_{10}$	$\wp_{10} \rightarrow \wp_{12}$							
$\wp_{11} \rightarrow \wp_{12}$	$\wp_{11} \rightarrow \wp_{13}$	$\wp_{11} \rightarrow \wp_{12}$						
$\wp_{12} \rightarrow \wp_{12}$	$\wp_{12} \rightarrow \wp_{11}$	$\wp_{12} \rightarrow \wp_{13}$	$\wp_{12} \rightarrow \wp_{11}$	$\wp_{12} \rightarrow \wp_{12}$	$\wp_{12} \rightarrow \wp_{12}$	$\wp_{12} \rightarrow \wp_{12}$	$\wp_{12} \rightarrow \wp_{12}$	$\wp_{12} \rightarrow \wp_{14}$
$\wp_{13} \rightarrow \wp_{12}$	$\wp_{13} \rightarrow \wp_{13}$	$\wp_{13} \rightarrow \wp_{12}$						

So the degree of score 4080.51 in $\wp_1$ is greater than $\wp_2$, so the circular intuitionistic value of 4080.51 is $\wp_1$. The remaining computed data are as follows:

Step VI: Circular intuitionistic logical relationships are in presented Table 2.

Step VII: We developed a collection of C-IFLRGs based on C-IFLRs, which are presented in Table 3.

B. Measure the forecasted value of circular intuitionistic fuzzy sets

The computed forecasted values are shown in Table 4. We are unable to predict what the anticipated value for November 1, 2001, will be since we do not have access to the beginning value for that day. The estimated value for the next day was computed similarly.

Figure 3 presents a graph that illustrates the true value as well as the observed value that make up the stock change index.

Table 4: Forecasted value of stock change index of order I

Date	True value	Forecasted value	Years	True value	Forecasted value
01-11-2001	3929.69	—	03-12-2001	4646.61	4,520
02-11-2001	3998.48	4,100	04-12-2001	4766.43	4,600
05-11-2001	4080.51	4,100	05-12-2001	4924.56	4,880
06-11-2001	4082.92	4,100	06-12-2001	5208.86	5,240
07-11-2001	4158.15	4,100	07-12-2001	5333.93	5,360
08-11-2001	4135.03	4,220	10-12-2001	5321.28	5,420
09-11-2001	4123.78	4,220	11-12-2001	5273.97	5,420
12-11-2001	4172.63	4,220	12-12-2001	5539.31	5,420
13-11-2001	4136.54	4,220	13-12-2001	5407.54	5,420
14-11-2001	4277.70	4,220	14-12-2001	5486.73	5,420
15-11-2001	4403.59	4,400	17-12-2001	5456.15	5,420
16-11-2001	4446.62	4,520	18-12-2001	5329.19	5,420
19-11-2001	4548.63	4,520	19-12-2001	5221.96	5,420
20-11-2001	4455.80	4,520	20-12-2001	5309.10	5,420
21-11-2001	4533.37	4,520	21-12-2001	5109.24	5,420
22-11-2001	4450.02	4,520	24-12-2001	5164.73	5,240
23-11-2001	4519.08	4,520	25-12-2001	5372.81	5,240
26-11-2001	4608.32	4,520	26-12-2001	5392.43	5,420
27-11-2001	4580.33	4,600	27-12-2001	5332.98	5,420
28-11-2001	4447.58	4,600	28-12-2001	5398.28	5,420
29-11-2001	4465.83	4,520	31-12-2001	5551.24	5,420
30-11-2001	4441.12	4,520			

Table 5: Circular intuitionistic logical relationship of order II

$\wp_1, \wp_1 \to \wp_1$	$\wp_1, \wp_1 \to \wp_1$	$\wp_1, \wp_1 \to \wp_2$	$\wp_1, \wp_2 \to \wp_2$	$\wp_2, \wp_2 \to \wp_2$	$\wp_2, \wp_2 \to \wp_2$
$\wp_2, \wp_2 \to \wp_2$	$\wp_2, \wp_2 \to \wp_3$	$\wp_2, \wp_3 \to \wp_4$	$\wp_3, \wp_4 \to \wp_4$	$\wp_4, \wp_4 \to \wp_5$	$\wp_4, \wp_5 \to \wp_4$
$\wp_5, \wp_4 \to \wp_5$	$\wp_4, \wp_5 \to \wp_4$	$\wp_5, \wp_4 \to \wp_5$	$\wp_4, \wp_5 \to \wp_6$	$\wp_5, \wp_6 \to \wp_6$	$\wp_6, \wp_6 \to \wp_4$
$\wp_6, \wp_4 \to \wp_5$	$\wp_4, \wp_5 \to \wp_4$	$\wp_5, \wp_4 \to \wp_6$	$\wp_4, \wp_6 \to \wp_7$	$\wp_6, \wp_7 \to \wp_8$	$\wp_7, \wp_8 \to \wp_{11}$
$\wp_8, \wp_{11} \to \wp_{12}$	$\wp_{11}, \wp_{12} \to \wp_{12}$	$\wp_{12}, \wp_{12} \to \wp_{11}$	$\wp_{12}, \wp_{11} \to \wp_{13}$	$\wp_{11}, \wp_{13} \to \wp_{12}$	$\wp_{13}, \wp_{12} \to \wp_{13}$
$\wp_{12}, \wp_{13} \to \wp_{13}$	$\wp_{13}, \wp_{13} \to \wp_{12}$	$\wp_{13}, \wp_{12} \to \wp_{11}$	$\wp_{12}, \wp_{11} \to \wp_{12}$	$\wp_{11}, \wp_{12} \to \wp_{10}$	$\wp_{12}, \wp_{10} \to \wp_{10}$
$\wp_{10}, \wp_{10} \to \wp_{12}$	$\wp_{10}, \wp_{12} \to \wp_{12}$	$\wp_{12}, \wp_{12} \to \wp_{12}$	$\wp_{12}, \wp_{12} \to \wp_{12}$	$\wp_{12}, \wp_{12} \to \wp_{14}$	

4.2 Circular intuitionistic logical relationship of order II

Now we will compute the circular intuitionistic logical relationship and its groups according to the provided approach to order II stock change index.

The circular intuitionistic logic relations of order II are in Table 5. Table 6 displays the circular intuitionistic logic relation groups that we created based on C-ILRs. These C-ILRs are of order II.

B. Measure the forecasted value of circular intuitionistic fuzzy sets.

Table 7 presents the results of the forecasting calculations that were performed with the help of the defuzzified criteria described in Section B. Figure 4 is a graph that depicts the true value, as well as the observed value, of the stock change index for order II.

C. Measurement of error by using RMSE and AFE

The formulas for determining errors are provided in Table 8 and are used to calculate RMSE and AFE, respectively. Comparing our results with other forecasting methods using RMSE and AFE can give an indication of the accuracy of your forecast. If the RMSE and AFE values are lower than those of other forecasting methods, it means that the proposed method is more accurate because of the lower error.

5 Discussion

Table 8 shows the difference in predicted performance between first- and second-order C-IFTSs. Compared to first-order C-IFTSs forecasting, second-order C-IFTSs forecasting uses error calculation formulas. When we calculate third-order C-IFTS forecasting, the error of the third order is less than the error of the second. Use the same formula to obtain the nth order. Higher-orders tend to have fewer errors. It is important to note that the

Table 6: Circular intuitionistic logical relationship groups of order II

$\wp_1, \wp_1 \to \wp_1$	$\wp_1, \wp_1 \to \wp_1$	$\wp_1, \wp_1 \to \wp_2$		$\wp_1, \wp_2 \to \wp_2$		
$\wp_2, \wp_2 \to \wp_2$	$\wp_2, \wp_2 \to \wp_2$	$\wp_2, \wp_2 \to \wp_2$	$\wp_2, \wp_2 \to \wp_3$	$\wp_2, \wp_3 \to \wp_4$		
$\wp_3, \wp_4 \to \wp_4$				$\wp_4, \wp_4 \to \wp_5$		
$\wp_4, \wp_5 \to \wp_4$	$\wp_4, \wp_5 \to \wp_4$	$\wp_4, \wp_5 \to \wp_6$	$\wp_4, \wp_5 \to \wp_4$	$\wp_5, \wp_4 \to \wp_5$	$\wp_5, \wp_4 \to \wp_5$	$\wp_5, \wp_4 \to \wp_6$
$\wp_5, \wp_6 \to \wp_6$				$\wp_6, \wp_6 \to \wp_4$		
$\wp_6, \wp_4 \to \wp_5$				$\wp_4, \wp_6 \to \wp_7$		
$\wp_6, \wp_7 \to \wp_8$				$\wp_7, \wp_8 \to \wp_{11}$		
$\wp_8, \wp_{11} \to \wp_{12}$				$\wp_{11}, \wp_{12} \to \wp_{12}$	$\wp_{11}, \wp_{12} \to \wp_{10}$	
$\wp_{12}, \wp_{12} \to \wp_{11}$	$\wp_{12}, \wp_{12} \to \wp_{12}$	$\wp_{12}, \wp_{12} \to \wp_{12}$	$\wp_{12}, \wp_{12} \to \wp_{14}$	$\wp_{12}, \wp_{11} \to \wp_{13}$	$\wp_{12}, \wp_{11} \to \wp_{12}$	
$\wp_{11}, \wp_{13} \to \wp_{12}$				$\wp_{13}, \wp_{12} \to \wp_{13}$	$\wp_{13}, \wp_{12} \to \wp_{11}$	
$\wp_{12}, \wp_{13} \to \wp_{13}$				$\wp_{13}, \wp_{13} \to \wp_{12}$		
$\wp_{12}, \wp_{10} \to \wp_{10}$				$\wp_{10}, \wp_{10} \to \wp_{12}$		
$\wp_{10}, \wp_{12} \to \wp_{12}$						

Table 7: Forecasted value of stock change index of order II

Date	True value	Forecasted value	Years	True value	Forecasted value
01-11-2001	3929.69	—	03-12-2001	4646.61	4,580
02-11-2001	3998.48	—	04-12-2001	4766.43	4,760
05-11-2001	4080.51	4,100	05-12-2001	4924.56	4,880
06-11-2001	4082.92	4,100	06-12-2001	5208.86	5,240
07-11-2001	4158.15	4,100	07-12-2001	5333.93	5,360
08-11-2001	4135.03	4,160	10-12-2001	5321.2	5,240
09-11-2001	4123.78	4,220	11-12-2001	5273.97	5,400
12-11-2001	4172.63	4,220	12-12-2001	5539.31	5,420
13-11-2001	4136.54	4,220	13-12-2001	5407.54	5,360
14-11-2001	4277.70	4,220	14-12-2001	5486.73	5,360
15-11-2001	4403.59	4,400	17-12-2001	5456.15	5,480
16-11-2001	4446.62	4,400	18-12-2001	5329.19	5,360
19-11-2001	4548.63	4,520	19-12-2001	5221.96	5,360
20-11-2001	4455.80	4,520	20-12-2001	5309.10	5,420
21-11-2001	4533.37	4,580	21-12-2001	5109.24	5,420
22-11-2001	4450.02	4,520	24-12-2001	5164.73	5,120
23-11-2001	4519.08	4,580	25-12-2001	5372.81	5,360
26-11-2001	4608.32	4,520	26-12-2001	5392.43	5,360
27-11-2001	4580.33	4,640	27-12-2001	5332.98	5,400
28-11-2001	4447.58	4,400	28-12-2001	5398.28	5,400
29-11-2001	4465.83	4,520	31-12-2001	5551.24	5,400
30-11-2001	4441.12	4,520			

Table 8: RMSE and AFE

Tools	Chen [38]	NASDAQ & Dow Jones [47]	Proposed method order I	Proposed method order II
RMSE	147.84	124.02	98.03	84.61
AFE	Not calculated	Not calculated	1.61	1.34

accuracy of the forecasts depends not only on the choice of forecasting method but also on the quality and completeness of the data.

The value of RMSE clearly shows the validity of my proposed algorithm to deal with this kind of problem in future prediction. As we discussed earlier, errors show the accuracy of your method. Researchers use this technique to invest the amount in stock index problems. A proposed algorithm is used to handle all these types of problems in fuzzy logic, in which a radius is also given for membership and nonmembership. In the past, no one could give any algorithm to solve this type of problem where the radius was also given. This algorithm is helpful for dealing with this kind of problem.

Radius tells us where the value of membership and nonmembership lies. The radius in C-IFS is important because it can impact the overall shape and size of the circular fuzzy set, which in turn can affect its ability to represent complex and uncertain information in a flexible and intuitive way.

6 Conclusion

With the circular radius, C-IFS membership and nonmembership degree totals are all equal to one or fewer, and they are rapidly gaining favor. It is obvious that utilizing intuitionistic fuzzy sets cannot address this issue; as a result, we employ C-IFS instead of IFSs if the total of the membership value and degree of the nonmembership value are equal. The suggested C-IFS approach was less complex and more straightforward since it employed a simple scoring formula. We used our method to verify the number of stock change indexes, and now we can forecast our data using the given criteria. In addition, we applied this approach to higher-order predictions and displayed the error, which indicates that higher-order forecasts have fewer errors and are thus more helpful for calculating future values. By using the advised strategy, we came up with a prediction for the next few years. Researchers may provide further solutions to the forecasting process in the future. One potential direction for future research could be to apply C-IFTS to various time series forecasting problems and compare their performance to existing methods.

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